SOLUTIONS TO

CONCEPTS OF PHYSICS

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Preface

It gives us immense pleasure to present 'Solutions To Concepts Of Physics'. This book contains solutions to all the exercise problems from 'Concepts Of Physics 1 and 2'. The problems have been illustrated in detail with diagrams.

You are advised to solve the problems yourself instead of using this book.

The book is not written by any of our members and is not meant for sale.

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SOLUTIONS TO CONCEPTS
CHAPTER – 1

1. a) Linear momentum : mv $= [MLT^{-1}]$

   b) Frequency : $\frac{1}{T} = [M^0L^0T^{-1}]$

   c) Pressure : $\frac{\text{Force}}{\text{Area}} = \frac{[MLT^{-2}]}{[L^2]} = [ML^{-1}T^{-2}]$

2. a) Angular speed $\omega = \theta/t = [M^0L^0T^{-1}]$

   b) Angular acceleration $\alpha = \frac{\omega}{t} = \frac{M^0L^0T^{-2}}{T} = [M^0L^0T^{-2}]$

   c) Torque $\tau = F\,r = [MLT^{-2}]\,[L] = [ML^2T^{-2}]$

   d) Moment of inertia $= Mr^2 = [M]\,[L^2] = [ML^2T^0]$

3. a) Electric field $E = F/q = \frac{MLT^{-2}}{[IT]} = [MLT^{-3}I^{-1}]$

   b) Magnetic field $B = \frac{F}{qv} = \frac{MLT^{-2}}{[IT][LT^{-1}]} = [MT^{-2}I^{-1}]$

   c) Magnetic permeability $\mu_0 = \frac{B \times 2\pi a}{I} = \frac{MT^{-2}I^{-1}] \times [L]}{[I]} = [MLT^{-2}I^{-2}]$

4. a) Electric dipole moment $P = qI = [IT] \times [L] = [LTI]$

   b) Magnetic dipole moment $M = IA = [I]\,[L^2]\,[L^2I]$

5. $E = h\nu$ where E = energy and ν = frequency.

   $h = \frac{E}{\nu} = \frac{[ML^2T^{-2}]}{[T^{-1}]}[ML^2T^{-1}]$

6. a) Specific heat capacity $= C = \frac{Q}{m\Delta T} = \frac{[ML^2T^{-2}]}{[M][K]} = [L^2T^{-2}K^{-1}]$

   b) Coefficient of linear expansion $= \alpha = \frac{L_1 - L_2}{L_0\Delta T} = \frac{[L]}{[L][R]} = [K^{-1}]$

   c) Gas constant $= R = \frac{PV}{nT} = \frac{[ML^{-1}T^{-2}][L^3]}{[(mol)][K]} = [ML^2T^{-2}K^{-1}(mol)^{-1}]$

7. Taking force, length and time as fundamental quantity

   a) Density $= \frac{m}{V} = \frac{(\text{force/acceleration})}{\text{Volume}} = \frac{[F/LT^{-2}]}{[L^2]} = \frac{F}{L^4T^{-2}} = [FL^{-4}T^2]$

   b) Pressure $= F/A = F/L^2 = [FL^{-2}]$

   c) Momentum = mv (Force / acceleration) × Velocity $= [F / LT^{-2}] \times [LT^{-1}] = [FT]$

   d) Energy $= \frac{1}{2}mv^2 = \frac{\text{Force}}{\text{acceleration}} \times (\text{velocity})^2$

   $= \left[\frac{F}{LT^{-2}}\right] \times [LT^{-1}]^2 = \left[\frac{F}{LT^{-2}}\right] \times [L^2T^{-2}] = [FL]$

8. $g = 10\frac{\text{metre}}{\text{sec}^2} = 36 \times 10^5\ \text{cm/min}^2$

9. The average speed of a snail is 0.02 mile/hr

   Converting to S.I. units, $\frac{0.02 \times 1.6 \times 1000}{3600}$ m/sec [1 mile = 1.6 km = 1600 m] $= 0.0089\ ms^{-1}$

   The average speed of leopard = 70 miles/hr

   In SI units = 70 miles/hour $= \frac{70 \times 1.6 \times 1000}{3600}$ = 31 m/s

10. Height h = 75 cm, Density of mercury = 13600 kg/m^3, g = 9.8 ms^{-2} then
    Pressure = hfg = 10×10^4 N/m^2 (approximately)
    In C.G.S. Units, P = 10×10^5 dyne/cm^2
11. In S.I. unit 100 watt = 100 Joule/sec
    In C.G.S. Unit = 10^9 erg/sec
12. 1 micro century = $10^4 \times 100$ years = $10^{-4} \times 365 \times 24 \times 60$ min
    So, 100 min = 10^5 / 52560 = 1.9 microcentury
13. Surface tension of water = 72 dyne/cm
    In S.I. Unit, 72 dyne/cm = 0.072 N/m
14. K = $kI^a \omega^b$ where k = Kinetic energy of rotating body and k = dimensionless constant
    Dimensions of left side are,
    K = $[ML^2T^{-2}]$
    Dimensions of right side are,
    $I^a = [ML^2]^a$, $\omega^b = [T^{-1}]^b$
    According to principle of homogeneity of dimension,
    $[ML^2T^{-2}] = [ML^2T^{-2}]\ [T^{-1}]^b$
    Equating the dimension of both sides,
    2 = 2a and –2 = –b $\Rightarrow$ a = 1 and b = 2
15. Let energy $E \propto M^aC^b$ where M = Mass, C = speed of light
    $\Rightarrow E = KM^aC^b$ (K = proportionality constant)
    Dimension of left side
    E = $[ML^2T^{-2}]$
    Dimension of right side
    $M^a = [M]^a$, $[C]^b = [LT^{-1}]^b$
    $\therefore [ML^2T^{-2}] = [M]^a[LT^{-1}]^b$
    $\Rightarrow$ a = 1; b = 2
    So, the relation is $E = KMC^2$
16. Dimensional formulae of R = $[ML^2T^{-3}I^{-2}]$
    Dimensional formulae of V = $[ML^2T^3I^{-1}]$
    Dimensional formulae of I = [I]
    $\therefore [ML^2T^3I^{-1}] = [ML^2T^{-3}I^{-2}]\ [I]$
    $\Rightarrow$ V = IR
17. Frequency f = $KL^aF^bM^c$ M = Mass/unit length, L = length, F = tension (force)
    Dimension of f = $[T^{-1}]$
    Dimension of right side,
    $L^a = [L^a]$, $F^b = [MLT^{-2}]^b$, $M^c = [ML^{-1}]^c$
    $\therefore [T^{-1}] = K[L]^a\ [MLT^{-2}]^b\ [ML^{-1}]^c$
    $M^0L^0T^{-1} = KM^{b+c}\ L^{a+b-c}\ T^{-2b}$
    Equating the dimensions of both sides,
    $\therefore$ b + c = 0 …(1)
    –c + a + b = 0 …(2)
    –2b = –1 …(3)
    Solving the equations we get,
    a = –1, b = 1/2 and c = –1/2
    $\therefore$ So, frequency f = $KL^{-1}F^{1/2}M^{-1/2} = \dfrac{K}{L}F^{1/2}M^{-1/2} = \dfrac{K}{L} = \sqrt{\dfrac{F}{M}}$

18. a) $h = \frac{2SCos\theta}{\rho rg}$

LHS = [L]

Surface tension = S = F/l = $\frac{MLT^{-2}}{L} = [MT^{-2}]$

Density = ρ = M/V = $[ML^{-3}T^0]$

Radius = r = [L], g = $[LT^{-2}]$

RHS = $\frac{2S\cos\theta}{\rho rg} = \frac{[MT^{-2}]}{[ML^{-3}T^0][L][LT^{-2}]} = [M^0L^1T^0] = [L]$

LHS = RHS

So, the relation is correct

b) $v = \sqrt{\frac{p}{\rho}}$ where v = velocity

LHS = Dimension of v = $[LT^{-1}]$

Dimension of p = F/A = $[ML^{-1}T^{-2}]$

Dimension of ρ = m/V = $[ML^{-3}]$

RHS = $\sqrt{\frac{p}{\rho}} = \sqrt{\frac{[ML^{-1}T^{-2}]}{[ML^{-3}]}} = [L^2T^{-2}]^{1/2} = [LT^{-1}]$

So, the relation is correct.

c) V = $(\pi pr^4t) / (8\eta l)$

LHS = Dimension of V = $[L^3]$

Dimension of p = $[ML^{-1}T^{-2}]$, $r^4 = [L^4]$, t = [T]

Coefficient of viscosity = $[ML^{-1}T^{-1}]$

RHS = $\frac{\pi pr^4t}{8\eta l} = \frac{[ML^{-1}T^{-2}][L^4][T]}{[ML^{-1}T^{-1}][L]}$

So, the relation is correct.

d) $v = \frac{1}{2\pi}\sqrt{(mgl/I)}$

LHS = dimension of v = $[T^{-1}]$

RHS = $\sqrt{(mgl/I)} = \sqrt{\frac{[M][LT^{-2}][L]}{[ML^2]}} = [T^{-1}]$

LHS = RHS

So, the relation is correct.

19. Dimension of the left side = $\int\frac{dx}{\sqrt{(a^2-x^2)}} = \int\frac{L}{\sqrt{(L^2-L^2)}} = [L^0]$

Dimension of the right side = $\frac{1}{a}\sin^{-1}\left(\frac{a}{x}\right) = [L^{-1}]$

So, the dimension of $\int\frac{dx}{\sqrt{(a^2-x^2)}} \neq \frac{1}{a}\sin^{-1}\left(\frac{a}{x}\right)$

So, the equation is dimensionally incorrect.

20. Important Dimensions and Units :

Physical quantity	Dimension	SI unit
Force (F)	$[M^1L^1T^{-2}]$	newton
Work (W)	$[M^1L^2T^{-2}]$	joule
Power (P)	$[M^1L^2T^{-3}]$	watt
Gravitational constant (G)	$[M^{-1}L^3T^{-2}]$	N-m^2/kg^2
Angular velocity (ω)	$[T^{-1}]$	radian/s
Angular momentum (L)	$[M^1L^2T^{-1}]$	kg-m^2/s
Moment of inertia (I)	$[M^1L^2]$	kg-m^2
Torque (τ)	$[M^1L^2T^{-2}]$	N-m
Young's modulus (Y)	$[M^1L^{-1}T^{-2}]$	N/m^2
Surface Tension (S)	$[M^1T^{-2}]$	N/m
Coefficient of viscosity (η)	$[M^1L^{-1}T^{-1}]$	N-s/m^2
Pressure (p)	$[M^1L^{-1}T^{-2}]$	N/m^2 (Pascal)
Intensity of wave (I)	$[M^1T^{-3}]$	watt/m^2
Specific heat capacity (c)	$[L^2T^{-2}K^{-1}]$	J/kg-K
Stefan's constant (σ)	$[M^1T^{-3}K^{-4}]$	watt/m^2-k^4
Thermal conductivity (k)	$[M^1L^1T^{-3}K^{-1}]$	watt/m-K
Current density (j)	$[I^1L^{-2}]$	ampere/m^2
Electrical conductivity (σ)	$[I^2T^3M^{-1}L^{-3}]$	Ω^{-1} m^{-1}
Electric dipole moment (p)	$[L^1I^1T^1]$	C-m
Electric field (E)	$[M^1L^1I^{-1}T^{-3}]$	V/m
Electrical potential (V)	$[M^1L^2I^{-1}T^{-3}]$	volt
Electric flux (Ψ)	$[M^1T^3I^{-1}L^{-3}]$	volt/m
Capacitance (C)	$[I^2T^4M^{-1}L^{-2}]$	farad (F)
Permittivity (ε)	$[I^2T^4M^{-1}L^{-3}]$	C^2/N-m^2
Permeability (μ)	$[M^1L^1I^{-2}T^{-3}]$	Newton/A^2
Magnetic dipole moment (M)	$[I^1L^2]$	N-m/T
Magnetic flux (ϕ)	$[M^1L^2I^{-1}T^{-2}]$	Weber (Wb)
Magnetic field (B)	$[M^1I^{-1}T^{-2}]$	tesla
Inductance (L)	$[M^1L^2I^{-2}T^{-2}]$	henry
Resistance (R)	$[M^1L^2I^{-2}T^{-3}]$	ohm (Ω)

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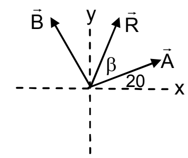
SOLUTIONS TO CONCEPTS
CHAPTER – 2

1. As shown in the figure,



The angle between $\vec{A}$ and $\vec{B}$ = 110° – 20° = 90°

$|\vec{A}|$ = 3 and $|\vec{B}|$ = 4m

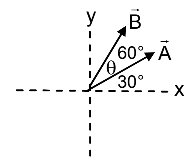
Resultant R = $\sqrt{A^2 + B^2 + 2AB\cos\theta}$ = 5 m

Let β be the angle between $\vec{R}$ and $\vec{A}$

$\beta = \tan^{-1}\left(\frac{4\sin 90°}{3 + 4\cos 90°}\right) = \tan^{-1}(4/3) = 53°$

∴ Resultant vector makes angle (53° + 20°) = 73° with x-axis.

2. Angle between $\vec{A}$ and $\vec{B}$ is θ = 60° – 30° =30°



$|\vec{A}|$ and $|\vec{B}|$ = 10 unit

R = $\sqrt{10^2 + 10^2 + 2.10.10.\cos 30°}$ = 19.3

β be the angle between $\vec{R}$ and $\vec{A}$

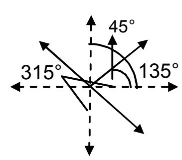
$\beta = \tan^{-1}\left(\frac{10\sin 30°}{10 + 10\cos 30°}\right) = \tan^{-1}\left(\frac{1}{2+\sqrt{3}}\right) = \tan^{-1}(0.26795) = 15°$

∴ Resultant makes 15° + 30° = 45° angle with x-axis.

3. x component of $\vec{A}$ = 100 cos 45° = $100/\sqrt{2}$ unit

x component of $\vec{B}$ = 100 cos 135° = $100/\sqrt{2}$

x component of $\vec{C}$ = 100 cos 315° = $100/\sqrt{2}$



Resultant x component = $100/\sqrt{2}$ – $100/\sqrt{2}$ + $100/\sqrt{2}$ = $100/\sqrt{2}$

y component of $\vec{A}$ = 100 sin 45° = $100/\sqrt{2}$ unit

y component of $\vec{B}$ = 100 sin 135° = $100/\sqrt{2}$

y component of $\vec{C}$ = 100 sin 315° = $-100/\sqrt{2}$

Resultant y component = $100/\sqrt{2}$ + $100/\sqrt{2}$ – $100/\sqrt{2}$ = $100/\sqrt{2}$

Resultant = 100

Tan α = $\frac{\text{y component}}{\text{x component}}$ = 1

⇒ α = $\tan^{-1}(1)$ = 45°

The resultant is 100 unit at 45° with x-axis.

4. $\vec{a} = 4\vec{i} + 3\vec{j}$, $\vec{b} = 3\vec{i} + 4\vec{j}$

a) $|\vec{a}| = \sqrt{4^2 + 3^2} = 5$

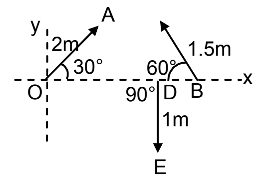
b) $|\vec{b}| = \sqrt{9 + 16} = 5$

c) $|\vec{a} + \vec{b}| = |7\vec{i} + 7\vec{j}| = 7\sqrt{2}$

d) $\vec{a} - \vec{b} = (-3+4)\hat{i} + (-4+3)\hat{j} = \hat{i} - \hat{j}$

$|\vec{a} - \vec{b}| = \sqrt{1^2 + (-1)^2} = \sqrt{2}$.

5. x component of $\overrightarrow{OA}$ = 2cos30° = $\sqrt{3}$



x component of $\overrightarrow{BC}$ = 1.5 cos 120° = –0.75

x component of $\overrightarrow{DE}$ = 1 cos 270° = 0

y component of $\overrightarrow{OA}$ = 2 sin 30° = 1

y component of $\overrightarrow{BC}$ = 1.5 sin 120° = 1.3

y component of $\overrightarrow{DE}$ = 1 sin 270° = –1

R_x = x component of resultant = $\sqrt{3} - 0.75 + 0$ = 0.98 m

R_y = resultant y component = 1 + 1.3 – 1 = 1.3 m

So, R = Resultant = 1.6 m

If it makes and angle α with positive x-axis

Tan α = $\dfrac{\text{y component}}{\text{x component}}$ = 1.32

$\Rightarrow \alpha = \tan^{-1} 1.32$

6. $|\vec{a}|$ = 3m $|\vec{b}|$ = 4

a) If R = 1 unit $\Rightarrow \sqrt{3^2 + 4^2 + 2.3.4.\cos\theta}$ = 1

θ = 180°

b) $\sqrt{3^2 + 4^2 + 2.3.4.\cos\theta}$ = 5

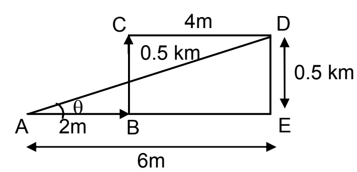
θ = 90°

c) $\sqrt{3^2 + 4^2 + 2.3.4.\cos\theta}$ = 7

θ = 0°

Angle between them is 0°.

7. $\overrightarrow{AD} = 2\hat{i} + 0.5\hat{J} + 4\hat{K} = 6\hat{i} + 0.5\hat{j}$



AD = $\sqrt{AE^2 + DE^2}$ = 6.02 KM

Tan θ = DE / AE = 1/12

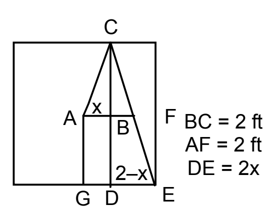
$\theta = \tan^{-1}$ (1/12)

The displacement of the car is 6.02 km along the distance $\tan^{-1}$ (1/12) with positive x-axis.

8. In ΔABC, tanθ = x/2 and in ΔDCE, tanθ = (2 – x)/4 tan θ = (x/2) = (2 – x)/4 = 4x

$\Rightarrow$ 4 – 2x = 4x

$\Rightarrow$ 6x = 4 $\Rightarrow$ x = 2/3 ft



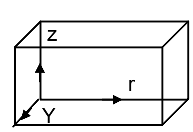
a) In ΔABC, AC = $\sqrt{AB^2 + BC^2}$ = $\dfrac{2}{3}\sqrt{10}$ ft

b) In ΔCDE, DE = 1 – (2/3) = 4/3 ft

CD = 4 ft. So, CE = $\sqrt{CD^2 + DE^2}$ = $\dfrac{4}{3}\sqrt{10}$ ft

c) In ΔAGE, AE = $\sqrt{AG^2 + GE^2}$ = $2\sqrt{2}$ ft.

9. Here the displacement vector $\vec{r} = 7\hat{i} + 4\hat{j} + 3\hat{k}$



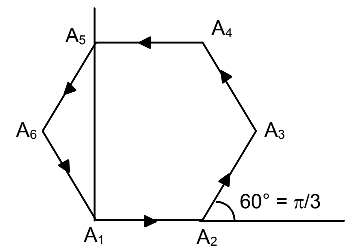
a) magnitude of displacement = $\sqrt{74}$ ft

b) the components of the displacement vector are 7 ft, 4 ft and 3 ft.

10. $\vec{a}$ is a vector of magnitude 4.5 unit due north.
 a) $3|\vec{a}| = 3 \times 4.5 = 13.5$
 $3\vec{a}$ is along north having magnitude 13.5 units.
 b) $-4|\vec{a}| = -4 \times 1.5 = -6$ unit
 $-4\vec{a}$ is a vector of magnitude 6 unit due south.

11. $|\vec{a}| = 2$ m, $|\vec{b}| = 3$ m
 angle between them $\theta = 60°$
 a) $\vec{a}\cdot\vec{b} = |\vec{a}|\cdot|\vec{b}|\cos 60° = 2 \times 3 \times 1/2 = 3\ m^2$
 b) $|\vec{a}\times\vec{b}| = |\vec{a}|\cdot|\vec{b}|\sin 60° = 2 \times 3 \times \sqrt{3/2} = 3\sqrt{3}\ m^2$.

12. We know that according to polygon law of vector addition, the resultant of these six vectors is zero.



 Here A = B = C = D = E = F (magnitude)
 So, Rx = A cosθ + A cos π/3 + A cos 2π/3 + A cos 3π/3 + A cos 4π/4 + A cos 5π/5 = 0
 [As resultant is zero. X component of resultant $R_x = 0$]
 = cos θ + cos π/3 + cos 2π/3 + cos 3π/3 + cos 4π/3 + cos 5π/3 = 0
 Note : Similarly it can be proved that,
 sin θ + sin π/3 + sin 2π/3 + sin 3π/3 + sin 4π/3 + sin 5π/3 = 0

13. $\vec{a} = 2\vec{i} + 3\vec{j} + 4\vec{k};\ \vec{b} = 3\vec{i} + 4\vec{j} + 5\vec{k}$

 $\vec{a}\cdot\vec{b} = ab\cos\theta \Rightarrow \theta = \cos^{-1}\dfrac{\vec{a}\cdot\vec{b}}{ab}$

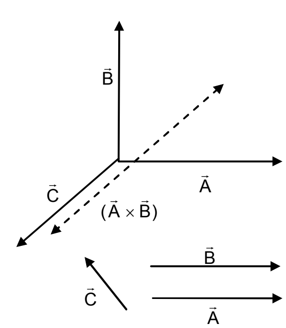
 $\Rightarrow \cos^{-1}\dfrac{2\times3+3\times4+4\times5}{\sqrt{2^2+3^2+4^2}\sqrt{3^2+4^2+5^2}} = \cos^{-1}\left(\dfrac{38}{\sqrt{1450}}\right)$

14. $\vec{A}\cdot(\vec{A}\times\vec{B}) = 0$ (claim)
 As, $\vec{A}\times\vec{B} = AB\sin\theta\hat{n}$
 AB sin θ $\hat{n}$ is a vector which is perpendicular to the plane containing $\vec{A}$ and $\vec{B}$, this implies that it is also perpendicular to $\vec{A}$. As dot product of two perpendicular vector is zero.
 Thus $\vec{A}\cdot(\vec{A}\times\vec{B}) = 0$.

15. $\vec{A} = 2\hat{i} + 3\hat{j} + 4\hat{k}$, $\vec{B} = 4\hat{i} + 3\hat{j} + 2\hat{k}$

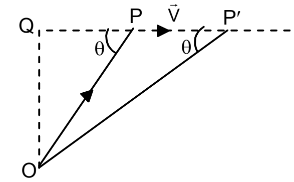
 $\vec{A}\times\vec{B} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 3 & 4\\ 4 & 3 & 2\end{vmatrix} \Rightarrow \hat{i}(6-12) - \hat{j}(4-16) + \hat{k}(6-12) = -6\hat{i} + 12\hat{j} - 6\hat{k}$.

16. Given that $\vec{A}$, $\vec{B}$ and $\vec{C}$ are mutually perpendicular



 $\vec{A} \times \vec{B}$ is a vector which direction is perpendicular to the plane containing $\vec{A}$ and $\vec{B}$.
 Also $\vec{C}$ is perpendicular to $\vec{A}$ and $\vec{B}$
 ∴ Angle between $\vec{C}$ and $\vec{A} \times \vec{B}$ is 0° or 180° (fig.1)
 So, $\vec{C} \times (\vec{A} \times \vec{B}) = 0$
 The converse is not true.
 For example, if two of the vector are parallel, (fig.2), then also
 $\vec{C} \times (\vec{A} \times \vec{B}) = 0$
 So, they need not be mutually perpendicular.

17. The particle moves on the straight line PP' at speed v.



From the figure,

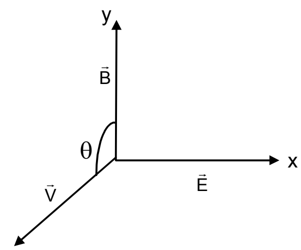
$\overrightarrow{OP} \times v = (OP)v\sin\theta\hat{n}$ = v(OP) sin θ $\hat{n}$ = v(OQ) $\hat{n}$

It can be seen from the figure, OQ = OP sin θ = OP' sin θ'

So, whatever may be the position of the particle, the magnitude and direction of $\overrightarrow{OP} \times \vec{v}$ remain constant.

$\therefore$ $\overrightarrow{OP} \times \vec{v}$ is independent of the position P.

18. Give $\vec{F} = q\vec{E} + q(\vec{v} \times \vec{B}) = 0$



$\Rightarrow \vec{E} = -(\vec{v} \times \vec{B})$

So, the direction of $\vec{v} \times \vec{B}$ should be opposite to the direction of $\vec{E}$. Hence, $\vec{v}$ should be in the positive yz-plane.

Again, E = vB sin θ ⇒ v = $\dfrac{E}{B\sin\theta}$

For v to be minimum, θ = 90° and so v_{min} = F/B

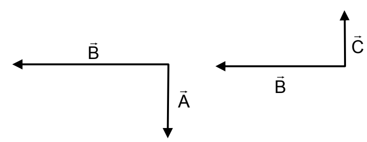
So, the particle must be projected at a minimum speed of E/B along +ve z-axis (θ = 90°) as shown in the figure, so that the force is zero.

19. For example, as shown in the figure,

$\vec{A} \perp \vec{B}$ $\vec{B}$ along west

$\vec{B} \perp \vec{C}$ $\vec{A}$ along south

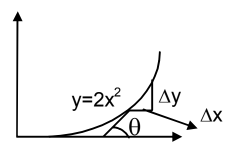
$\vec{C}$ along north



$\vec{A} \cdot \vec{B}$ = 0 $\therefore$ $\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{C}$

$\vec{B} \cdot \vec{C}$ = 0 But $\vec{B} \neq \vec{C}$

20. The graph y = $2x^2$ should be drawn by the student on a graph paper for exact results.



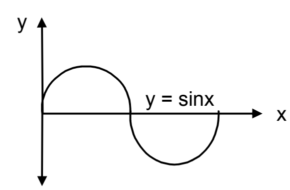
To find slope at any point, draw a tangent at the point and extend the line to meet x-axis. Then find tan θ as shown in the figure.

It can be checked that,

Slope = tan θ = $\dfrac{dy}{dx} = \dfrac{d}{dx}(2x^2)$ = 4x

Where x = the x-coordinate of the point where the slope is to be measured.

21. y = sinx



So, y + Δy = sin (x + Δx)

Δy = sin (x + Δx) – sin x

= $\left(\dfrac{\pi}{3} + \dfrac{\pi}{100}\right) - \sin\dfrac{\pi}{3}$ = 0.0157.

22. Given that, i = $i_0 e^{-t/RC}$

$\therefore$ Rate of change of current = $\dfrac{di}{dt} = \dfrac{d}{dt} i_0 e^{-i/RC} = i_0 \dfrac{d}{dt} e^{-t/RC}$ = $\dfrac{-i_0}{RC} \times e^{-t/RC}$

When a) t = 0, $\dfrac{di}{dt} = \dfrac{-i}{RC}$

b) when t = RC, $\dfrac{di}{dt} = \dfrac{-i}{RCe}$

c) when t = 10 RC, $\dfrac{di}{dt} = \dfrac{-i_0}{RCe^{10}}$

23. Equation i = $i_0 e^{-t/RC}$

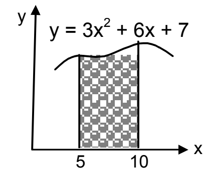
i_0 = 2A, R = 6 × 10^{-5} Ω, C = 0.0500 × 10^{-6} F = 5 × 10^{-7} F

a) i = $2 \times e^{\left(\frac{-0.3}{6\times0^3\times5\times10^{-7}}\right)} = 2 \times e^{\left(\frac{-0.3}{0.3}\right)} = \frac{2}{e}\text{amp}$.

b) $\frac{di}{dt} = \frac{-i_0}{RC} e^{-t/RC}$ when t = 0.3 sec ⇒ $\frac{di}{dt} = -\frac{2}{0.30} e^{(-0.3/0.3)} = \frac{-20}{3e}\text{Amp/sec}$

c) At t = 0.31 sec, i = $2e^{(-0.3/0.3)} = \frac{5.8}{3e}\text{Amp}$.

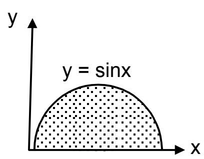
24. y = $3x^2 + 6x + 7$



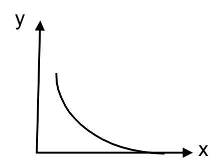
∴ Area bounded by the curve, x axis with coordinates with x = 5 and x = 10 is given by,

Area = $\int_0^y dy = \int_5^{10} (3x^2 + 6x + 7)dx = 3\left.\frac{x^3}{3}\right]_5^{10} + 5\left.\frac{x^2}{3}\right]_5^{10} + 7x]_5^{10}$ = 1135 sq.units.

25. Area = $\int_0^y dy = \int_0^{\pi} \sin x dx = -[\cos x]_0^{\pi}$ = 2



26. The given function is y = e^{-x}



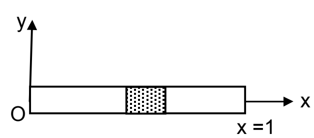
When x = 0, y = e^{-0} = 1

x increases, y value deceases and only at x = ∞, y = 0.

So, the required area can be found out by integrating the function from 0 to ∞.

So, Area = $\int_0^{\infty} e^{-x}dx = -[e^{-x}]_0^{\infty} = 1$.

27. $\rho = \frac{\text{mass}}{\text{length}} = a + bx$



a) S.I. unit of 'a' = kg/m and SI unit of 'b' = kg/m^2 (from principle of homogeneity of dimensions)

b) Let us consider a small element of length 'dx' at a distance x from the origin as shown in the figure.

∴ dm = mass of the element = ρ dx = (a + bx) dx

So, mass of the rod = m = $\int dm = \int_0^L (a + bx)dx = \left[ax + \frac{bx^2}{2}\right]_0^L = aL + \frac{bL^2}{2}$

28. $\frac{dp}{dt}$ = (10 N) + (2 N/S)t

momentum is zero at t = 0

∴ momentum at t = 10 sec will be

dp = [(10 N) + 2Ns t]dt

$\int_0^p dp = \int_0^{10} 10dt + \int_0^{10} (2tdt) = 10t]_0^{10} + 2\left.\frac{t^2}{2}\right]_0^{10}$ = 200 kg m/s.

29. The change in a function of y and the independent variable x are related as $\frac{dy}{dx} = x^2$.

$\Rightarrow$ dy = x^2 dx

Taking integration of both sides,

$\int dy = \int x^2 dx \Rightarrow y = \frac{x^3}{3} + c$

$\therefore$ y as a function of x is represented by $y = \frac{x^3}{3} + c$.

30. The number significant digits

a) 1001 No.of significant digits = 4

b) 100.1 No.of significant digits = 4

c) 100.10 No.of significant digits = 5

d) 0.001001 No.of significant digits = 4

31. The metre scale is graduated at every millimeter.

1 m = 100 mm

The minimum no.of significant digit may be 1 (e.g. for measurements like 5 mm, 7 mm etc) and the maximum no.of significant digits may be 4 (e.g.1000 mm)

So, the no.of significant digits may be 1, 2, 3 or 4.

32. a) In the value 3472, after the digit 4, 7 is present. Its value is greater than 5.

So, the next two digits are neglected and the value of 4 is increased by 1.

$\therefore$ value becomes 3500

b) value = 84

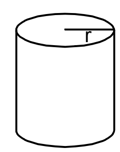
c) 2.6

d) value is 28.

33. Given that, for the cylinder

Length = l = 4.54 cm, radius = r = 1.75 cm

Volume = $\pi r^2 l = \pi \times (4.54) \times (1.75)^2$



Since, the minimum no.of significant digits on a particular term is 3, the result should have 3 significant digits and others rounded off.

So, volume V = $\pi r^2 l = (3.14) \times (1.75) \times (1.75) \times (4.54) = 43.6577\ cm^3$

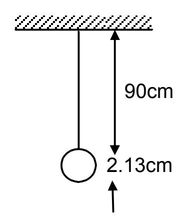
Since, it is to be rounded off to 3 significant digits, V = 43.7 cm^3.

34. We know that,

Average thickness = $\frac{2.17 + 2.17 + 2.18}{3}$ = 2.1733 mm

Rounding off to 3 significant digits, average thickness = 2.17 mm.

35. As shown in the figure,



Actual effective length = (90.0 + 2.13) cm

But, in the measurement 90.0 cm, the no. of significant digits is only 2.

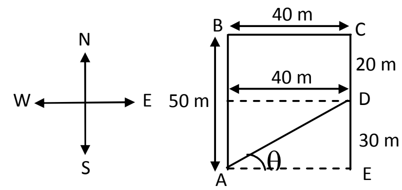
So, the addition must be done by considering only 2 significant digits of each measurement.

So, effective length = 90.0 + 2.1 = 92.1 cm.

* * * *

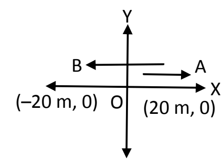
SOLUTIONS TO CONCEPTS
CHAPTER – 3

1. a) Distance travelled = 50 + 40 + 20 = 110 m
   b) AF = AB – BF = AB – DC = 50 – 20 = 30 M
   His displacement is AD
   AD = $\sqrt{AF^2 - DF^2} = \sqrt{30^2 + 40^2} = 50m$
   In ΔAED tan θ = DE/AE = 30/40 = 3/4
   $\Rightarrow \theta = \tan^{-1}(3/4)$
   His displacement from his house to the field is 50 m,
   $\tan^{-1}$ (3/4) north to east.

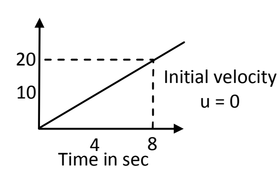


A → Initial point
(starting point)

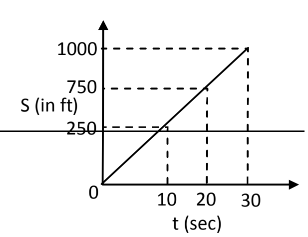
2. O → Starting point origin.
   i) Distance travelled = 20 + 20 + 20 = 60 m
   ii) Displacement is only OB = 20 m in the negative direction.
   Displacement → Distance between final and initial position.



3. a) V_{ave} of plane (Distance/Time) = 260/0.5 = 520 km/hr.
   b) V_{ave} of bus = 320/8 = 40 km/hr.
   c) plane goes in straight path
   velocity = $\vec{V}_{ave}$ = 260/0.5 = 520 km/hr.
   d) Straight path distance between plane to Ranchi is equal to the displacement of bus.
   ∴ Velocity = $\vec{V}_{ave}$ = 260/8 = 32.5 km/hr.
4. a) Total distance covered 12416 – 12352 = 64 km in 2 hours.
   Speed = 64/2 = 32 km/h
   b) As he returns to his house, the displacement is zero.
   Velocity = (displacement/time) = 0 (zero).
5. Initial velocity u = 0 (∴ starts from rest)
   Final velocity v = 18 km/hr = 5 sec
   (i.e. max velocity)
   Time interval t = 2 sec.
   ∴ Acceleration = $a_{ave} = \frac{v-u}{t} = \frac{5}{2}$ = 2.5 m/s^2.
6. In the interval 8 sec the velocity changes from 0 to 20 m/s.
   Average acceleration = 20/8 = 2.5 m/s^2 $\left(\frac{\text{change in velocity}}{\text{time}}\right)$
   Distance travelled S = ut + 1/2 at^2
   $\Rightarrow$ 0 + 1/2(2.5)8^2 = 80 m.



7. In 1st 10 sec S_1 = ut + 1/2 at^2 $\Rightarrow$ 0 + (1/2 × 5 × 10^2) = 250 ft.
   At 10 sec v = u + at = 0 + 5 × 10 = 50 ft/sec.
   ∴ From 10 to 20 sec (Δt = 20 – 10 = 10 sec) it moves with uniform velocity 50 ft/sec,



Distance $S_2 = 50 \times 10 = 500$ ft

Between 20 sec to 30 sec acceleration is constant i.e. -5 ft/s^2. At 20 sec velocity is 50 ft/sec.

$t = 30 - 20 = 10$ s

$S_3 = ut + 1/2\, at^2$

$= 50 \times 10 + (1/2)(-5)(10)^2 = 250$ m

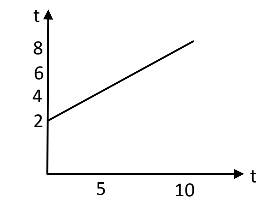
Total distance travelled is 30 sec = $S_1 + S_2 + S_3 = 250 + 500 + 250 = 1000$ ft.

8. a) Initial velocity u = 2 m/s.

final velocity v = 8 m/s

time = 10 sec,

acceleration = $\frac{v-u}{ta} = \frac{8-2}{10} = 0.6$ m/s^2



b) $v^2 - u^2 = 2aS$

$\Rightarrow$ Distance S = $\frac{v^2-u^2}{2a} = \frac{8^2-2^2}{2\times 0.6} = 50$ m.

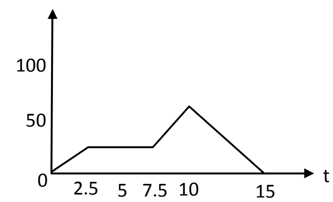
c) Displacement is same as distance travelled.

Displacement = 50 m.

9. a) Displacement in 0 to 10 sec is 1000 m.

time = 10 sec.

$V_{ave} = s/t = 100/10 = 10$ m/s.



(slope of the graph at t = 2 sec)

b) At 2 sec it is moving with uniform velocity 50/2.5 = 20 m/s.

at 2 sec. $V_{inst} = 20$ m/s.

At 5 sec it is at rest.

V_{inst} = zero.

At 8 sec it is moving with uniform velocity 20 m/s

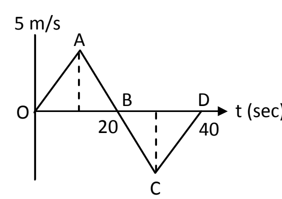
$V_{inst} = 20$ m/s

At 12 sec velocity is negative as it move towards initial position. $V_{inst} = -20$ m/s.

10. Distance in first 40 sec is, Δ OAB + ΔBCD

$= \frac{1}{2} \times 5 \times 20 + \frac{1}{2} \times 5 \times 20 = 100$ m.

Average velocity is 0 as the displacement is zero.



11. Consider the point B, at t = 12 sec

At t = 0 ; s = 20 m

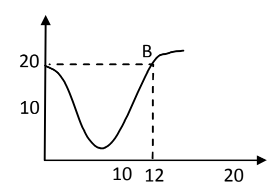
and t = 12 sec s = 20 m

So for time interval 0 to 12 sec

Change in displacement is zero.

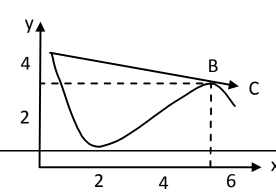
So, average velocity = displacement/ time = 0

$\therefore$ The time is 12 sec.



12. At position B instantaneous velocity has direction along $\overrightarrow{BC}$. For average velocity between A and B.

V_{ave} = displacement / time = $(\overrightarrow{AB}/t)$ t = time



We can see that $\overrightarrow{AB}$ is along $\overrightarrow{BC}$ i.e. they are in same direction.

The point is B (5m, 3m).

13. u = 4 m/s, a = 1.2 m/s^2, t = 5 sec

Distance = s = $ut + \frac{1}{2}at^2$

= 4(5) + 1/2 (1.2)5^2 = 35 m.

14. Initial velocity u = 43.2 km/hr = 12 m/s

u = 12 m/s, v = 0

a = –6 m/s^2 (deceleration)

Distance S = $\frac{v^2 - u^2}{2(-6)}$ = 12 m

15. Initial velocity u = 0
    Acceleration a = 2 m/s^2. Let final velocity be v (before applying breaks)
    t = 30 sec
    v = u + at $\Rightarrow$ 0 + 2 × 30 = 60 m/s
    a) $S_1 = ut + \frac{1}{2}at^2 = 900$ m
       when breaks are applied u′ = 60 m/s
       v′ = 0, t = 60 sec (1 min)
       Declaration a′ = (v – u)/t = = (0 – 60)/60 = –1 m/s^2.
       $S_2 = \frac{v'^2 - u'^2}{2a'}$ = 1800 m
       Total S = $S_1 + S_2$ = 1800 + 900 = 2700 m = 2.7 km.
    b) The maximum speed attained by train v = 60 m/s
    c) Half the maximum speed = 60/2= 30 m/s
       Distance S = $\frac{v^2 - u^2}{2a} = \frac{30^2 - 0^2}{2 \times 2}$ = 225 m from starting point
       When it accelerates the distance travelled is 900 m. Then again declarates and attain 30 m/s.
       $\therefore$ u = 60 m/s, v = 30 m/s, a = –1 m/s^2
       Distance = $\frac{v^2 - u^2}{2a} = \frac{30^2 - 60^2}{2(-1)}$ = 1350 m
       Position is 900 + 1350 = 2250 = 2.25 km from starting point.
16. u = 16 m/s (initial), v = 0, s = 0.4 m.
    Deceleration a = $\frac{v^2 - u^2}{2s}$ = –320 m/s^2.
    Time = t = $\frac{v - u}{a} = \frac{0 - 16}{-320}$ = 0.05 sec.
17. u = 350 m/s, s = 5 cm = 0.05 m, v = 0
    Deceleration = a = $\frac{v^2 - u^2}{2s} = \frac{0 - (350)^2}{2 \times 0.05}$ = –12.2 × 10^5 m/s^2.
    Deceleration is 12.2 × 10^5 m/s^2.
18. u = 0, v = 18 km/hr = 5 m/s, t = 5 sec
    a = $\frac{v - u}{t} = \frac{5 - 0}{5}$ = 1 m/s^2.
    s = $ut + \frac{1}{2}at^2$ = 12.5 m
    a) Average velocity V_{ave} = (12.5)/5 = 2.5 m/s.
    b) Distance travelled is 12.5 m.
19. In reaction time the body moves with the speed 54 km/hr = 15 m/sec (constant speed)
    Distance travelled in this time is S_1 = 15 × 0.2 = 3 m.
    When brakes are applied,
       u = 15 m/s, v = 0, a = –6 m/s^2 (deceleration)

$S_2 = \frac{v^2 - u^2}{2a} = \frac{0 - 15^2}{2(-6)} = 18.75$ m

Total distance $s = s_1 + s_2 = 3 + 18.75 = 21.75 = 22$ m.

20.

	Driver X Reaction time 0.25	Driver Y Reaction time 0.35
A (deceleration on hard braking = 6 m/s^2)	Speed = 54 km/h Braking distance a= 19 m Total stopping distance b = 22 m	Speed = 72 km/h Braking distance c = 33 m Total stopping distance d = 39 m.
B (deceleration on hard braking = 7.5 m/s^2)	Speed = 54 km/h Braking distance e = 15 m Total stopping distance f = 18 m	Speed = 72 km/h Braking distance g = 27 m Total stopping distance h = 33 m.

$$a = \frac{0^2 - 15^2}{2(-6)} = 19 \text{ m}$$

So, b = 0.2 × 15 + 19 = 33 m

Similarly other can be calculated.

Braking distance : Distance travelled when brakes are applied.

Total stopping distance = Braking distance + distance travelled in reaction time.

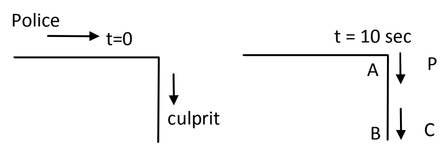
21. V_P = 90 km/h = 25 m/s.

V_C = 72 km/h = 20 m/s.

In 10 sec culprit reaches at point B from A.

Distance converted by culprit S = vt = 20 × 10 = 200 m.

At time t = 10 sec the police jeep is 200 m behind the culprit.



Time = s/v = 200 / 5 = 40 s. (Relative velocity is considered).

In 40 s the police jeep will move from A to a distance S, where

S = vt = 25 × 40 = 1000 m = 1.0 km away.

∴ The jeep will catch up with the bike, 1 km far from the turning.

22. v_1 = 60 km/hr = 16.6 m/s.

v_2 = 42 km/h = 11.6 m/s.

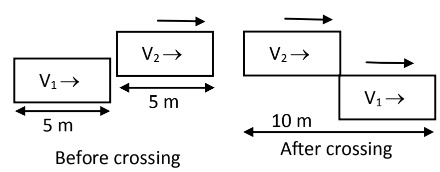
Relative velocity between the cars = (16.6 – 11.6) = 5 m/s.

Distance to be travelled by first car is 5 + t = 10 m.

Time = t = s/v = 0/5 = 2 sec to cross the 2nd car.

In 2 sec the 1st car moved = 16.6 × 2 = 33.2 m

H also covered its own length 5 m.



∴ Total road distance used for the overtake = 33.2 + 5 = 38 m.

23. u = 50 m/s, g = –10 m/s^2 when moving upward, v = 0 (at highest point).

a) $S = \frac{v^2 - u^2}{2a} = \frac{0 - 50^2}{2(-10)} = 125 \text{ m}$

maximum height reached = 125 m

b) t = (v – u)/a = (0 – 50)/–10 = 5 sec

c) s′ = 125/2 = 62.5 m, u = 50 m/s, a = –10 m/s^2,

$v^2 - u^2 = 2as$

$\Rightarrow v = \sqrt{(u^2 + 2as)} = \sqrt{50^2 + 2(-10)(62.5)}$ = 35 m/s.

24. Initially the ball is going upward

$u = -7$ m/s, s = 60 m, a = g = 10 m/s^2

$s = ut + \frac{1}{2}at^2 \Rightarrow 60 = -7t + 1/2\ 10t^2$

$\Rightarrow 5t^2 - 7t - 60 = 0$

$t = \frac{7 \pm \sqrt{49 - 4.5(-60)}}{2 \times 5} = \frac{7 \pm 35.34}{10}$

taking positive sign $t = \frac{7 + 35.34}{10}$ = 4.2 sec ($\therefore\ t \neq -ve$)

Therefore, the ball will take 4.2 sec to reach the ground.

25. u = 28 m/s, v = 0, a = –g = –9.8 m/s^2

a) $S = \frac{v^2 - u^2}{2a} = \frac{0^2 - 28^2}{2(9.8)}$ = 40 m

b) time $t = \frac{v - u}{a} = \frac{0 - 28}{-9.8}$ = 2.85

$t' = 2.85 - 1 = 1.85$

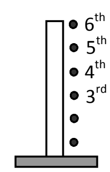
$v' = u + at' = 28 - (9.8)(1.85) = 9.87$ m/s.

$\therefore$ The velocity is 9.87 m/s.

c) No it will not change. As after one second velocity becomes zero for any initial velocity and deceleration is g = 9.8 m/s^2 remains same. Fro initial velocity more than 28 m/s max height increases.

26. For every ball, u = 0, a = g = 9.8 m/s^2

$\therefore$ 4th ball move for 2 sec, 5th ball 1 sec and 3rd ball 3 sec when 6th ball is being dropped.



For 3rd ball t = 3 sec

$S_3 = ut + \frac{1}{2}at^2 = 0 + 1/2\ (9.8)3^2$ = 4.9 m below the top.

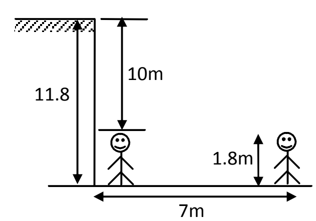
For 4th ball, t = 2 sec

$S_2 = 0 + 1/2\ gt^2 = 1/2\ (9.8)2^2$ = 19.6 m below the top (u = 0)

For 5th ball, t = 1 sec

$S_3 = ut + 1/2\ at^2 = 0 + 1/2\ (9.8)t^2$ = 4.98 m below the top.

27. At point B (i.e. over 1.8 m from ground) the kid should be catched.



For kid initial velocity u = 0

Acceleration = 9.8 m/s^2

Distance S = 11.8 – 1.8 = 10 m

$S = ut + \frac{1}{2}at^2 \Rightarrow 10 = 0 + 1/2\ (9.8)t^2$

$\Rightarrow t^2 = 2.04 \Rightarrow t = 1.42$.

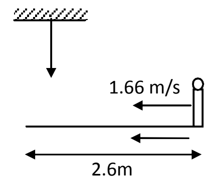
In this time the man has to reach at the bottom of the building.

Velocity s/t = 7/1.42 = 4.9 m/s.

28. Let the true of fall be 't' initial velocity u = 0

Acceleration a = 9.8 m/s^2

Distance S = 12/1 m



$\therefore$ S = $ut + \frac{1}{2}at^2$

$\Rightarrow$ 12.1 = 0 + 1/2 (9.8) × t^2

$\Rightarrow t^2 = \frac{12.1}{4.9}$ = 2.46 $\Rightarrow$ t = 1.57 sec

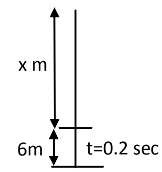
For cadet velocity = 6 km/hr = 1.66 m/sec

Distance = vt = 1.57 × 1.66 = 2.6 m.

The cadet, 2.6 m away from tree will receive the berry on his uniform.

29. For last 6 m distance travelled s = 6 m, u = ?

t = 0.2 sec, a = g = 9.8 m/s^2



S = $ut + \frac{1}{2}at^2 \Rightarrow$ 6 = u(0.2) + 4.9 × 0.04

$\Rightarrow$ u = 5.8/0.2 = 29 m/s.

For distance x, u = 0, v = 29 m/s, a = g = 9.8 m/s^2

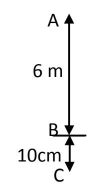
S = $\frac{v^2 - u^2}{2a} = \frac{29^2 - 0^2}{2 \times 9.8}$ = 42.05 m

Total distance = 42.05 + 6 = 48.05 = 48 m.

30. Consider the motion of ball form A to B.

B $\rightarrow$ just above the sand (just to penetrate)

u = 0, a = 9.8 m/s^2, s = 5 m



S = $ut + \frac{1}{2}at^2$

$\Rightarrow$ 5 = 0 + 1/2 (9.8)t^2

$\Rightarrow t^2$ = 5/4.9 = 1.02 $\Rightarrow$ t = 1.01.

$\therefore$ velocity at B, v = u + at = 9.8 × 1.01 (u = 0) =9.89 m/s.

From motion of ball in sand

u_1 = 9.89 m/s, v_1 = 0, a = ?, s = 10 cm = 0.1 m.

a = $\frac{v_1^2 - u_1^2}{2s} = \frac{0-(9.89)^2}{2 \times 0.1}$ = – 490 m/s^2

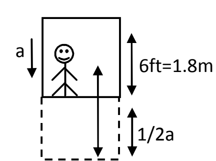
The retardation in sand is 490 m/s^2.

31. For elevator and coin u = 0

As the elevator descends downward with acceleration a′ (say)

The coin has to move more distance than 1.8 m to strike the floor. Time taken t = 1 sec.

$S_c = ut + \frac{1}{2}a't^2$ = 0 + 1/2 $g(1)^2$ = 1/2 g



$S_e = ut + \frac{1}{2}at^2$ = u + 1/2 $a(1)^2$ = 1/2 a

Total distance covered by coin is given by = 1.8 + 1/2 a = 1/2 g

$\Rightarrow$ 1.8 +a/2 = 9.8/2 = 4.9

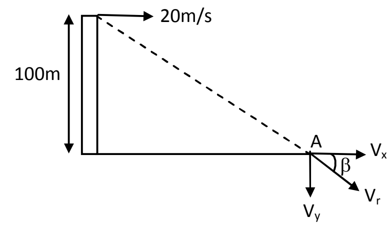
$\Rightarrow$ a = 6.2 m/s^2 = 6.2 × 3.28 = 20.34 ft/s^2.

32. It is a case of projectile fired horizontally from a height.

h = 100 m, g = 9.8 m/s^2

a) Time taken to reach the ground t = $\sqrt{(2h/g)}$

= $\sqrt{\frac{2 \times 100}{9.8}}$ = 4.51 sec.



b) Horizontal range x = ut = 20 × 4.5 = 90 m.

c) Horizontal velocity remains constant through out the motion.

At A, V = 20 m/s

A V_y = u + at = 0 + 9.8 × 4.5 = 44.1 m/s.

Resultant velocity V_r = $\sqrt{(44.1)^2 + 20^2}$ = 48.42 m/s.

Tan β = $\frac{V_y}{V_x} = \frac{44.1}{20}$ = 2.205

⇒ β = $\tan^{-1}$ (2.205) = 60°.

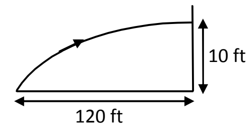
The ball strikes the ground with a velocity 48.42 m/s at an angle 66° with horizontal.

33. u = 40 m/s, a = g= 9.8 m/s^2, θ = 60° Angle of projection.

a) Maximum height h = $\frac{u^2 \sin^2 \theta}{2g} = \frac{40^2 (\sin 60°)^2}{2 \times 10}$ = 60 m

b) Horizontal range X = (u^2 sin 2θ) / g = (40^2 sin 2(60°)) / 10 = $80\sqrt{3}$ m.

34. $g = 9.8\ m/s^2$, $32.2\ ft/s^2$; 40 yd = 120 ft



horizontal range x = 120 ft, u = 64 ft/s, $\theta = 45°$

We know that horizontal range X = u cos θt

$$\Rightarrow t = \frac{x}{u\cos\theta} = \frac{120}{64\cos 45°} = 2.65 \text{ sec.}$$

$$y = u \sin\theta(t) - 1/2\ gt^2 = 64\frac{1}{\sqrt{2}(2.65)} - \frac{1}{2}(32.2)(2.65)^2$$

= 7.08 ft which is less than the height of goal post.

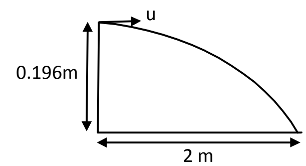
In time 2.65, the ball travels horizontal distance 120 ft (40 yd) and vertical height 7.08 ft which is less than 10 ft. The ball will reach the goal post.

35. The goli move like a projectile.

Here h = 0.196 m

Horizontal distance X = 2 m

Acceleration $g = 9.8\ m/s^2$.



Time to reach the ground i.e.

$$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2\times 0.196}{9.8}} = 0.2 \text{ sec}$$

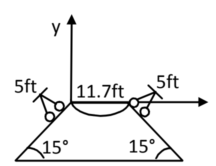
Horizontal velocity with which it is projected be u.

$\therefore$ x = ut

$$\Rightarrow u = \frac{x}{t} = \frac{2}{0.2} = 10 \text{ m/s.}$$

36. Horizontal range X = 11.7 + 5 = 16.7 ft covered by te bike.

$g = 9.8\ m/s^2 = 32.2\ ft/s^2$.



$$y = x\tan\theta - \frac{gx^2\sec^2\theta}{2u^2}$$

To find, minimum speed for just crossing, the ditch

y = 0 ($\therefore$ A is on the x axis)

$$\Rightarrow x\tan\theta = \frac{gx^2\sec^2\theta}{2u^2} \Rightarrow u^2 = \frac{gx^2\sec^2\theta}{2x\tan\theta} = \frac{gx}{2\sin\theta\cos\theta} = \frac{gx}{\sin 2\theta}$$

$$\Rightarrow u = \sqrt{\frac{(32.2)(16.7)}{1/2}} \text{ (because sin 30° = 1/2)}$$

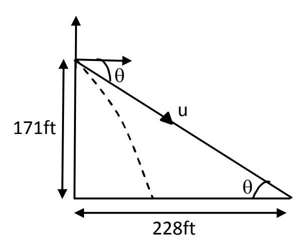
$\Rightarrow$ u = 32.79 ft/s = 32 ft/s.

37. $\tan\theta = 171/228 \Rightarrow \theta = \tan^{-1}(171/228)$

The motion of projectile (i.e. the packed) is from A. Taken reference axis at A.

$\therefore\ \theta = -37°$ as u is below x-axis.

u = 15 ft/s, $g = 32.2\ ft/s^2$, y = –171 ft



$$y = x\tan\theta - \frac{x^2 g\sec^2\theta}{2u^2}$$

$$\therefore -171 = -x\,(0.7536) - \frac{x^2 g(1.568)}{2(225)}$$

$\Rightarrow 0.1125x^2 + 0.7536\,x - 171 = 0$

x = 35.78 ft (can be calculated)

Horizontal range covered by the packet is 35.78 ft.

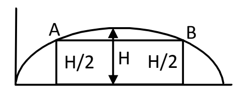
So, the packet will fall 228 – 35.78 = 192 ft short of his friend.

38. Here u = 15 m/s, θ = 60°, g = 9.8 m/s^2

Horizontal range X = $\frac{u^2 \sin 2\theta}{g} = \frac{(15)^2 \sin(2\times 60°)}{9.8}$ = 19.88 m

In first case the wall is 5 m away from projection point, so it is in the horizontal range of projectile. So the ball will hit the wall. In second case (22 m away) wall is not within the horizontal range. So the ball would not hit the wall.

39. Total of flight T = $\frac{2u\sin\theta}{g}$



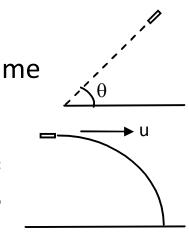
Average velocity = $\frac{\text{change in displacement}}{\text{time}}$

From the figure, it can be said AB is horizontal. So there is no effect of vertical component of the velocity during this displacement.

So because the body moves at a constant speed of 'u cos θ' in horizontal direction.

The average velocity during this displacement will be u cos θ in the horizontal direction.

40. During the motion of bomb its horizontal velocity u remains constant and is same as that of aeroplane at every point of its path. Suppose the bomb explode i.e. reach the ground in time t. Distance travelled in horizontal direction by bomb = ut = the distance travelled by aeroplane. So bomb explode vertically below the aeroplane.

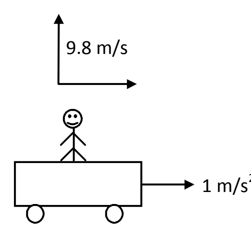


Suppose the aeroplane move making angle θ with horizontal. For both bomb and aeroplane, horizontal distance is u cos θ t. t is time for bomb to reach the ground.

So in this case also, the bomb will explode vertically below aeroplane.

41. Let the velocity of car be u when the ball is thrown. Initial velocity of car is = Horizontal velocity of ball.

Distance travelled by ball B S_b = ut (in horizontal direction)



And by car S_c = ut + 1/2 at^2 where t $\rightarrow$ time of flight of ball in air.

$\therefore$ Car has travelled extra distance $S_c - S_b$ = 1/2 at^2.

Ball can be considered as a projectile having θ = 90°.

$\therefore$ t = $\frac{2u\sin\theta}{g} = \frac{2\times 9.8}{9.8}$ = 2 sec.

$\therefore$ $S_c - S_b$ = 1/2 at^2 = 2 m

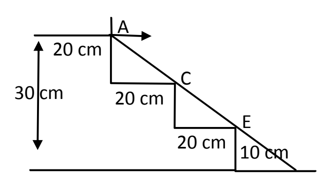
$\therefore$ The ball will drop 2m behind the boy.

42. At minimum velocity it will move just touching point E reaching the ground.

A is origin of reference coordinate.

If u is the minimum speed.

X = 40, Y = –20, θ = 0°



$\therefore$ Y = x tan θ – g$\frac{x^2 \sec^2\theta}{2u^2}$ (because g = 10 m/s^2 = 1000 cm/s^2)

$\Rightarrow$ –20 = x tan θ – $\frac{1000\times 40^2 \times 1}{2u^2}$

$\Rightarrow$ u = 200 cm/s = 2 m/s.

$\therefore$ The minimum horizontal velocity is 2 m/s.

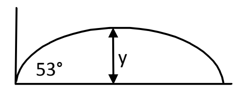
43. a) As seen from the truck the ball moves vertically upward comes back. Time taken = time taken by truck to cover 58.8 m.

$\therefore$ time = $\frac{s}{v} = \frac{58.8}{14.7}$ = 4 sec. (V = 14.7 m/s of truck)

u = ?, v = 0, g = –9.8 m/s^2 (going upward), t = 4/2 = 2 sec.

v = u + at $\Rightarrow$ 0 = u – 9.8 × 2 $\Rightarrow$ u = 19.6 m/s. (vertical upward velocity).

b) From road it seems to be projectile motion.



Total time of flight = 4 sec

In this time horizontal range covered 58.8 m = x

$\therefore$ X = u cos θ t

$\Rightarrow$ u cos θ = 14.7 ...(1)

Taking vertical component of velocity into consideration.

y = $\frac{0^2 - (19.6)^2}{2 \times (-9.8)}$ = 19.6 m [from (a)]

$\therefore$ y = u sin θ t – 1/2 gt^2

$\Rightarrow$ 19.6 = u sin θ (2) – 1/2 (9.8)2^2 $\Rightarrow$ 2u sin θ = 19.6 × 2

$\Rightarrow$ u sin θ = 19.6 ...(ii)

$\frac{u \sin\theta}{u \cos\theta}$ = tan θ $\Rightarrow$ $\frac{19.6}{14.7}$ = 1.333

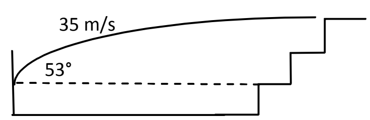
$\Rightarrow$ θ = $\tan^{-1}$ (1.333) = 53°

Again u cos θ = 14.7

$\Rightarrow$ u = $\frac{14.7}{u \cos 53°}$ = 24.42 m/s.

The speed of ball is 42.42 m/s at an angle 53° with horizontal as seen from the road.

44. θ = 53°, so cos 53° = 3/5



Sec^2 θ = 25/9 and tan θ = 4/3

Suppose the ball lands on nth bench

So, y = (n – 1)1 ...(1) [ball starting point 1 m above ground]

Again y = x tan θ – $\frac{gx^2 \sec^2\theta}{2u^2}$ [x = 110 + n – 1 = 110 + y]

$\Rightarrow$ y = (110 + y)(4/3) – $\frac{10(110+y)^2(25/9)}{2 \times 35^2}$

$\Rightarrow$ $\frac{440}{3} + \frac{4}{3}y - \frac{250(110+y)^2}{18 \times 35^2}$

From the equation, y can be calculated.

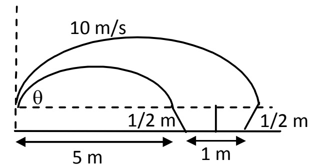
$\therefore$ y = 5

$\Rightarrow$ n – 1 = 5 $\Rightarrow$ n = 6.

The ball will drop in sixth bench.

45. When the apple just touches the end B of the boat.

x = 5 m, u = 10 m/s, g = 10 m/s^2, θ = ?



$x = \frac{u^2 \sin 2\theta}{g}$

$\Rightarrow 5 = \frac{10^2 \sin 2\theta}{10} \Rightarrow 5 = 10 \sin 2\theta$

$\Rightarrow \sin 2\theta = 1/2 \Rightarrow \sin 30° \text{ or } \sin 150°$

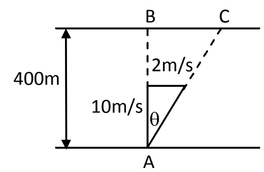
$\Rightarrow \theta = 15° \text{ or } 75°$

Similarly for end C, x = 6 m

Then $2\theta_1 = \sin^{-1}(gx/u^2) = \sin^{-1}(0.6) = 182°$ or $71°$.

So, for a successful shot, θ may very from 15° to 18° or 71° to 75°.

46. a) Here the boat moves with the resultant velocity R. But the vertical component 10 m/s takes him to the opposite shore.



$\tan\theta = 2/10 = 1/5$

Velocity = 10 m/s

distance = 400 m

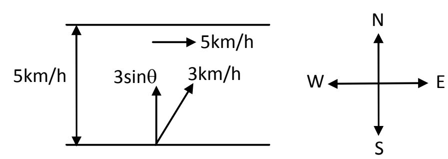
Time = 400/10 = 40 sec.

b) The boat will reach at point C.

In ΔABC, $\tan\theta = \frac{BC}{AB} = \frac{BC}{400} = \frac{1}{5}$

$\Rightarrow BC = 400/5 = 80$ m.

47. a) The vertical component 3 sin θ takes him to opposite side.

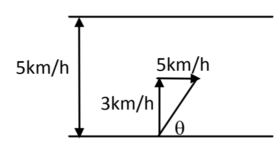


Distance = 0.5 km, velocity = 3 sin θ km/h

$\text{Time} = \frac{\text{Distance}}{\text{Velocity}} = \frac{0.5}{3\sin\theta}\text{hr}$

= 10/sinθ min.

b) Here vertical component of velocity i.e. 3 km/hr takes him to opposite side.



$\text{Time} = \frac{\text{Distance}}{\text{Velocity}} = \frac{0.5}{3} = 0.16 \text{ hr}$

∴ 0.16 hr = 60 × 0.16 = 9.6 = 10 minute.

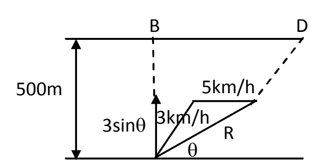
48. Velocity of man $\vec{V}_m = 3$ km/hr

BD horizontal distance for resultant velocity R.

X-component of resultant $R_x = 5 + 3\cos\theta$

$t = 0.5 / 3\sin\theta$

which is same for horizontal component of velocity.



$H = BD = (5 + 3\cos\theta)(0.5 / 3\sin\theta) = \frac{5 + 3\cos\theta}{6\sin\theta}$

For H to be min $(dH/d\theta) = 0$

$\Rightarrow \frac{d}{d\theta}\left(\frac{5 + 3\cos\theta}{6\sin\theta}\right) = 0$

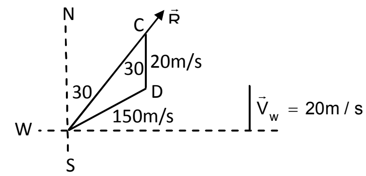
$\Rightarrow -18(\sin^2\theta + \cos^2\theta) - 30\cos\theta = 0$

$\Rightarrow -30\cos\theta = 18 \Rightarrow \cos\theta = -18/30 = -3/5$

$\text{Sin}\,\theta = \sqrt{1-\cos^2\theta} = 4/5$

$\therefore$ H = $\dfrac{5+3\cos\theta}{6\sin\theta} = \dfrac{5+3(-3/5)}{6\times(4/5)}$ = $\dfrac{2}{3}$ km.

49. In resultant direction $\vec{R}$ the plane reach the point B.



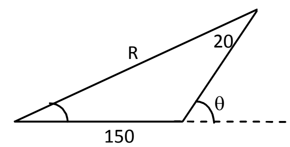
Velocity of wind $\vec{V}_w$ = 20 m/s

Velocity of aeroplane $\vec{V}_a$ = 150 m/s

In ΔACD according to sine formula

$\therefore \dfrac{20}{\sin A} = \dfrac{150}{\sin 30°} \Rightarrow \sin A = \dfrac{20}{150}\sin 30° = \dfrac{20}{150}\times\dfrac{1}{2} = \dfrac{1}{15}$

$\Rightarrow$ A = $\sin^{-1}$ (1/15)



a) The direction is $\sin^{-1}$ (1/15) east of the line AB.

b) $\sin^{-1}$ (1/15) = 3°48′

$\Rightarrow$ 30° + 3°48′ = 33°48′

R = $\sqrt{150^2 + 20^2 + 2(150)20\cos 33°48'}$ = 167 m/s.

Time = $\dfrac{s}{v} = \dfrac{500000}{167}$ = 2994 sec = 49 = 50 min.

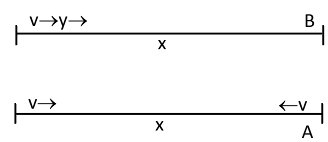
50. Velocity of sound v, Velocity of air u, Distance between A and B be x.

In the first case, resultant velocity of sound = v + u

$\Rightarrow$ (v + u) t_1 = x

$\Rightarrow$ v + u = x/t_1 ...(1)

In the second case, resultant velocity of sound = v – u



$\therefore$(v – u) t_2 = x

$\Rightarrow$ v – u = x/t_2 ...(2)

From (1) and (2) 2v = $\dfrac{x}{t_1} + \dfrac{x}{t_2} = x\left(\dfrac{1}{t_1} + \dfrac{1}{t_2}\right)$

$\Rightarrow$ v = $\dfrac{x}{2}\left(\dfrac{1}{t_1} + \dfrac{1}{t_2}\right)$

From (i) u = $\dfrac{x}{t_1} - v = \dfrac{x}{t_1} - \left(\dfrac{x}{2t_1} + \dfrac{x}{2t_2}\right)$ = $\dfrac{x}{2}\left(\dfrac{1}{t_1} - \dfrac{1}{t_2}\right)$

$\therefore$ Velocity of air V = $\dfrac{x}{2}\left(\dfrac{1}{t_1} + \dfrac{1}{t_2}\right)$

And velocity of wind u = $\dfrac{x}{2}\left(\dfrac{1}{t_1} - \dfrac{1}{t_2}\right)$

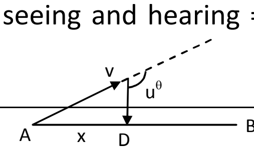
51. Velocity of sound v, velocity of air u

Velocity of sound be in direction AC so it can reach B with resultant velocity AD.

Angle between v and u is $\theta > \pi/2$.

Resultant $\overrightarrow{AD} = \sqrt{(v^2 - u^2)}$

Here time taken by light to reach B is neglected. So time lag between seeing and hearing = time to here the drum sound.



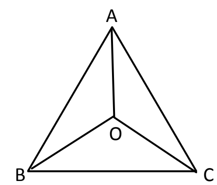
$$t = \frac{\text{Displacement}}{\text{velocity}} = \frac{x}{\sqrt{v^2 - u^2}}$$

$$\Rightarrow \frac{x}{\sqrt{(v+u)(v-u)}} = \frac{x}{\sqrt{(x/t_1)(x/t_2)}}$$ [from question no. 50]

$$= \sqrt{t_1 t_2}\ .$$

52. The particles meet at the centroid O of the triangle. At any instant the particles will form an equilateral ΔABC with the same centroid.

Consider the motion of particle A. At any instant its velocity makes angle 30°. This component is the rate of decrease of the distance AO.



Initially AO = $\frac{2}{3}\sqrt{a^2 - \left(\frac{a}{2}\right)^2} = \frac{a}{\sqrt{3}}$

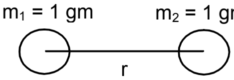
Therefore, the time taken for AO to become zero.

$$= \frac{a/\sqrt{3}}{v\cos 30^\circ} = \frac{2a}{\sqrt{3}v \times \sqrt{3}} = \frac{2a}{3v}\ .$$

* * * *

SOLUTIONS TO CONCEPTS
CHAPTER – 4

1. m = 1 gm = 1/1000 kg



$F = 6.67 \times 10^{-17}$ N $\Rightarrow F = \dfrac{Gm_1m_2}{r^2}$

$\therefore 6.67 \times 20^{-17} = \dfrac{6.67 \times 10^{-11} \times (1/1000) \times (1/1000)}{r^2}$

$\Rightarrow r^2 = \dfrac{6.67 \times 10^{-11} \times 10^{-6}}{6.64 \times 10^{-17}} = \dfrac{10^{-17}}{10^{-17}} = 1$

$\Rightarrow$ r = $\sqrt{1}$ = 1 metre.

So, the separation between the particles is 1 m.

2. A man is standing on the surface of earth

The force acting on the man = mg ………(i)

Assuming that, m = mass of the man = 50 kg

And g = acceleration due to gravity on the surface of earth = 10 m/s^2

W = mg = 50× 10= 500 N = force acting on the man

So, the man is also attracting the earth with a force of 500 N

3. The force of attraction between the two charges

$= \dfrac{1}{4\pi\varepsilon_o} \dfrac{q_1 q_2}{r^2} = 9 \times 10^9 \dfrac{1}{r^2}$

The force of attraction is equal to the weight

$Mg = \dfrac{9 \times 10^9}{r^2}$

$\Rightarrow r^2 = \dfrac{9 \times 10^9}{m \times 10} = \dfrac{9 \times 10^8}{m}$ [Taking g=10 m/s^2]

$\Rightarrow r = \sqrt{\dfrac{9 \times 10^8}{m}} = \dfrac{3 \times 10^4}{\sqrt{m}}$ mt

For example, Assuming m= 64 kg,

$r = \dfrac{3 \times 10^4}{\sqrt{64}} = \dfrac{3}{8} 10^4$ = 3750 m

4. mass = 50 kg

r = 20 cm = 0.2 m

$F_G = G\dfrac{m_1m_2}{r^2} = \dfrac{6.67 \times 10^{-11} \times 2500}{0.04}$

Coulomb's force $F_C = \dfrac{1}{4\pi\varepsilon_o} \dfrac{q_1q_2}{r^2}$ = 9 × $10^9 \dfrac{q^2}{0.04}$

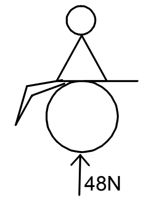
Since, $F_G = F_c = \dfrac{6.7 \times 10^{-11} \times 2500}{0.04} = \dfrac{9 \times 10^9 \times q^2}{0.04}$

$\Rightarrow q^2 = \dfrac{6.7 \times 10^{-11} \times 2500}{0.04} = \dfrac{6.7 \times 10^{-9}}{9 \times 10^9} \times 25$

= 18.07 × 10^{-18}

q = $\sqrt{18.07 \times 10^{-18}}$ = 4.3 × 10^{-9} C.

5. The limb exerts a normal force 48 N and frictional force of 20 N. Resultant magnitude of the force,



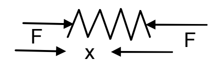
$R = \sqrt{(48)^2 + (20)^2}$

$= \sqrt{2304 + 400}$

$= \sqrt{2704}$

= 52 N

6. The body builder exerts a force = 150 N.



Compression x = 20 cm = 0.2 m

$\therefore$ Total force exerted by the man = f = kx

$\Rightarrow$ kx = 150

$\Rightarrow k = \frac{150}{0.2} = \frac{1500}{2} = 750$ N/m

7. Suppose the height is h.

At earth station $F = GMm/R^2$

M = mass of earth

m = mass of satellite

R = Radius of earth

$F = \frac{GMm}{(R+h)^2} = \frac{GMm}{2R^2}$

$\Rightarrow 2R^2 = (R + h)^2 \Rightarrow R^2 - h^2 - 2Rh = 0$

$\Rightarrow h^2 + 2Rh - R^2 = 0$

$H = \frac{\left(-2R \pm \sqrt{4R^2 + 4R^2}\right)}{2} = \frac{-2R \pm 2\sqrt{2R}}{2}$

$= -R \pm \sqrt{2R} = R\left(\sqrt{2} - 1\right)$

= 6400 × (0.414)

= 2649.6 = 2650 km

8. Two charged particle placed at a sehortion 2m. exert a force of 20m.

F_1 = 20 N. $\quad r_1$ = 20 cm

F_2 = ? $\quad r_2$ = 25 cm

Since, $F = \frac{1}{4\pi\varepsilon_o}\frac{q_1q_2}{r^2}$, $\quad F \propto \frac{1}{r^2}$

$\frac{F_1}{F_2} = \frac{r_2^2}{r_1^2} \Rightarrow F_2 = F_1 \times \left(\frac{r_1}{r_2}\right)^2 = 20 \times \left(\frac{20}{25}\right)^2 = 20 \times \frac{16}{25} = \frac{64}{5}$ = 12.8 N = 13 N.

9. The force between the earth and the moon, $F = G\frac{m_m m_c}{r^2}$

$F = \frac{6.67 \times 10^{-11} \times 7.36 \times 10^{22} \times 6 \times 10^{24}}{\left(3.8 \times 10^8\right)^2} = \frac{6.67 \times 7.36 \times 10^{35}}{(3.8)^2 \times 10^{16}}$

$= 20.3 \times 10^{19} = 2.03 \times 10^{20}$ N $= 2 \times 10^{20}$ N

10. Charge on proton $= 1.6 \times 10^{-19}$

$\therefore F_{electrical} = \frac{1}{4\pi\varepsilon_o} \times \frac{q_1q_2}{r^2} = \frac{9 \times 10^9 \times (1.6)^2 \times 10^{-38}}{r^2}$

mass of proton $= 1.732 \times 10^{-27}$ kg

$$F_{gravity} = G\frac{m_1m_2}{r^2} = \frac{6.67\times10^{-11}\times(1.732)\times10^{-54}}{r^2}$$

$$\frac{F_e}{F_g} = \frac{\dfrac{9\times10^9\times(1.6)^2\times10^{-38}}{r^2}}{\dfrac{6.67\times10^{-11}\times(1.732)\times10^{-54}}{r^2}} = \frac{9\times(1.6)^2\times10^{-29}}{6.67\ (1.732)^2\ 10^{-65}} = 1.24\times10^{36}$$

11. The average separation between proton and electron of Hydrogen atom is r= 5.3 10^{-11}m.

    a) Coulomb's force = F = $9\times10^9\times\frac{q_1q_2}{r^2} = \frac{9\times10^9\times(1.0\times10^{-19})^2}{(5.3\times10^{-11})^2} = 8.2\times10^{-8}$ N.

    b) When the average distance between proton and electron becomes 4 times that of its ground state

    Coulomb's force F = $\frac{1}{4\pi\varepsilon_o}\times\frac{q_1q_2}{(4r)^2} = \frac{9\times10^9\times(1.6\times10^{-19})^2}{16\times(5.3)^2\times10^{-22}} = \frac{9\times(1.6)^2}{16\times(5.3)^2}\times10^{-7}$

    $= 0.0512\times10^{-7} = 5.1\times10^{-9}$ N.

12. The geostationary orbit of earth is at a distance of about 36000km.
    We know that, $g' = GM / (R+h)^2$
    At h = 36000 km. $g' = GM / (36000+6400)^2$
    $\therefore \frac{g`}{g} = \frac{6400\times6400}{42400\times42400} = \frac{256}{106\times106} = 0.0227$
    $\Rightarrow g' = 0.0227\times9.8 = 0.223$
    [taking g = 9.8 m/s^2 at the surface of the earth]
    A 120 kg equipment placed in a geostationary satellite will have weight
    Mg` = 0.233 × 120 = 26.79 = 27 N

* * * *

SOLUTIONS TO CONCEPTS
CHAPTER – 5

1. $m = 2kg$
   $S = 10m$
   Let, acceleration = a, Initial velocity u = 0.
   $S = ut + 1/2\ at^2$
   $\Rightarrow 10 = ½\ a\ (2^2) \Rightarrow 10 = 2a \Rightarrow a = 5\ m/s^2$
   Force: F = ma = 2 × 5 = 10N (Ans)

2. $u = 40\ km/hr = \dfrac{40000}{3600} = 11.11\ m/s.$
   m = 2000 kg ; v = 0 ; s = 4m
   acceleration 'a' $= \dfrac{v^2 - u^2}{2s} = \dfrac{0^2 - (11.11)^2}{2 \times 4} = -\dfrac{123.43}{8} = -15.42\ m/s^2$ (deceleration)
   So, braking force = F = ma = 2000 × 15.42 = 30840 = $3.08\ 10^4$ N (Ans)

3. Initial velocity u = 0 (negligible)
   $v = 5 \times 10^6\ m/s.$
   $s = 1cm = 1 \times 10^{-2} m.$
   acceleration $a = \dfrac{v^2 - u^2}{2s} = \dfrac{(5 \times 10^6)^2 - 0}{2 \times 1 \times 10^{-2}} = \dfrac{25 \times 10^{12}}{2 \times 10^{-2}} = 12.5 \times 10^{14} ms^{-2}$
   $F = ma = 9.1 \times 10^{-31} \times 12.5 \times 10^{14} = 113.75 \times 10^{-17} = 1.1 \times 10^{-15}$ N.

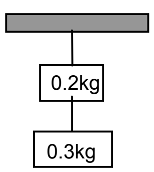
4.


fig 1

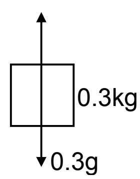


fig 2

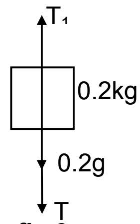


fig 3

   $g = 10m/s^2$ $\quad$ $T - 0.3g = 0 \Rightarrow T = 0.3g = 0.3 \times 10 = 3$ N
   $T_1 - (0.2g + T) = 0 \Rightarrow T_1 = 0.2g + T = 0.2 \times 10 + 3 = 5N$
   $\therefore$ Tension in the two strings are 5N & 3N respectively.

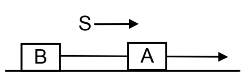
5.


Fig 1

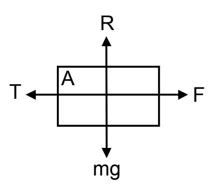


Fig 2

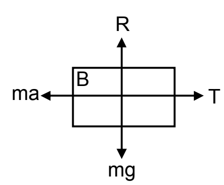
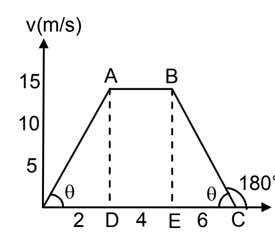


Fig 3

   T + ma – F = 0 $\qquad$ T – ma = 0 $\Rightarrow$ T = ma ………..(i)
   $\Rightarrow$ F= T + ma $\Rightarrow$ F= T + T $\qquad$ from (i)
   $\Rightarrow$ 2T = F $\Rightarrow$ T = F / 2

6. $m = 50g = 5 \times 10^{-2}$ kg
   As shown in the figure,
   Slope of OA = $\tan\theta\ \dfrac{AD}{OD} = \dfrac{15}{3} = 5\ m/s^2$
   So, at t = 2sec acceleration is $5m/s^2$
   Force = ma = $5 \times 10^{-2} \times 5$ = 0.25N along the motion



At $t = 4$ sec slope of AB = 0, acceleration = 0 [$\tan 0° = 0$]

$\therefore$ Force = 0

At $t = 6$ sec, acceleration = slope of BC.

In $\Delta BEC = \tan\theta = \frac{BE}{EC} = \frac{15}{3} = 5$.

Slope of BC = $\tan(180° - \theta) = -\tan\theta = -5\ m/s^2$ (deceleration)

Force = ma = $5 \times 10^{-2}\ 5 = 0.25$ N. Opposite to the motion.

7. Let, F $\rightarrow$ contact force between m_A & m_B.

And, f $\rightarrow$ force exerted by experimenter.

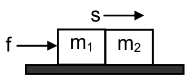


Fig 1

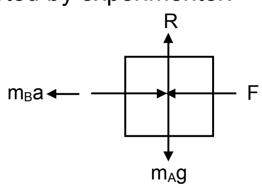


Fig 2

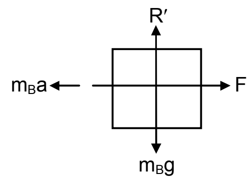


Fig 3

$F + m_A a - f = 0$ $m_B a - f = 0$

$\Rightarrow F = f - m_A a$ ……….(i) $\Rightarrow F = m_B a$ ……...(ii)

From eqn (i) and eqn (ii)

$\Rightarrow f - m_A a = m_B a \Rightarrow f = m_B a + m_A a \Rightarrow f = a(m_A + m_B)$.

$\Rightarrow f = \frac{F}{m_B}(m_B + m_A) = F\left(1 + \frac{m_A}{m_B}\right)$ [because $a = F/m_B$]

$\therefore$ The force exerted by the experimenter is $F\left(1 + \frac{m_A}{m_B}\right)$

8. $r = 1mm = 10^{-3}$

'm' = 4mg = 4×10^{-6}kg

$s = 10^{-3}$m.

$v = 0$

$u = 30$ m/s.

So, $a = \frac{v^2 - u^2}{2s} = \frac{-30 \times 30}{2 \times 10^{-3}} = -4.5 \times 10^5\ m/s^2$ (decelerating)

Taking magnitude only deceleration is $4.5 \times 10^5\ m/s^2$

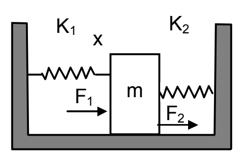
So, force $F = 4 \times 10^{-6} \times 4.5 \times 10^5 = 1.8$ N

9. x = 20 cm = 0.2m, k = 15 N/m, m = 0.3kg.

Acceleration $a = \frac{F}{m} = \frac{-kx}{x} = \frac{-15(0.2)}{0.3} = -\frac{3}{0.3} = -10 m/s^2$ (deceleration)

So, the acceleration is 10 m/s^2 opposite to the direction of motion

10. Let, the block m towards left through displacement x.



$F_1 = k_1 x$ (compressed)

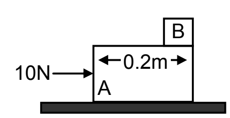
$F_2 = k_2 x$ (expanded)

They are in same direction.

Resultant $F = F_1 + F_2 \Rightarrow F = k_1 x + k_2 x \Rightarrow F = x(k_1 + k_2)$

So, a = acceleration = $\frac{F}{m} = \frac{x(k_1 + k_2)}{m}$ opposite to the displacement.

11. m = 5 kg of block A.



ma = 10 N

$\Rightarrow a\ 10/5 = 2\ m/s^2$.

As there is no friction between A & B, when the block A moves, Block B remains at rest in its position.

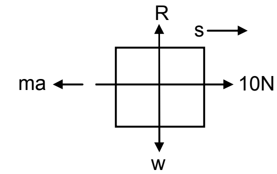
Initial velocity of A = u = 0.

Distance to cover so that B separate out s = 0.2 m.

Acceleration a = 2 m/s^2

$\therefore$ s= ut + ½ at^2

$\Rightarrow 0.2 = 0 + (½) \times 2 \times t^2 \Rightarrow t^2 = 0.2 \Rightarrow t = 0.44$ sec $\Rightarrow$ t= 0.45 sec.



12. a) at any depth let the ropes make angle θ with the vertical

From the free body diagram

$F\cos\theta + F\cos\theta - mg = 0$

$\Rightarrow 2F\cos\theta = mg \Rightarrow F = \dfrac{mg}{2\cos\theta}$

As the man moves up. θ increases i.e. cos θ decreases. Thus F increases.

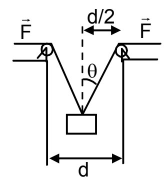


Fig-1

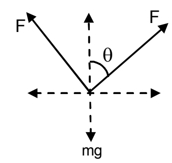
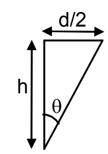


Fig-2

b) When the man is at depth h

$$\cos\theta = \frac{h}{\sqrt{(d/2)^2 + h^2}}$$

$$\text{Force} = \frac{mg}{\dfrac{h}{\sqrt{\dfrac{d^2}{4} + h^2}}} = \frac{mg}{4h}\sqrt{d^2 + 4h^2}$$



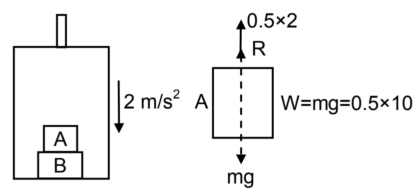
13. From the free body diagram

$\therefore R + 0.5 \times 2 - w = 0$

$\Rightarrow R = w - 0.5 \times 2$

= 0.5 (10 – 2) = 4N.

So, the force exerted by the block A on the block B, is 4N.



14. a) The tension in the string is found out for the different conditions from the free body diagram as shown below.

$T - (W + 0.06 \times 1.2) = 0$

$\Rightarrow T = 0.05 \times 9.8 + 0.05 \times 1.2$

= 0.55 N.

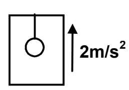


Fig-1

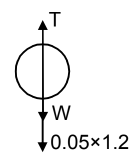


Fig-2

b) $\therefore T + 0.05 \times 1.2 - 0.05 \times 9.8 = 0$

$\Rightarrow T = 0.05 \times 9.8 - 0.05 \times 1.2$

= 0.43 N.

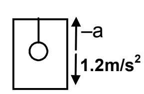


Fig-3

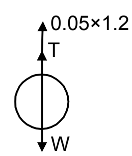


Fig-4

c) When the elevator makes uniform motion

$T - W = 0$

$\Rightarrow T = W = 0.05 \times 9.8$

= 0.49 N

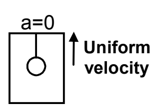


Fig-5

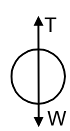


Fig-6

d) $T + 0.05 \times 1.2 - W = 0$

$\Rightarrow T = W - 0.05 \times 1.2$

= 0.43 N.

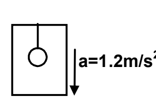


Fig-7

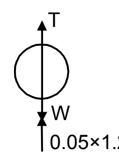


Fig-8

e) $T - (W + 0.05 \times 1.2) = 0$

$\Rightarrow T = W + 0.05 \times 1.2$

= 0.55 N

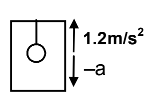


Fig-9

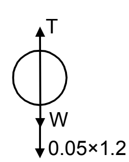


Fig-10

f) When the elevator goes down with uniform velocity acceleration = 0

$T - W = 0$

$\Rightarrow T = W = 0.05 \times 9.8$

$= 0.49$ N.

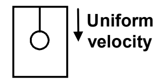


Fig-11

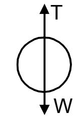


Fig-12

15. When the elevator is accelerating upwards, maximum weight will be recorded.

$R - (W + ma) = 0$

$\Rightarrow R = W + ma = m(g + a)$ max.wt.

When decelerating upwards, maximum weight will be recorded.

$R + ma - W = 0$

$\Rightarrow R = W - ma = m(g - a)$

So, $m(g + a) = 72 \times 9.9$ …(1)

$m(g - a) = 60 \times 9.9$ …(2)

Now, $mg + ma = 72 \times 9.9 \Rightarrow mg - ma = 60 \times 9.9$

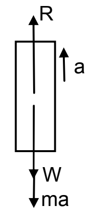
$\Rightarrow 2mg = 1306.8$

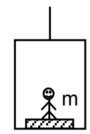
$\Rightarrow m = \dfrac{1306.8}{2 \times 9.9} = 66$ Kg

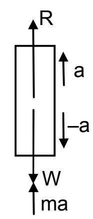
So, the true weight of the man is 66 kg.

Again, to find the acceleration, $mg + ma = 72 \times 9.9$

$\Rightarrow a = \dfrac{72 \times 9.9 - 66 \times 9.9}{66} = \dfrac{9.9}{11} = 0.9\ \text{m/s}^2$.







16. Let the acceleration of the 3 kg mass relative to the elevator is ‘a’ in the downward direction.

As, shown in the free body diagram

$T - 1.5\,g - 1.5(g/10) - 1.5\,a = 0$ from figure (1)

and, $T - 3g - 3(g/10) + 3a = 0$ from figure (2)

$\Rightarrow T = 1.5\,g + 1.5(g/10) + 1.5a$ … (i)

And $T = 3g + 3(g/10) - 3a$ … (ii)

Equation (i) × 2 $\Rightarrow 3g + 3(g/10) + 3a = 2T$

Equation (ii) × 1 $\Rightarrow 3g + 3(g/10) - 3a = T$

Subtracting the above two equations we get, $T = 6a$

Subtracting $T = 6a$ in equation (ii)

$6a = 3g + 3(g/10) - 3a$.

$\Rightarrow 9a = \dfrac{33g}{10} \Rightarrow a = \dfrac{(9.8)33}{10} = 32.34$

$\Rightarrow a = 3.59\ \therefore\ T = 6a = 6 \times 3.59 = 21.55$

$T^1 = 2T = 2 \times 21.55 = 43.1$ N cut is T_1 shown in spring.

Mass $= \dfrac{wt}{g} = \dfrac{43.1}{9.8} = 4.39 = 4.4$ kg

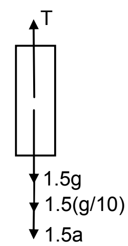


Fig-1

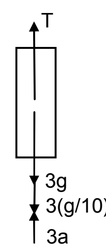


Fig-2

17. Given, m = 2 kg, k = 100 N/m

From the free body diagram, $kl - 2g = 0 \Rightarrow kl = 2g$

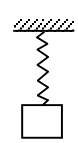
$\Rightarrow l = \dfrac{2g}{k} = \dfrac{2 \times 9.8}{100} = \dfrac{19.6}{100} = 0.196 = 0.2$ m

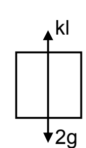
Suppose further elongation when 1 kg block is added be x,

Then $k(1 + x) = 3g$

$\Rightarrow kx = 3g - 2g = g = 9.8$ N

$\Rightarrow x = \dfrac{9.8}{100} = 0.098 = 0.1$ m





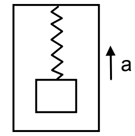
18. $a = 2\ m/s^2$

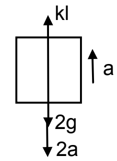
$kl - (2g + 2a) = 0$

$\Rightarrow kl = 2g + 2a$

$= 2 \times 9.8 + 2 \times 2 = 19.6 + 4$

$\Rightarrow l = \dfrac{23.6}{100} = 0.236\ m = 0.24\ m$



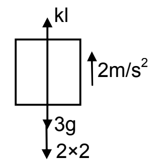


When 1 kg body is added total mass (2 + 1)kg = 3kg.

elongation be l_1

$kl_1 = 3g + 3a = 3 \times 9.8 + 6$

$\Rightarrow l_1 = \dfrac{33.4}{100} = 0.0334 = 0.36$



Further elongation = $l_1 - l = 0.36 - 0.12$ m.

19. Let, the air resistance force is F and Buoyant force is B.

Given that

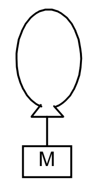
$F_a \propto v$, where $v \rightarrow$ velocity

$\Rightarrow F_a = kv$, where $k \rightarrow$ proportionality constant.

When the balloon is moving downward,

$B + kv = mg$ …(i)

$\Rightarrow M = \dfrac{B + kv}{g}$



For the balloon to rise with a constant velocity v, (upward)

let the mass be m

Here, $B - (mg + kv) = 0$ …(ii)

$\Rightarrow B = mg + kv$

$\Rightarrow m = \dfrac{B - kw}{g}$

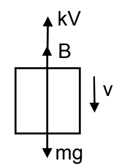


Fig-1

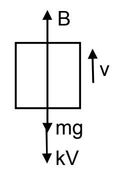


Fig-2

So, amount of mass that should be removed = M – m.

$= \dfrac{B + kv}{g} - \dfrac{B - kv}{g} = \dfrac{B + kv - B + kv}{g} = \dfrac{2kv}{g} = \dfrac{2(Mg - B)}{G} = 2\{M - (B/g)\}$

20. When the box is accelerating upward,

$U - mg - m(g/6) = 0$

$\Rightarrow U = mg + mg/6 = m\{g + (g/6)\} = 7\ mg/7$ …(i)

$\Rightarrow m = 6U/7g$.

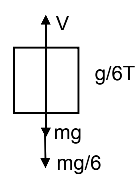


Fig-1

When it is accelerating downward, let the required mass be M.

$U - Mg + Mg/6 = 0$

$\Rightarrow U = \dfrac{6Mg - Mg}{6} = \dfrac{5Mg}{6} \Rightarrow M = \dfrac{6U}{5g}$

Mass to be added = $M - m = \dfrac{6U}{5g} - \dfrac{6U}{7g} = \dfrac{6U}{g}\left(\dfrac{1}{5} - \dfrac{1}{7}\right)$

$= \dfrac{6U}{g}\left(\dfrac{2}{35}\right) = \dfrac{12}{35}\left(\dfrac{U}{g}\right)$

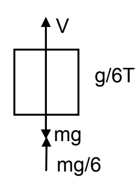


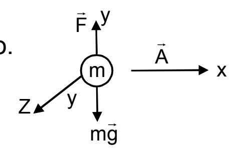
Fig-2

$= \dfrac{12}{35}\left(\dfrac{7mg}{6} \times \dfrac{1}{g}\right)$ from (i)

= 2/5 m.

$\therefore$ The mass to be added is 2m/5.

21. Given that, $\vec{F} = \vec{u} \times \vec{A}$ and $\overrightarrow{mg}$ act on the particle.



For the particle to move undeflected with constant velocity, net force should be zero.

$\therefore\ (\vec{u} \times \vec{A}) + \overrightarrow{mg} = 0$

$\therefore\ (\vec{u} \times \vec{A}) = -\overrightarrow{mg}$

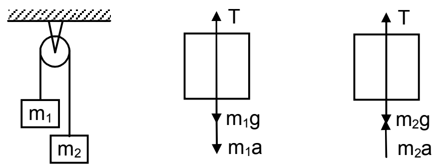
Because, $(\vec{u} \times \vec{A})$ is perpendicular to the plane containing $\vec{u}$ and $\vec{A}$, $\vec{u}$ should be in the xz-plane.

Again, u A sin θ = mg

$\therefore\ u = \dfrac{mg}{A \sin\theta}$

u will be minimum, when sin θ = 1 $\Rightarrow$ θ = 90°

$\therefore\ u_{min} = \dfrac{mg}{A}$ along Z-axis.

22. 

m_1 = 0.3 kg, m_2 = 0.6 kg

$T - (m_1g + m_1a) = 0$ …(i) $\Rightarrow T = m_1g + m_1a$

$T + m_2a - m_2g = 0$ …(ii) $\Rightarrow T = m_2g - m_2a$

From equation (i) and equation (ii)

$m_1g + m_1a + m_2a - m_2g = 0$, from (i)

$\Rightarrow a(m_1 + m_2) = g(m_2 - m_1)$

$\Rightarrow a = f\left(\dfrac{m_2 - m_1}{m_1 + m_2}\right) = 9.8\left(\dfrac{0.6 - 0.3}{0.6 + 0.3}\right) = 3.266\ ms^{-2}$.

a) t = 2 sec acceleration = 3.266 ms^{-2}

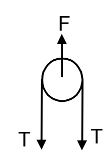
Initial velocity u = 0

So, distance travelled by the body is,

$S = ut + 1/2\ at^2 \Rightarrow 0 + ½(3.266)\ 2^2 = 6.5$ m

b) From (i) $T = m_1(g + a) = 0.3\ (9.8 + 3.26) = 3.9$ N

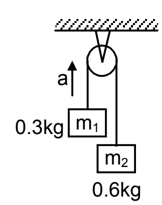
c) The force exerted by the clamp on the pully is given by



F – 2T = 0

F = 2T = 2 × 3.9 = 7.8 N.

23. a = 3.26 m/s^2



T = 3.9 N

After 2 sec mass m_1 the velocity

V = u + at = 0 + 3.26 × 2 = 6.52 m/s upward.

At this time m_2 is moving 6.52 m/s downward.

At time 2 sec, m_2 stops for a moment. But m_1 is moving upward with velocity 6.52 m/s.

It will continue to move till final velocity (at highest point) because zero.

Here, v = 0 ; u = 6.52

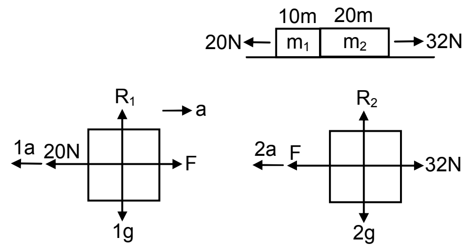
A = –g = –9.8 m/s^2 [moving up ward m_1]

$V = u + at \Rightarrow 0 = 6.52 + (-9.8)t$

$\Rightarrow t = 6.52/9.8 = 0.66 = 2/3$ sec.

During this period 2/3 sec, m_2 mass also starts moving downward. So the string becomes tight again after a time of 2/3 sec.

24. Mass per unit length 3/30 kg/cm = 0.10 kg/cm.
Mass of 10 cm part = m_1 = 1 kg
Mass of 20 cm part = m_2 = 2 kg.
Let, F = contact force between them.
From the free body diagram



$F – 20 – 10 = 0$ …(i)
And, $32 – F – 2a = 0$ …(ii)
From eqa (i) and (ii) $3a – 12 = 0 \Rightarrow a = 12/3 = 4\ m/s^2$
Contact force F = 20 + 1a = 20 + 1 × 4 = 24 N.

25.

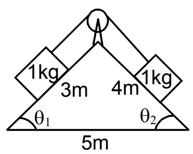


Fig-1

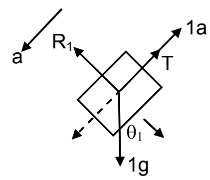


Fig-2

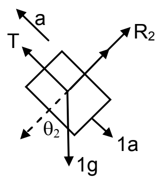


Fig-3

Sin θ_1 = 4/5
sin θ_2 = 3/5

$g \sin \theta_1 – (a + T) = 0$
$\Rightarrow g$ sing $\theta_1 = a + T$ …(i)
$\Rightarrow T + a – g \sin \theta_1 = 0$

$T – g \sin \theta_2 – a = 0$
$\Rightarrow T = g \sin \theta_2 + a$ …(ii)

From eqn (i) and (ii), $g \sin \theta_2 + a + a – g \sin \theta_1 = 0$

$\Rightarrow 2a = g \sin \theta_1 – g \sin \theta_2 = g \left(\frac{4}{5} - \frac{3}{5}\right) = g / 5$

$\Rightarrow a = \frac{g}{5} \times \frac{1}{2} = \frac{g}{10}$

26.

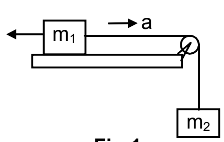


Fig-1

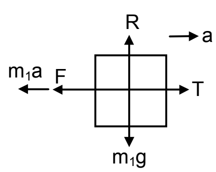


Fig-2

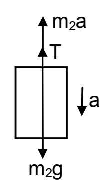


Fig-3

From the above Free body diagram
$M_1a + F – T = 0 \Rightarrow T = m_1a + F$ …(i)

From the above Free body diagram
$m_2a + T – m_2g = 0$ ….(ii)
$\Rightarrow m_2a + m_1a + F – m_2g = 0$ (from (i))
$\Rightarrow a(m_1 + m_2) + m_2g/2 – m_2g = 0$ {because $f = m^2g/2$}
$\Rightarrow a(m_1 + m_2) – m_2g = 0$
$\Rightarrow a(m_1 + m_2) = m_2g/2 \Rightarrow a = \frac{m_2 g}{2(m_1 = m_2)}$

Acceleration of mass m_1 is $\frac{m_2 g}{2(m_1 = m_2)}$ towards right.

27.

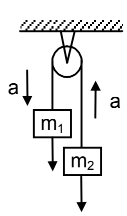


Fig-1

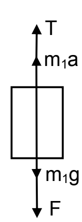


Fig-2

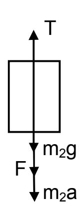


Fig-3

From the above free body diagram
$T + m_1a – m(m_1g + F) = 0$

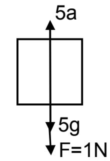
From the free body diagram
$T – (m_2g + F + m_2a) = 0$

$\Rightarrow T = m_1g + F - m_1a \Rightarrow T = 5g + 1 - 5a$ …(i)

$\Rightarrow T = m_2g + F + m_2a \Rightarrow T = 2g + 1 + 2a$ …(ii)

From the eqn (i) and eqn (ii)

$5g + 1 - 5a = 2g + 1 + 2a \Rightarrow 3g - 7a = 0 \Rightarrow 7a = 3g$



$\Rightarrow a = \frac{3g}{7} = \frac{29.4}{7} = 4.2\ m/s^2$ [g= 9.8m/s^2]

a) acceleration of block is 4.2 m/s^2

b) After the string breaks m_1 move downward with force F acting down ward.

$m_1a = F + m_1g = (1 + 5g) = 5(g + 0.2)$ Force = 1N, acceleration = 1/5= 0.2m/s.

So, acceleration = $\frac{\text{Force}}{\text{mass}} = \frac{5(g+0.2)}{5}$ = (g + 0.2) m/s^2

28.

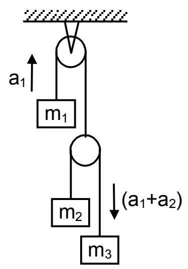


Fig-1

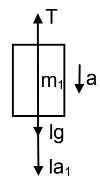


Fig-2

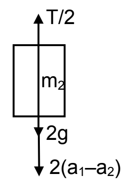


Fig-3

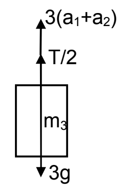


Fig-4

Let the block m+1+ moves upward with acceleration a, and the two blocks m_2 an m_3 have relative acceleration a_2 due to the difference of weight between them. So, the actual acceleration at the blocks m_1, m_2 and m_3 will be a_1.

$(a_1 - a_2)$ and $(a_1 + a_2)$ as shown

$T = 1g - 1a_2 = 0$ ...(i) from fig (2)

$T/2 - 2g - 2(a_1 - a_2) = 0$ ...(ii) from fig (3)

$T/2 - 3g - 3(a_1 + a_2) = 0$ ...(iii) from fig (4)

From eqn (i) and eqn (ii), eliminating T we get, $1g + 1a_2 = 4g + 4(a_1 + a_2) \Rightarrow 5a_2 - 4a_1 = 3g$ (iv)

From eqn (ii) and eqn (iii), we get $2g + 2(a_1 - a_2) = 3g - 3(a_1 - a_2) \Rightarrow 5a_1 + a_2 =$ (v)

Solving (iv) and (v) $a_1 = \frac{2g}{29}$ and $a_2 = g - 5a_1 = g - \frac{10g}{29} = \frac{19g}{29}$

So, $a_1 - a_2 = \frac{2g}{29} - \frac{19g}{29} = -\frac{17g}{29}$

$a_1 + a_2 = \frac{2g}{29} + \frac{19g}{29} = \frac{21g}{29}$ So, acceleration of m_1, m_2, m_3 ae $\frac{19g}{29}$(up) $\frac{17g}{29}$ (doan) $\frac{21g}{29}$(down) respectively.

Again, for m_1, u = 0, s= 20cm=0.2m and $a_2 = \frac{19}{29}g$ [g = 10m/s^2]

$\therefore S = ut + ½ at^2 = 0.2 = \frac{1}{2} \times \frac{19}{29} gt^2 \Rightarrow t = 0.25$sec.

29.

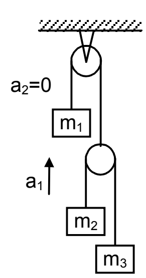


Fig-1

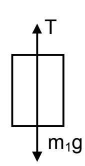


Fig-2

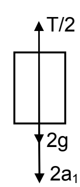


Fig-3

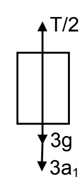


Fig-4

m_1 should be at rest.

$T - m_1g = 0$ $T/2 - 2g - 2a_1 = 0$ $T/2 - 3g - 3a_1 = 0$

$\Rightarrow T = m_1g$ …(i) $\Rightarrow T - 4g - 4a_1 = 0$ …(ii) $\Rightarrow T = 6g - 6a_1$ …(iii)

From eqn (ii) & (iii) we get

$3T - 12g = 12g - 2T \Rightarrow T = 24g/5 = 408g.$

Putting yhe value of T eqn (i) we get, $m_1 = 4.8$kg.

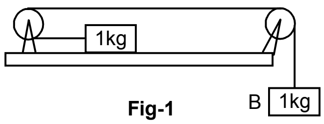
30. 

Fig-1

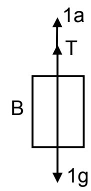


Fig-2

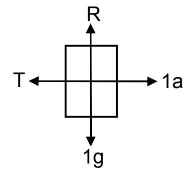


Fig-3

$T + 1a = 1g$ …(i) $T - 1a = 0 \Rightarrow T = 1a$ (ii)

From eqn (i) and (ii), we get

$$1a + 1a = 1g \Rightarrow 2a = g \Rightarrow a = \frac{g}{2} = \frac{10}{2} = 5\text{m/s}^2$$

From (ii) T = 1a = 5N.

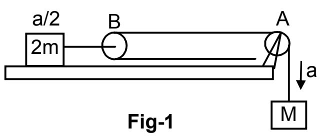
31. 

Fig-1

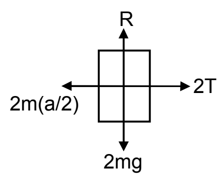


Fig-2

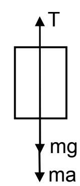


Fig-3

$Ma - 2T = 0$ $T + Ma - Mg = 0$

$\Rightarrow Ma = 2T \Rightarrow T = Ma/2.$ $\Rightarrow Ma/2 + ma = Mg.$ (because $T = Ma/2$)

$\Rightarrow 3\,Ma = 2\,Mg \Rightarrow a = 2g/3$

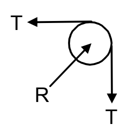
a) acceleration of mass M is 2g/3.

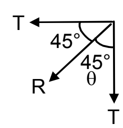
b) Tension $T = \frac{Ma}{2} = \frac{M}{2} = \frac{2g}{3} = \frac{Mg}{3}$

c) Let, R^1 = resultant of tensions = force exerted by the clamp on the pulley

$R^1 = \sqrt{T^2 + T^2} = \sqrt{2}T$

$\therefore R = \sqrt{2}T = \sqrt{2}\frac{Mg}{3} = \frac{\sqrt{2}Mg}{3}$





Again, $\tan\theta = \frac{T}{T} = 1 \Rightarrow \theta = 45°.$

So, it is $\frac{\sqrt{2}Mg}{3}$ at an angle of 45° with horizontal.

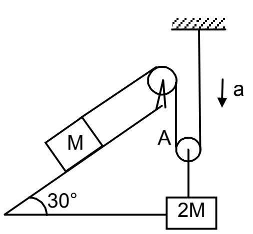
32. 

Fig-1

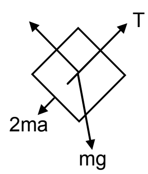


Fig-2

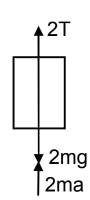


Fig-3

$2Ma + Mg \sin\theta - T = 0$

$\Rightarrow T = 2Ma + Mg \sin \theta$ …(i)

$2T + 2Ma - 2Mg = 0$

$\Rightarrow 2(2Ma + Mg \sin \theta) + 2Ma - 2Mg = 0$ [From (i)]

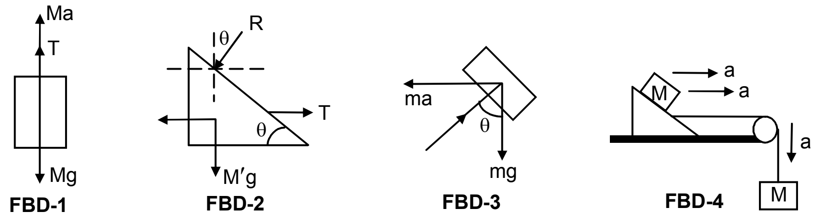
$\Rightarrow 4Ma + 2Mg\sin\theta + 2\,Ma - 2Mg = 0$

$\Rightarrow 6Ma + 2Mg \sin 30° - 2Mg = 0$

$\Rightarrow 6Ma = Mg \Rightarrow a = g/6$.

Acceleration of mass M is 2a = s × g/6 = g/3 up the plane.

33.



As the block 'm' does not slinover M′, ct will have same acceleration as that of M′

From the freebody diagrams.

$T + Ma - Mg = 0$ ...(i) (From FBD – 1)

$T - M'a - R \sin \theta = 0$ ...(ii) (From FBD -2)

$R \sin \theta - ma = 0$ ...(iii) (From FBD -3)

$R \cos \theta - mg = 0$ ...(iv) (From FBD -4)

Eliminating T, R and a from the above equation, we get $M = \dfrac{M' + m}{\cot\theta - 1}$

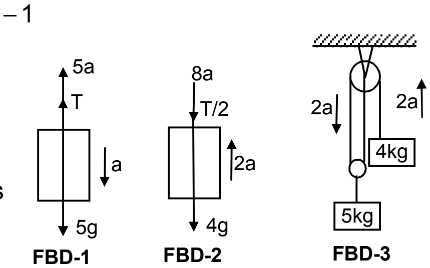
34. a) $5a + T - 5g = 0 \Rightarrow T = 5g - 5a$ ...(i) (From FBD-1)

Again $(1/2) - 4g - 8a = 0 \Rightarrow T = 8g - 16a$ ...(ii) (from FBD-2)

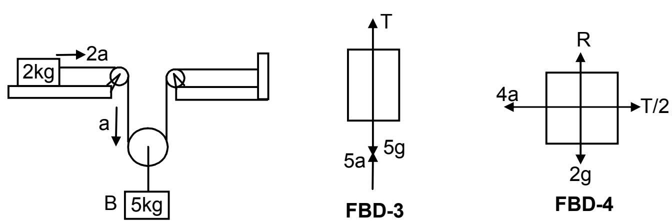
From equn (i) and (ii), we get

$5g - 5a = 8g + 16a \Rightarrow 21a = -3g \Rightarrow a = -1/7g$

So, acceleration of 5 kg mass is g/7 upward and that of 4 kg mass is 2a = 2g/7 (downward).



b)



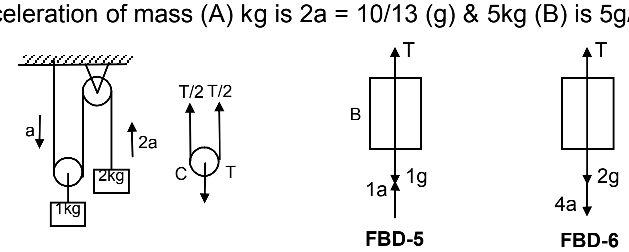
$4a - t/2 = 0 \Rightarrow 8a - T = 0 \Rightarrow T = 8a$ … (ii) [From FBD -4]

Again, $T + 5a - 5g = 0 \Rightarrow 8a + 5a - 5g = 0$

$\Rightarrow 13a - 5g = 0 \Rightarrow a = 5g/13$ downward. (from FBD -3)

Acceleration of mass (A) kg is 2a = 10/13 (g) & 5kg (B) is 5g/13.

c)



$T + 1a - 1g = 0 \Rightarrow T = 1g - 1a$ …(i) [From FBD – 5]

Again, $\dfrac{T}{2} - 2g - 4a = 0 \Rightarrow T - 4g - 8a = 0$ …(ii) [From FBD -6]

$\Rightarrow 1g - 1a - 4g - 8a = 0$ [From (i)]

$\Rightarrow$ a = –(g/3) downward.

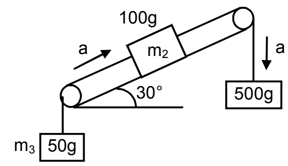
Acceleration of mass 1kg(b) is g/3 (up)

Acceleration of mass 2kg(A) is 2g/3 (downward).

35. m_1 = 100g = 0.1kg

m_2 = 500g = 0.5kg

m_3 = 50g = 0.05kg.



T + 0.5a – 0.5g = 0 ...(i)

T_1 – 0.5a – 0.05g = a ...(ii)

T_1 + 0.1a – T + 0.05g = 0 ...(iii)

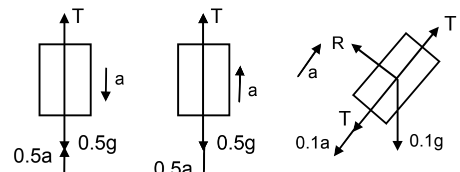
From equn (ii) T_1 = 0.05g + 0.05a ...(iv)

From equn (i) T_1 = 0.5g – 0.5a ...(v)

Equn (iii) becomes T_1 + 0.1a – T + 0.05g = 0

$\Rightarrow$ 0.05g + 0.05a + 0.1a – 0.5g + 0.5a + 0.05g = 0 [From (iv) and (v)]

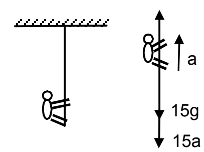
$\Rightarrow 0.65a = 0.4g \Rightarrow a = \frac{0.4}{0.65} = \frac{40}{65}g = \frac{8}{13}g$ downward



FBD-1 FBD-2 FBD-3

Acceleration of 500gm block is 8g/13g downward.

36. m = 15 kg of monkey. a = 1 m/s^2.



From the free body diagram

$\therefore$ T – [15g + 15(1)] = 0 $\Rightarrow$ T = 15 (10 + 1) $\Rightarrow$ T = 15 × 11 $\Rightarrow$ T = 165 N.

The monkey should apply 165N force to the rope.

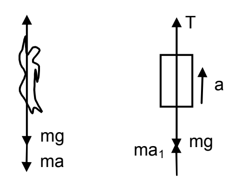
Initial velocity u = 0 ; acceleration a = 1m/s^2 ; s = 5m.

$\therefore$ s = ut + ½ at^2

5 = 0 + (1/2)1 t^2 $\Rightarrow$ t^2 = 5 × 2 $\Rightarrow$ t = $\sqrt{10}$ sec.

Time required is $\sqrt{10}$ sec.

37. Suppose the monkey accelerates upward with acceleration 'a' & the block, accelerate downward with acceleration a_1. Let Force exerted by monkey is equal to 'T'



From the free body diagram of monkey

$\therefore$ T – mg – ma = 0 ...(i)

$\Rightarrow$ T = mg + ma.

Again, from the FBD of the block,

T = ma_1 – mg = 0.

$\Rightarrow$ mg + ma + ma_1 – mg = 0 [From (i)] $\Rightarrow$ ma = –ma_1 $\Rightarrow$ a = a_1.

Acceleration '–a' downward i.e. 'a' upward.

$\therefore$The block & the monkey move in the same direction with equal acceleration.

If initially they are rest (no force is exertied by monkey) no motion of monkey of block occurs as they have same weight (same mass). Their separation will not change as time passes.

38. Suppose A move upward with acceleration a, such that in the tail of A maximum tension 30N produced.

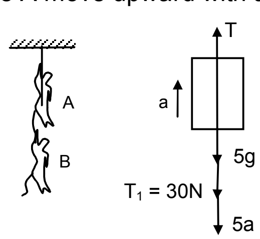


Fig-2

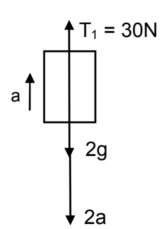


Fig-3

T – 5g – 30 – 5a = 0 ...(i)

$\Rightarrow$ T = 50 + 30 +(5 × 5) = 105 N (max)

30 – 2g – 2a = 0 ...(ii)

$\Rightarrow$ 30 – 20 – 2a = 0 $\Rightarrow$ a = 5 m/s^2

So, A can apply a maximum force of 105 N in the rope to carry the monkey B with it.

For minimum force there is no acceleration of monkey 'A' and B. $\Rightarrow$ a = 0

Now equation (ii) is $T'_1 - 2g = 0 \Rightarrow T'_1 = 20$ N (wt. of monkey B)

Equation (i) is $T - 5g - 20 = 0$ [As $T'_1 = 20$ N]

$\Rightarrow T = 5g + 20 = 50 + 20 = 70$ N.

$\therefore$ The monkey A should apply force between 70 N and 105 N to carry the monkey B with it.

39. (i) Given, Mass of man = 60 kg.

Let R′ = apparent weight of man in this case.

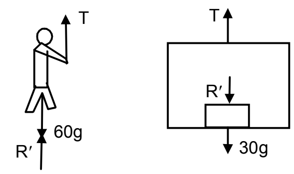
Now, $R' + T - 60g = 0$ [From FBD of man]

$\Rightarrow T = 60g - R'$ ...(i)

$T - R' - 30g = 0$ ...(ii) [From FBD of box]

$\Rightarrow 60g - R' - R' - 30g = 0$ [From (i)]

$\Rightarrow R' = 15g$ The weight shown by the machine is 15kg.



(ii) To get his correct weight suppose the applied force is 'T' and so, acclerates upward with 'a'.

In this case, given that correct weight = R = 60g, where g = acc^n due to gravity

From the FBD of the man

$T^1 + R - 60g - 60a = 0$

$\Rightarrow T^1 - 60a = 0$ [$\therefore R = 60g$]

$\Rightarrow T^1 = 60a$ ...(i)

From the FBD of the box

$T^1 - R - 30g - 30a = 0$

$\Rightarrow T^1 - 60g - 30g - 30a = 0$

$\Rightarrow T^1 - 30a = 90g = 900$

$\Rightarrow T^1 = 30a - 900$ ...(ii)

From eqn (i) and eqn (ii) we get $T^1 = 2T^1 - 1800 \Rightarrow T^1 = 1800$N.

$\therefore$ So, he should exert 1800 N force on the rope to get correct reading.

40. The driving force on the block which n the body to move sown the plane is F = mg sin θ,

So, acceleration = g sin θ

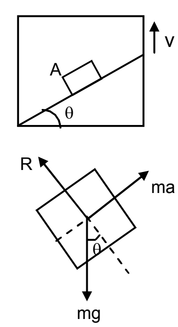
Initial velocity of block u = 0.

$s = \ell$, $a = g \sin\theta$

Now, $S = ut + ½ at^2$

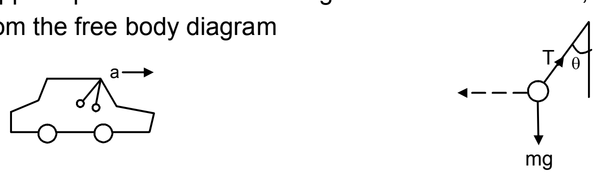
$\Rightarrow \ell = 0 + ½ (g \sin\theta) t^2 \Rightarrow g^2 = \dfrac{2\ell}{g\sin\theta} \Rightarrow t = \sqrt{\dfrac{2\ell}{g\sin\theta}}$

Time taken is $\sqrt{\dfrac{2\,\ell}{g\sin\theta}}$



41. Suppose pendulum makes θ angle with the vertical. Let, m = mass of the pendulum.

From the free body diagram



$T\cos\theta - mg = 0$

$\Rightarrow T\cos\theta = mg$

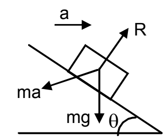
$\Rightarrow T = \dfrac{mg}{\cos\theta}$ ...(i)

$ma - T\sin\theta = 0$

$\Rightarrow ma = T\sin\theta$

$\Rightarrow t = \dfrac{ma}{\sin\theta}$ ...(ii)

From (i) & (ii) $\frac{mg}{\cos\theta} = \frac{ma}{\sin\theta} \Rightarrow \tan\theta = \frac{a}{g} \Rightarrow \theta = \tan^{-1}\frac{a}{g}$

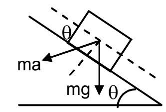


The angle is $\text{Tan}^{-1}(a/g)$ with vertical.

(ii) m → mass of block.

Suppose the angle of incline is 'θ'

From the diagram

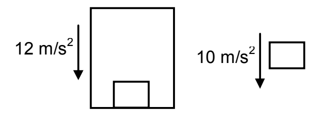


$ma\cos\theta - mg\sin\theta = 0 \Rightarrow ma\cos\theta = mg\sin\theta \Rightarrow \frac{\sin\theta}{\cos\theta} = \frac{a}{g}$

$\Rightarrow \tan\theta = a/g \Rightarrow \theta = \tan^{-1}(a/g)$.

42. Because, the elevator is moving downward with an acceleration 12 m/s^2 (>g), the bodygets separated. So, body moves with acceleration g = 10 m/s^2 [freely falling body] and the elevator move with acceleration 12 m/s^2

Now, the block has acceleration = g = 10 m/s^2



u = 0

t = 0.2 sec

So, the distance travelled by the block is given by.

$\therefore s = ut + ½ at^2$

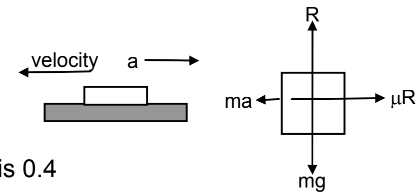
$= 0 + (½)\, 10\, (0.2)^2 = 5 \times 0.04 = 0.2$ m = 20 cm.

The displacement of body is 20 cm during first 0.2 sec.

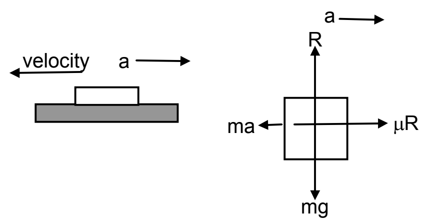
* * * *

SOLUTIONS TO CONCEPTS
CHAPTER 6

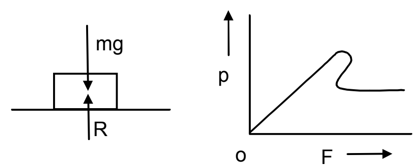
1. Let m = mass of the block
From the freebody diagram,
$R - mg = 0 \Rightarrow R = mg$ ...(1)
Again $ma - \mu R = 0 \Rightarrow ma = \mu R = \mu mg$ (from (1))
$\Rightarrow a = \mu g \Rightarrow 4 = \mu g \Rightarrow \mu = 4/g = 4/10 = 0.4$
The co-efficient of kinetic friction between the block and the plane is 0.4



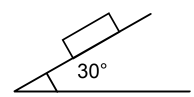
2. Due to friction the body will decelerate
Let the deceleration be 'a'
$R - mg = 0 \Rightarrow R = mg$ ...(1)
$ma - \mu R = 0 \Rightarrow ma = \mu R = \mu mg$ (from (1))
$\Rightarrow a = \mu g = 0.1 \times 10 = 1m/s^2.$
Initial velocity u = 10 m/s
Final velocity v = 0 m/s
$a = -1m/s^2$ (deceleration)
$S = \frac{v^2 - u^2}{2a} = \frac{0 - 10^2}{2(-1)} = \frac{100}{2} = 50m$
It will travel 50m before coming to rest.

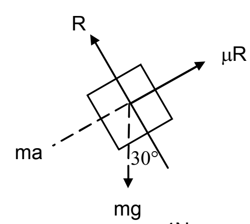


3. Body is kept on the horizontal table.
If no force is applied, no frictional force will be there
$f \rightarrow$ frictional force
$F \rightarrow$ Applied force
From grap it can be seen that when applied force is zero, frictional force is zero.

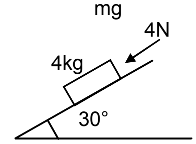


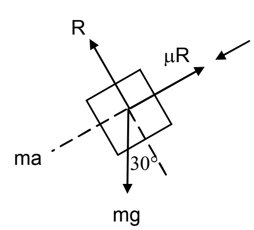
4. From the free body diagram,
$R - mg \cos\theta = 0 \Rightarrow R = mg \cos\theta$ ..(1)
For the block
U = 0, s = 8m, t = 2sec.
$\therefore s = ut + ½ at^2 \Rightarrow 8 = 0 + ½ a 2^2 \Rightarrow a = 4m/s^2$
Again, $\mu R + ma - mg \sin\theta = 0$
$\Rightarrow \mu mg \cos\theta + ma - mg \sin\theta = 0$ [from (1)]
$\Rightarrow m(\mu g \cos\theta + a - g \sin\theta) = 0$
$\Rightarrow \mu \times 10 \times \cos 30° = g \sin 30° - a$
$\Rightarrow \mu \times 10 \times \sqrt{(3/3)} = 10 \times (1/2) - 4$
$\Rightarrow (5/\sqrt{3})\mu = 1 \Rightarrow \mu = 1/(5/\sqrt{3}) = 0.11$
$\therefore$ Co-efficient of kinetic friction between the two is 0.11.



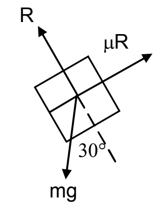


5. From the free body diagram
$4 - 4a - \mu R + 4g \sin 30° = 0$ …(1)
$R - 4g \cos 30° = 0$ ...(2)
$\Rightarrow R = 4g \cos 30°$
Putting the values of R is & in equn. (1)
$4 - 4a - 0.11 \times 4g \cos 30° + 4g \sin 30° = 0$
$\Rightarrow 4 - 4a - 0.11 \times 4 \times 10 \times (\sqrt{3}/2) + 4 \times 10 \times (1/2) = 0$
$\Rightarrow 4 - 4a - 3.81 + 20 = 0 \Rightarrow a \approx 5 m/s^2$
For the block u =0, t = 2sec, $a = 5m/s^2$
Distance $s = ut + ½ at^2 \Rightarrow s = 0 + (1/2)\, 5 \times 2^2 = 10m$
The block will move 10m.





6. To make the block move up the incline, the force should be equal and opposite to the net force acting down the incline = μ R + 2 g sin 30°



(body moving down)

$= 0.2 \times (9.8)\sqrt{3} + 2$ I $9.8 \times (1/2)$ [from (1)]

= 3.39 + 9.8 = 13N

With this minimum force the body move up the incline with a constant velocity as net force on it is zero.

b) Net force acting down the incline is given by,

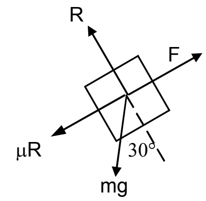
F = 2 g sin 30° – μR

= 2 × 9.8 × (1/2) – 3.39 = 6.41N

Due to F = 6.41N the body will move down the incline with acceleration.

No external force is required.

$\therefore$ Force required is zero.



(body moving us)

7. From the free body diagram

$g = 10 m/s^2$, m = 2kg, $\theta = 30°$, $\mu = 0.2$

$R - mg\cos\theta - F\sin\theta = 0$

$\Rightarrow R = mg\cos\theta + F\sin\theta$ ...(1)

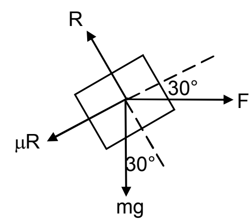
And $mg\sin\theta + \mu R - F\cos\theta = 0$

$\Rightarrow mg\sin\theta + \mu(mg\cos\theta + F\sin\theta) - F\cos\theta = 0$

$\Rightarrow mg\sin\theta + \mu\, mg\cos\theta + \mu F\sin\theta - F\cos\theta = 0$

$$\Rightarrow F = \frac{(mg\sin\theta - \mu mg\cos\theta)}{(\mu\sin\theta - \cos\theta)}$$

$$\Rightarrow F = \frac{2\times10\times(1/2)+0.2\times2\times10\times(\sqrt{3}/2)}{0.2\times(1/2)-(\sqrt{3}/2)} = \frac{13.464}{0.76} = 17.7N \approx 17.5N$$



8. m $\rightarrow$ mass of child

$R - mg\cos 45° = 0$

$\Rightarrow R = mg\cos 45° = mg/v^2$ ...(1)

Net force acting on the boy due to which it slides down is mg sin 45° - μR

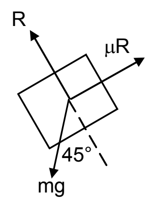
= mg sin 45° - μ mg cos 45°

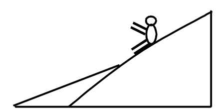
$= m \times 10\,(1/\sqrt{2}) - 0.6 \times m \times 10 \times (1/\sqrt{2})$

$= m\,[(5/\sqrt{2}) - 0.6 \times (5/\sqrt{2})]$

$= m(2\sqrt{2})$

$$\text{acceleration} = \frac{\text{Force}}{\text{mass}} = \frac{m(2\sqrt{2})}{m} = 2\sqrt{2}\ m/s^2$$





9. Suppose, the body is accelerating down with acceleration 'a'.

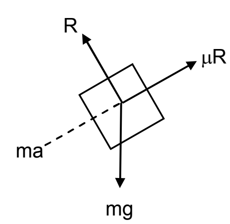
From the free body diagram

$R - mg\cos\theta = 0$

$\Rightarrow R = mg\cos\theta$ ...(1)

$ma + mg\sin\theta - \mu R = 0$

$$\Rightarrow a = \frac{mg(\sin\theta - \mu\cos\theta)}{m} = g(\sin\theta - \mu\cos\theta)$$



For the first half mt. u = 0, s = 0.5m, t = 0.5 sec.

So, v = u + at = 0 + (0.5)4 = 2 m/s

$S = ut + \frac{1}{2}at^2 \Rightarrow 0.5 = 0 + \frac{1}{2}a(0/5)^2 \Rightarrow a = 4 m/s^2$ ...(2)

For the next half metre

u\` = 2m/s, $a = 4 m/s^2$, s= 0.5.

$\Rightarrow 0.5 = 2t + (1/2)\,4\,t^2 \Rightarrow 2t^2 + 2t - 0.5 = 0$

$\Rightarrow 4t^2 + 4t - 1 = 0$

$\therefore = \frac{-4 \pm \sqrt{16+16}}{2 \times 4} = \frac{1.656}{8} = 0.207sec$

Time taken to cover next half meter is 0.21sec.

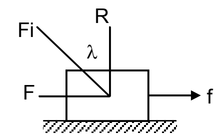
10. $f \rightarrow$ applied force

$F_i \rightarrow$ contact force

$F \rightarrow$ frictional force

$R \rightarrow$ normal reaction

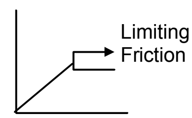
$\mu = \tan\lambda = F/R$



When F = μR, F is the limiting friction (max friction). When applied force increase, force of friction increase upto limiting friction (μR)

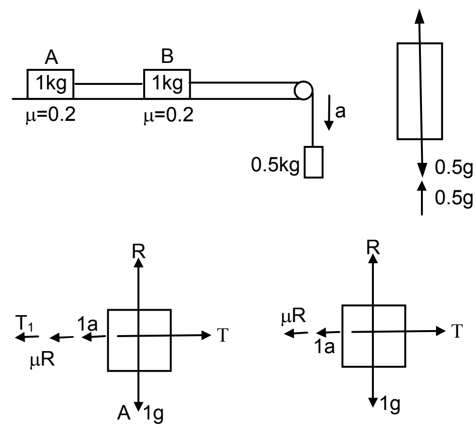
Before reaching limiting friction

$F < \mu R$



$\therefore \tan\lambda = \frac{F}{R} \le \frac{\mu R}{R} \Rightarrow \tan\lambda \le \mu \Rightarrow \lambda \le \tan^{-1}\mu$

11. From the free body diagram



$T + 0.5a - 0.5g = 0$ ...(1)

$\mu R + 1a + T_1 - T = 0$ ...(2)

$\mu R + 1a - T_1 = 0$

$\mu R + 1a = T_1$ ...(3)

From (2) & (3) $\Rightarrow \mu R + a = T - T_1$

$\therefore T - T_1 = T_1$

$\Rightarrow T = 2T_1$

Equation (2) becomes $\mu R + a + T_1 - 2T_1 = 0$

$\Rightarrow \mu R + a - T_1 = 0$

$\Rightarrow T_1 = \mu R + a = 0.2g + a$ ...(4)

Equation (1) becomes $2T_1 + 0/5a - 0.5g = 0$

$\Rightarrow T_1 = \frac{0.5g - 0.5a}{2} = 0.25g - 0.25a$ ...(5)

From (4) & (5) $0.2g + a = 0.25g - 0.25a$

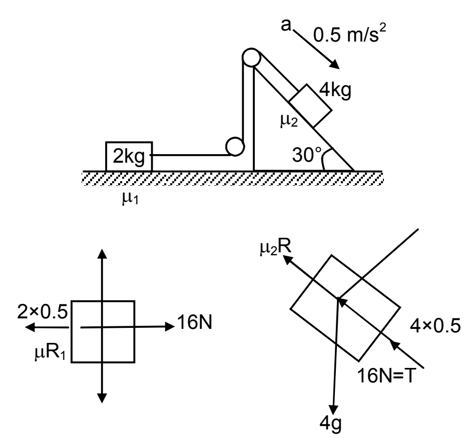
$\Rightarrow a = \frac{0.05}{1.25} \times 10 = 0.04 \text{ I } 10 = 0.4m/s^2$

a) Accln of 1kg blocks each is $0.4m/s^2$

b) Tension $T_1 = 0.2g + a + 0.4 = 2.4N$

c) $T = 0.5g - 0.5a = 0.5 \times 10 - 0.5 \times 0.4 = 4.8N$

12. From the free body diagram



$\mu_1 R + 1 - 16 = 0$

$\Rightarrow \mu_1 (2g) + (-15) = 0$

$\Rightarrow \mu_1 = 15/20 = 0.75$

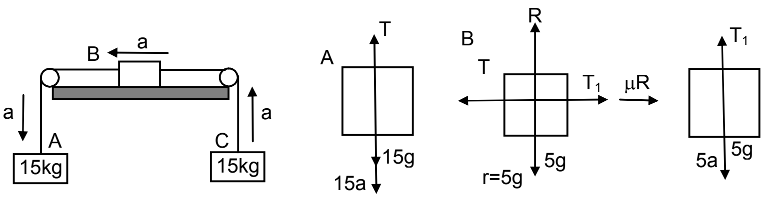
$\mu_2 R_1 + 4 \times 0.5 + 16 - 4g \sin 30° = 0$

$\Rightarrow \mu_2 (20\sqrt{3}) + 2 + 16 - 20 = 0$

$\Rightarrow \mu_2 = \frac{2}{20\sqrt{3}} = \frac{1}{17.32} = 0.057 \approx 0.06$

$\therefore$Co-efficient of friction $\mu_1 = 0.75$ & $\mu_2 = 0.06$

13.



From the free body diagram

$T + 15a - 15g = 0$

$\Rightarrow T = 15g - 15a$ ...(i)

$T - (T_1 + 5a + \mu R) = 0$

$\Rightarrow T - (5g + 5a + 5a + \mu R) = 0$

$\Rightarrow T = 5g + 10a + \mu R$ …(ii)

$T_1 - 5g - 5a = 0$

$\Rightarrow T_1 = 5g + 5a$ …(iii)

From (i) & (ii) $15g - 15a = 5g + 10a + 0.2\ (5g)$

$\Rightarrow 25a = 90 \Rightarrow a = 3.6\text{m/s}^2$

Equation (ii) $\Rightarrow T = 5 \times 10 + 10 \times 3.6 + 0.2 \times 5 \times 10$

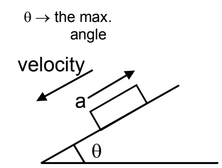
$\Rightarrow$ 96N in the left string

Equation (iii) $T_1 = 5g + 5a = 5 \times 10 + 5 \times 3.6 =$ 68N in the right string.

14. $s = 5\text{m}$, $\mu = 4/3$, $g = 10\text{m/s}^2$

$u = 36\text{km/h} = 10\text{m/s}$, $v = 0$,

$$a = \frac{v^2 - u^2}{2s} = \frac{0 - 10^2}{2 \times 5} = -10\text{m/s}^2$$



From the freebody diagrams,

$R - mg\cos\theta = 0$; $g = 10\text{m/s}^2$

$\Rightarrow R = mg\cos\theta$ ….(i) ; $\mu = 4/3$.

Again, $ma + mg\sin\theta - \mu R = 0$

$\Rightarrow ma + mg\sin\theta - \mu\, mg\cos\theta = 0$

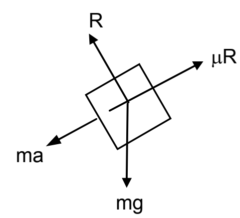
$\Rightarrow a + g\sin\theta - mg\cos\theta = 0$

$\Rightarrow 10 + 10\sin\theta - (4/3) \times 10\cos\theta = 0$

$\Rightarrow 30 + 30\sin\theta - 40\cos\theta = 0$

$\Rightarrow 3 + 3\sin\theta - 4\cos\theta = 0$

$\Rightarrow 4\cos\theta - 3\sin\theta = 3$



$\Rightarrow 4\sqrt{1 - \sin^2\theta} = 3 + 3\sin\theta$

$\Rightarrow 16\ (1 - \sin^2\theta) = 9 + 9\sin^2\theta + 18\sin\theta$

$$\sin\theta = \frac{-18 \pm \sqrt{18^2 - 4(25)(-7)}}{2 \times 25} = \frac{-18 \pm 32}{50} = \frac{14}{50} = 0.28 \text{ [Taking +ve sign only]}$$

$\Rightarrow \theta = \sin^{-1}(0.28) = 16°$

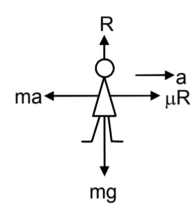
Maximum incline is $\theta = 16°$

15. to reach in minimum time, he has to move with maximum possible acceleration.

Let, the maximum acceleration is 'a'

$\therefore ma - \mu R = 0 \Rightarrow ma = \mu\, mg$

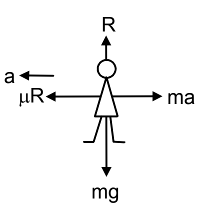
$\Rightarrow a = \mu g = 0.9 \times 10 = 9\text{m/s}^2$



a) Initial velocity $u = 0$, $t = ?$

$a = 9\text{m/s}^2$, $s = 50\text{m}$

$$s = ut + \tfrac{1}{2}at^2 \Rightarrow 50 = 0 + (1/2)\, 9\, t^2 \Rightarrow t = \sqrt{\frac{100}{9}} = \frac{10}{3} \text{ sec.}$$



b) After overing 50m, velocity of the athelete is

$V = u + at = 0 + 9 \times (10/3) = 30\text{m/s}$

He has to stop in minimum time. So deceleration ia $-a = -9\text{m/s}^2$ (max)

$$\begin{bmatrix} R = ma \\ ma = \mu R(\text{max frictional force}) \\ \Rightarrow a = \mu g = 9m/s^2(\text{Deceleration}) \end{bmatrix}$$

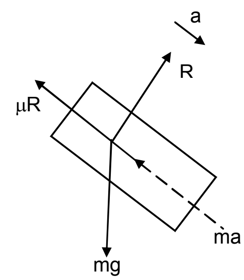
u^1 = 30m/s, v^1 = 0

$t = \frac{v^1 - u^1}{a} = \frac{0-30}{-a} = \frac{-30}{-a} = \frac{10}{3}$ sec.

16. Hardest brake means maximum force of friction is developed between car's type & road.

Max frictional force = μR

From the free body diagram



$R - mg\cos\theta = 0$

$\Rightarrow R = mg\cos\theta$ ...(i)

and $\mu R + ma - mg \sin$) = 0 …(ii)

$\Rightarrow \mu mg\cos\theta + ma - mg\sin\theta = 0$

$\Rightarrow \mu g\cos\theta + a - 10 \times (1/2) = 0$

$\Rightarrow a = 5 - \{1 - (2\sqrt{3})\} \times 10\ (\sqrt{3}/2) = 2.5\ m/s^2$

When, hardest brake is applied the car move with acceleration 2.5m/s^2

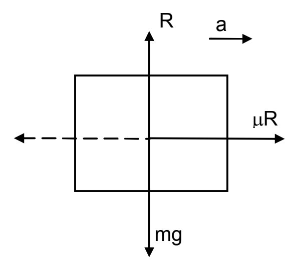
S = 12.8m, u = 6m/s

S0, velocity at the end of incline

$V = \sqrt{u^2 + 2as} = \sqrt{6^2 + 2(2.5)(12.8)} = \sqrt{36 + 64}$ = 10m/s = 36km/h

Hence how hard the driver applies the brakes, that car reaches the bottom with least velocity 36km/h.

17. Let, , a maximum acceleration produced in car.



$\therefore ma = \mu R$ [For more acceleration, the tyres will slip]

$\Rightarrow ma = \mu\ mg \Rightarrow a = \mu g = 1 \times 10 = 10m/s^2$

For crossing the bridge in minimum time, it has to travel with maximum acceleration

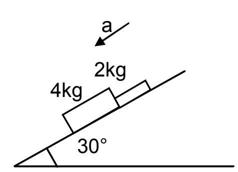
u = 0, s = 500m, a = 10m/s^2

$s = ut + ½\ at^2$

$\Rightarrow 500 = 0 + (1/2)\ 10\ t^2 \Rightarrow t = 10$ sec.

If acceleration is less than 10m/s^2, time will be more than 10sec. So one can't drive through the bridge in less than 10sec.

18. From the free body diagram



$R = 4g\cos 30° = 4 \times 10 \times \sqrt{3}/2 = 20\sqrt{3}$ ...(i)

$\mu_2 R + 4a - P - 4g\sin 30° = 0 \Rightarrow 0.3\ (40)\cos 30° + 4a - P - 40\sin 20° = 0$ …(ii)

$P + 2a + \mu_1 R_1 - 2g\sin 30° = 0$ …(iii)

$R_1 = 2g\cos 30° = 2 \times 10 \times \sqrt{3}/2 = 10\sqrt{3}$ ...(iv)

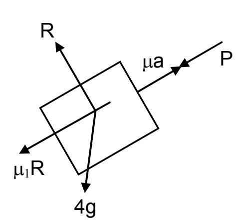
Equn. (ii) $6\sqrt{3} + 4a - P - 20 = 0$

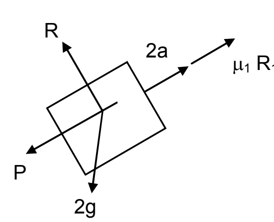
Equn (iv) $P + 2a + 2\sqrt{3} - 10 = 0$

From Equn (ii) & (iv) $6\sqrt{3} + 6a - 30 + 2\sqrt{3} = 0$

$\Rightarrow 6a = 30 - 8\sqrt{3} = 30 - 13.85 = 16.15$

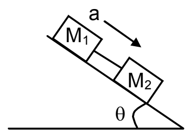
$\Rightarrow a = \frac{16.15}{6} = 2.69 = 2.7m/s^2$

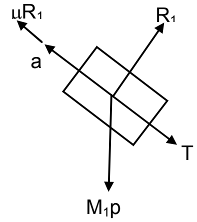


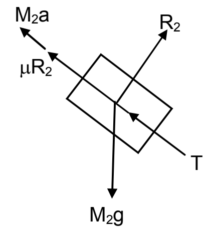


b) can be solved. In this case, the 4 kg block will travel with more acceleration because, coefficient of friction is less than that of 2kg. So, they will move separately. Drawing the free body diagram of 2kg mass only, it can be found that, a = 2.4m/s^2.

19. From the free body diagram







$R_1 = M_1 g \cos\theta$ ...(i)

$R_2 = M_2 g \cos\theta$ ...(ii)

$T + M_1 g \sin\theta - m_1 a - \mu R_1 = 0$ ...(iii)

$T - M_2 - M_2 a + \mu R_2 = 0$ ...(iv)

Equn (iii) $\Rightarrow$ $T + M_1 g \sin\theta - M_1 a - \mu M_1 g \cos\theta = 0$

Equn (iv) $\Rightarrow$ $T - M_2 g \sin\theta + M_2 a + \mu M_2 g \cos\theta = 0$ ...(v)

Equn (iv) & (v) $\Rightarrow$ $g \sin\theta (M_1 + M_2) - a(M_1 + M_2) - \mu g \cos\theta (M_1 + M_2) = 0$

$\Rightarrow a (M_1 + M_2) = g \sin\theta (M_1 + M_2) - \mu g \cos\theta (M_1 + M_2)$

$\Rightarrow a = g(\sin\theta - \mu \cos\theta)$

$\therefore$ The blocks (system has acceleration $g(\sin\theta - \mu\cos\theta)$

The force exerted by the rod on one of the blocks is tension.

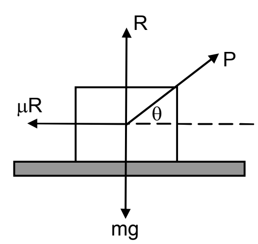
Tension $T = -M_1 g \sin\theta + M_1 a + \mu M_1 g \sin\theta$

$\Rightarrow T = -M_1 g \sin\theta + M_1(g \sin\theta - \mu g \cos\theta) + \mu M_1 g \cos\theta$

$\Rightarrow T = 0$

20. Let 'p' be the force applied to at an angle θ

From the free body diagram



$R + P \sin\theta - mg = 0$

$\Rightarrow R = -P \sin\theta + mg$ ...(i)

$\mu R - p \cos\theta$ ...(ii)

Equn. (i) is $\mu(mg - P \sin\theta) - P \cos\theta = 0$

$\Rightarrow \mu mg = \mu \rho \sin\theta - P \cos\theta \Rightarrow \rho = \dfrac{\mu mg}{\mu \sin\theta + \cos\theta}$

Applied force P should be minimum, when $\mu \sin\theta + \cos\theta$ is maximum.

Again, $\mu \sin\theta + \cos\theta$ is maximum when its derivative is zero.

$\therefore d/d\theta\,(\mu \sin\theta + \cos\theta) = 0$

$\Rightarrow \mu \cos\theta - \sin\theta = 0 \Rightarrow \theta = \tan^{-1}\mu$

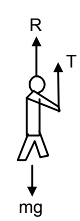
$$\text{So, } P = \frac{\mu mg}{\mu \sin\theta + \cos\theta} = \frac{\mu mg/\cos\theta}{\frac{\mu \sin\theta}{\cos\theta} + \frac{\cos\theta}{\cos\theta}} = \frac{\mu mg \sec\theta}{1 + \mu \tan\theta} = \frac{\mu mg \sec\theta}{1 + \tan^2\theta}$$

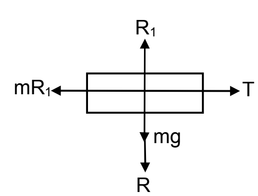
$$= \frac{\mu mg}{\sec\theta} = \frac{\mu mg}{\sqrt{(1 + \tan^2\theta}} = \frac{\mu mg}{\sqrt{1 + \mu^2}}$$

Minimum force is $\dfrac{\mu mg}{\sqrt{1+\mu^2}}$ at an angle $\theta = \tan^{-1}\mu$.

21. Let, the max force exerted by the man is T.

From the free body diagram





$R + T - Mg = 0$

$\Rightarrow R = Mg - T$ ...(i)

$R_1 - R - mg = 0$

$\Rightarrow R_1 = R + mg$ ...(ii)

And $T - \mu R_1 = 0$

$\Rightarrow T - \mu (R + mg) = 0$ [From equn. (ii)]

$\Rightarrow T - \mu R - \mu mg = 0$

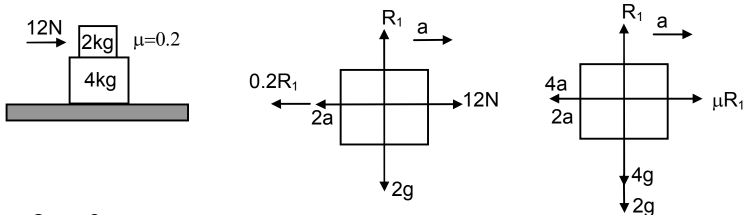
$\Rightarrow T - \mu (Mg + T) - \mu mg = 0$ [from (i)]

$\Rightarrow T (1 + \mu) = \mu Mg + \mu mg$

$\Rightarrow T = \frac{\mu(M+m)g}{1+\mu}$

Maximum force exerted by man is $\frac{\mu(M+m)g}{1+\mu}$

22.



$R_1 - 2g = 0$

$\Rightarrow R_1 = 2 \times 10 = 20$

$2a + 0.2 R_1 - 12 = 0$

$\Rightarrow 2a + 0.2(20) = 12$

$\Rightarrow 2a = 12 - 4 = 8$

$\Rightarrow a = 4m/s^2$

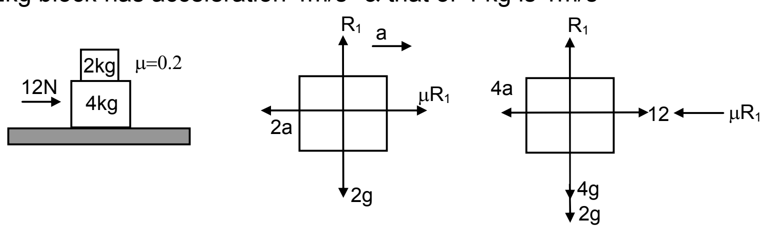
$4a_1 - \mu R_1 = 0$

$\Rightarrow 4a_1 = \mu R_1 = 0.2 (20)$

$\Rightarrow 4a_1 = 4$

$\Rightarrow a_1 = 1m/s^2$

2kg block has acceleration $4m/s^2$ & that of 4 kg is $1m/s^2$



(ii) $R_1 = 2g = 20$

$Ma - \mu R_1 = 0$

$\Rightarrow 2a = 0.2 (20) = 4$

$\Rightarrow a = 2m/s^2$

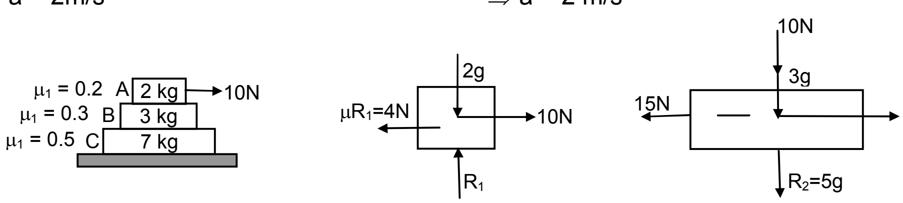
$4a + 0.2 \times 2 \times 10 - 12 = 0$

$\Rightarrow 4a + 4 = 12$

$\Rightarrow 4a = 8$

$\Rightarrow a = 2 m/s^2$

23.



a) When the 10N force applied on 2kg block, it experiences maximum frictional force

$\mu R_1 = \mu \times 2kg = (0.2) \times 20 = 4N$ from the 3kg block.

So, the 2kg block experiences a net force of 10 – 4 = 6N

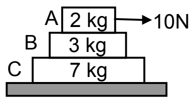
So, $a_1 = 6/2 = 3 m/s^2$

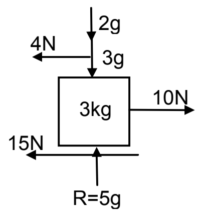
But for the 3kg block, (fig-3) the frictional force from 2kg block (4N) becomes the driving force and the maximum frictional force between 3kg and 7 kg block is

$\mu_2 R_2 = (0.3) \times 5kg = 15N$

So, the 3kg block cannot move relative to the 7kg block. The 3kg block and 7kg block both will have same acceleration ($a_2 = a_3$) which will be due to the 4N force because there is no friction from the floor.

$\therefore a_2 = a_3 = 4/10 = 0.4m/s^2$





b) When the 10N force is applied to the 3kg block, it can experience maximum frictional force of 15 + 4 = 19N from the 2kg block & 7kg block.

So, it can not move with respect to them.

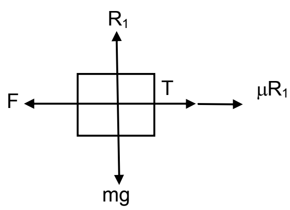
As the floor is frictionless, all the three bodies will move together

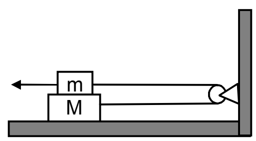
$\therefore a_1 = a_2 = a_3 = 10/12 = (5/6)m/s^2$

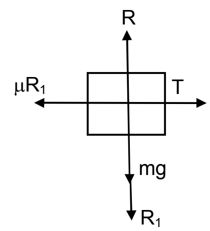
c) Similarly, it can be proved that when the 10N force is applied to the 7kg block, all the three blocks will move together.

Again $a_1 = a_2 = a_3 = (5/6)m/s^2$

24. Both upper block & lower block will have acceleration $2m/s^2$







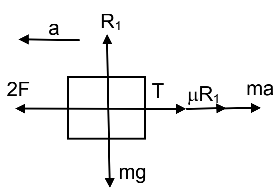
$R_1 = mg$ ...(i)

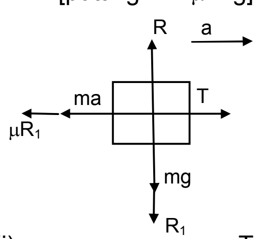
$F - \mu R_1 - T = 0 \Rightarrow F - \mu mg - T = 0$ ...(ii)

$\therefore F = \mu\, mg + \mu\, mg = 2\,\mu\, mg$ [putting $T = \mu$ mg]

$T - \mu R_1 = 0$

$\Rightarrow T = \mu mg$





b) $2F - T - \mu\, mg - ma = 0$ …(i)

$T - Ma - \mu\, mg = 0$ [$\therefore R_1 = mg$]

$\Rightarrow T = Ma + \mu\, mg$

Putting value of T in (i)

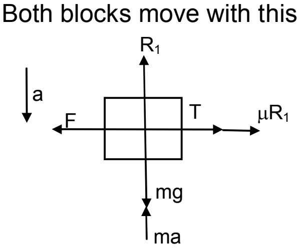
$2f - Ma - \mu mg - \mu\, mg - ma = 0$

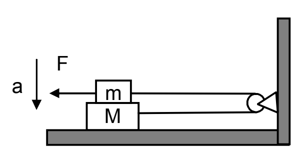
$\Rightarrow 2(2\mu mg) - 2\,\mu\, mg = a(M + m)$ [Putting $F = 2\,\mu mg$]

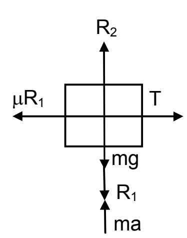
$\Rightarrow 4\mu\, mg - 2\,\mu\, mg = a\,(M + m)$ $\Rightarrow a = \dfrac{2\mu mg}{M+m}$

Both blocks move with this acceleration 'a' in opposite direction.

25.







$T = mR_1$
$= m\,(mg–ma)$

$R_1 + ma - mg = 0$

$\Rightarrow R_1 = m(g–a) = mg - ma$ ...(i)

$T - \mu\, R_1 = 0 \Rightarrow T = m\,(mg - ma)$ ...(ii)

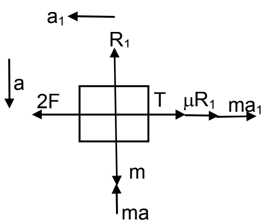
Again, $F - T - \mu\, R_1 = 0$

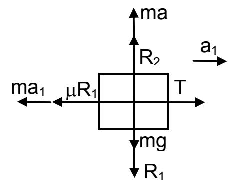
$\Rightarrow F - \{\mu(mg - ma)\} - u(mg - ma) = 0$

$\Rightarrow F - \mu\, mg + \mu\, ma - \mu\, mg + \mu\, ma = 0$

$\Rightarrow F = 2\,\mu\, mg - 2\mu\, ma \Rightarrow F = 2\mu\, m(g - a)$

b) Acceleration of the block be a_1





$R_1 = mg - ma$ ...(i)

$2F - T - \mu R_1 - ma_1 = 0$

$\Rightarrow 2F - t - \mu mg + \mu a - ma_1 = 0$ ...(ii)

$T - \mu R_1 - M\, a_1 = 0$

$\Rightarrow T = \mu R_1 + M\, a_1$

$\Rightarrow T = \mu\,(mg - ma) + Ma_1$

$\Rightarrow T = \mu\, mg - \mu\, ma + M\, a_1$

Subtracting values of F & T, we get

$2(2\mu m(g - a)) - 2(\mu mg - \mu ma + Ma_1) - \mu mg + \mu\, ma - \mu\, a_1 = 0$

$\Rightarrow 4\mu\, mg - 4\,\mu\, ma - 2\,\mu\, mg + 2\mu\, ma = ma_1 + M\, a_1$ $\Rightarrow a_1 = \dfrac{2\mu m(g - a)}{M + m}$

Both blocks move with this acceleration but in opposite directions.

26. $R_1 + QE - mg = 0$

$R_1 = mg - QE$ ...(i)

$F - T - \mu R_1 = 0$

$\Rightarrow F - T\ \ \mu(mg - QE) = 0$

$\Rightarrow F - T - \mu\, mg + \mu QE = 0$ …(2)

$T - \mu\, R_1 = 0$

$\Rightarrow T = \mu\, R_1 = \mu\,(mg - QE) = \mu\, mg - \mu QE$

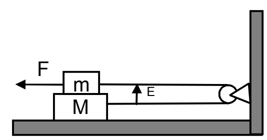
Now equation (ii) is $F - mg + \mu\, QE - \mu\, mg + \mu\, QE = 0$

$\Rightarrow F - 2\,\mu\, mg + 2\mu\, QE = 0$

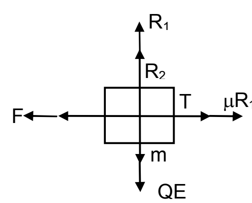
$\Rightarrow F = 2\mu mg - 2\mu\, QE$

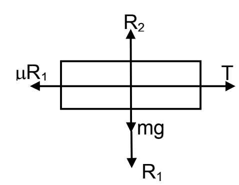
$\Rightarrow F = 2\mu(mg - QE)$

Maximum horizontal force that can be applied is $2\mu(mg - QE)$.



F=QE





27. Because the block slips on the table, maximum frictional force acts on it.

From the free body diagram

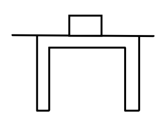
$R = mg$

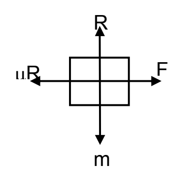
$\therefore F - \mu\, R = 0 \Rightarrow F = \mu R = \mu\, mg$

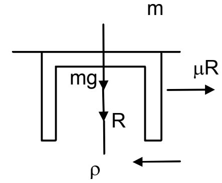
But the table is at rest. So, frictional force at the legs of the table is not $\mu\, R_1$. Let be f, so form the free body diagram.

$f_o - \mu\, R = 0 \Rightarrow f_o = \mu R = \mu\, mg.$

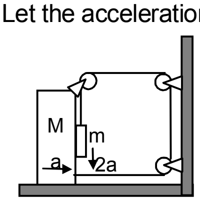
Total frictional force on table by floor is μ mg.

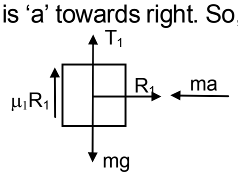




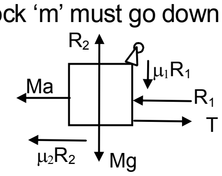


28. Let the acceleration of block M is ‘a’ towards right. So, the block ‘m’ must go down with an acceleration ‘2a’.





(FBD-1)



(FBD-2)

As the block ‘m’ is in contact with the block ‘M’, it will also have acceleration ‘a’ towards right. So, it will experience two inertia forces as shown in the free body diagram-1.

From free body diagram -1

$R_1 – ma = 0 \Rightarrow R_1 = ma$ ...(i)

Again, $2ma + T – mg + \mu_1 R_1 = 0$

$\Rightarrow T = mg – (2 – \mu_1)ma$ …(ii)

From free body diagram-2

$T + \mu_1 R_1 + mg – R_2 = 0$

$\Rightarrow R_2 = T + \mu_1 \, ma + Mg$ [Putting the value of R_1 from (i)]

$= (mg – 2ma – \mu_1 \, ma) + \mu_1 \, ma + Mg$ [Putting the value of T from (ii)]

$\therefore R_2 = Mg + mg – 2ma$ …(iii)

Again, form the free body diagram -2

$T + T – R – Ma – \mu_2 R_2 = 0$

$\Rightarrow 2T – MA – mA – \mu_2 (Mg + mg – 2ma) = 0$ [Putting the values of R_1 and R_2 from (i) and (iii)]

$\Rightarrow 2T = (M + m) + \mu_2(Mg + mg – 2ma)$ ...(iv)

From equation (ii) and (iv)

$2T = 2\,mg – 2(2 + \mu_1)mg = (M + m)a + \mu_2(Mg + mg – 2ma)$

$\Rightarrow 2mg – \mu_2(M + m)g = a\,(M + m – 2\mu_2 m + 4m + 2\mu_1 m)$

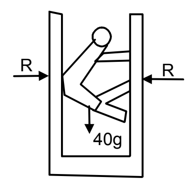
$\Rightarrow a = \dfrac{[2m - \mu_2(M+m)]g}{M + m[5 + 2(\mu_1 - \mu_2)]}$

29. Net force = *(202 + (15)2 – (0.5) × 40 = 25 – 20 = 5N

$\therefore \tan\theta = 20/15 = 4/3 \Rightarrow \mu = \tan^{-1}(4/3) = 53°$

So, the block will move at an angle 53 ° with an 15N force

30. a) Mass of man = 50kg. $g = 10\ m/s^2$



Frictional force developed between hands, legs & back side with the wall the wt of man. So he remains in equilibrium.

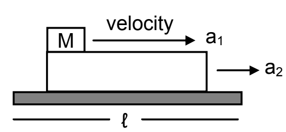
He gives equal force on both the walls so gets equal reaction R from both the walls. If he applies unequal forces R should be different he can't rest between the walls. Frictional force $2\mu R$ balance his wt.

From the free body diagram

$\mu R + \mu R = 40g \Rightarrow 2\,\mu R = 40 \times 10 \Rightarrow R = \dfrac{40 \times 10}{2 \times 0.8} = 250N$

b) The normal force is 250 N.

31. Let a_1 and a_2 be the accelerations of ma and M respectively.



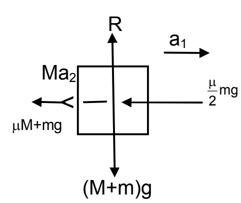
Here, $a_1 > a_2$ so that m moves on M

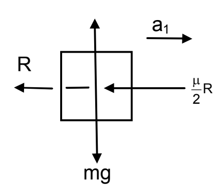
Suppose, after time 't' m separate from M.

In this time, m covers $vt + ½\, a_1 t^2$ and $S_M = vt + ½\, a_2 t^2$

For 'm' to m to 'm' separate from M. $vt + ½\, a_1 t^2 = vt + ½\, a_2 t^2 + \ell$ ...(1)

Again from free body diagram





$Ma_1 + \mu/2\, R = 0$

$\Rightarrow ma_1 = – (\mu/2)\, mg = – (\mu/2)m \times 10 \Rightarrow a_1 = –5\mu$

Again,

$Ma_2 + \mu\,(M + m)g – (\mu/2)mg = 0$

$\Rightarrow 2Ma_2 + 2\mu\,(M + m)g – \mu\, mg = 0$

$\Rightarrow 2\,M\,a_2 = \mu\, mg – 2\mu Mg – 2\,\mu mg$

$\Rightarrow a_2 \ \dfrac{-\mu mg - 2\mu Mg}{2M}$

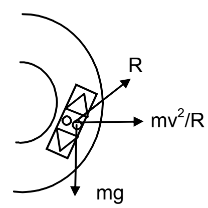
Putting values of a_1 & a_2 in equation (1) we can find that

$T = \sqrt{\left(\dfrac{4ml}{(M+m)\mu g}\right)}$

♣ ♣ ♣ ♣ ♣

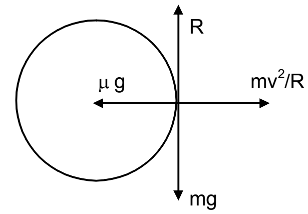
SOLUTIONS TO CONCEPTS circular motion;;
CHAPTER 7

1. Distance between Earth & Moon
   $r = 3.85 \times 10^5$ km $= 3.85 \times 10^8$m
   $T = 27.3$ days $= 24 \times 3600 \times (27.3)$ sec $= 2.36 \times 10^6$ sec
   $v = \frac{2\pi r}{T} = \frac{2 \times 3.14 \times 3.85 \times 10^8}{2.36 \times 10^6} = 1025.42$m/sec
   $a = \frac{v^2}{r} = \frac{(1025.42)^2}{3.85 \times 10^8} = 0.00273$m/sec^2 $= 2.73 \times 10^{-3}$m/sec^2
2. Diameter of earth = 12800km
   Radius R = 6400km $= 64 \times 10^5$ m
   $V = \frac{2\pi R}{T} = \frac{2 \times 3.14 \times 64 \times 10^5}{24 \times 3600}$ m/sec = 465.185
   $a = \frac{V^2}{R} = \frac{(46.5185)^2}{64 \times 10^5} = 0.0338$m/sec^2
3. V = 2t, r = 1cm
   a) Radial acceleration at t = 1 sec.
   $a = \frac{v^2}{r} = \frac{2^2}{1} = 4$cm/sec^2
   b) Tangential acceleration at t = 1sec.
   $a = \frac{dv}{dt} = \frac{d}{dt}(2t) = 2$cm/sec^2
   c) Magnitude of acceleration at t = 1sec
   $a = \sqrt{4^2 + 2^2} = \sqrt{20}$ cm/sec^2
4. Given that m = 150kg,
   v= 36km/hr = 10m/sec, r = 30m
   Horizontal force needed is $\frac{mv^2}{r} = \frac{150 \times (10)^2}{30} = \frac{150 \times 100}{30} = 500$N
5. in the diagram



   $R \cos\theta = mg$ ..(i)
   $R \sin\theta = \frac{mv^2}{r}$ ..(ii)
   Dividing equation (i) with equation (ii)
   $\text{Tan}\,\theta = \frac{mv^2}{rmg} = \frac{v^2}{rg}$
   v = 36km/hr = 10m/sec, r = 30m
   $\text{Tan}\,\theta = \frac{v^2}{rg} = \frac{100}{30 \times 10} = (1/3)$
   $\Rightarrow \theta = \tan^{-1}(1/3)$
6. Radius of Park = r = 10m
   speed of vehicle = 18km/hr = 5 m/sec
   Angle of banking $\tan\theta = \frac{v^2}{rg}$
   $\Rightarrow \theta = \tan^{-1}\frac{v^2}{rg} = \tan^{-1}\frac{25}{100} = \tan^{-1}(1/4)$

7. The road is horizontal (no banking)



$\frac{mv^2}{R} = \mu N$

and N = mg

So $\frac{mv^2}{R} = \mu\, mg$ v = 5m/sec, R = 10m

$\Rightarrow \frac{25}{10} = \mu g \Rightarrow \mu = \frac{25}{100} = 0.25$

8. Angle of banking = θ = 30°

Radius = r = 50m

$\tan\theta = \frac{v^2}{rg} \Rightarrow \tan 30° = \frac{v^2}{rg}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{v^2}{rg} \Rightarrow v^2 = \frac{rg}{\sqrt{3}} = \frac{50\times10}{\sqrt{3}}$

$\Rightarrow v = \sqrt{\frac{500}{\sqrt{3}}} = 17\text{m/sec.}$

9. Electron revolves around the proton in a circle having proton at the centre.

Centripetal force is provided by coulomb attraction.

r = 5.3 →t 10^{-11}m m = mass of electron = 9.1 × 10^{-3}kg.

charge of electron = 1.6 × 10^{-19}c.

$\frac{mv^2}{r} = k\frac{q^2}{r^2} \Rightarrow v^2 = \frac{kq^2}{rm} = \frac{9\times10^9\times1.6\times1.6\times10^{-38}}{5.3\times10^{-11}\times9.1\times10^{-31}} = \frac{23.04}{48.23}\times10^{13}$

$\Rightarrow v^2 = 0.477 \times 10^{13} = 4.7 \times 10^{12}$

$\Rightarrow v = \sqrt{4.7\times10^{12}} = 2.2 \times 10^6$ m/sec

10. At the highest point of a vertical circle

$\frac{mv^2}{R} = mg$

$\Rightarrow v^2 = Rg \Rightarrow v = \sqrt{Rg}$

11. A celling fan has a diameter = 120cm.

∴Radius = r = 60cm = 0/6m

Mass of particle on the outer end of a blade is 1g.

n = 1500 rev/min = 25 rev/sec

$\omega = 2\,\pi\, n = 2\,\pi \times 25 = 157.14$

Force of the particle on the blade = $Mr\omega^2$ = (0.001) × 0.6 × (157.14) = 14.8N

The fan runs at a full speed in circular path. This exerts the force on the particle (inertia). The particle also exerts a force of 14.8N on the blade along its surface.

12. A mosquito is sitting on an L.P. record disc & rotating on a turn table at $33\frac{1}{3}$ rpm.

$n = 33\frac{1}{3}\text{ rpm} = \frac{100}{3\times60}\text{ rps}$

$\therefore \omega = 2\,\pi\, n = 2\,\pi \times \frac{100}{180} = \frac{10\pi}{9}$ rad/sec

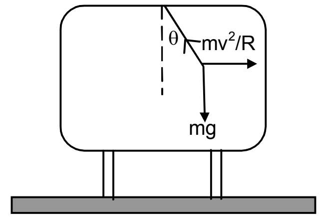
r = 10cm =0.1m, g = 10m/sec^2

$\mu mg \geq mr\omega^2 \Rightarrow \mu = \frac{r\omega^2}{g} \geq \frac{0.1\times\left(\frac{10\pi}{9}\right)^2}{10}$

$\Rightarrow \mu \geq \frac{\pi^2}{81}$

13. A pendulum is suspended from the ceiling of a car taking a turn

$r = 10\text{m}$, $v = 36\text{km/hr} = 10\text{ m/sec}$, $g = 10\text{m/sec}^2$



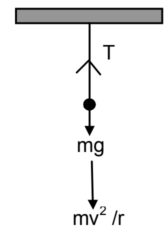
From the figure $T\sin\theta = \dfrac{mv^2}{r}$ ..(i)

$T\cos\theta = mg$ ..(ii)

$$\Rightarrow \frac{\sin\theta}{\cos\theta} = \frac{mv^2}{rmg} \Rightarrow \tan\theta = \frac{v^2}{rg} \Rightarrow \theta = \tan^{-1}\left(\frac{v^2}{rg}\right)$$

$$= \tan^{-1}\frac{100}{10\times 10} = \tan^{-1}(1) \Rightarrow \theta = 45°$$

14. At the lowest pt.



$$T = mg + \frac{mv^2}{r}$$

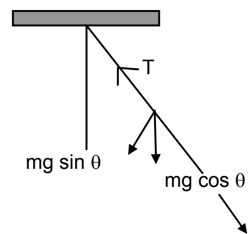
Here $m = 100\text{g} = 1/10\text{ kg}$, $r = 1\text{m}$, $v = 1.4\text{ m/sec}$

$$T = mg + \frac{mv^2}{r} = \frac{1}{10}\times 9.8\times\frac{(1.4)^2}{10} = 0.98 + 0.196 = 1.176 = 1.2\text{ N}$$

15. Bob has a velocity 1.4m/sec, when the string makes an angle of 0.2 radian.

$m = 100\text{g} = 0.1\text{kg}$, $r = 1\text{m}$, $v = 1.4\text{m/sec}$.

From the diagram,



$$T - mg\cos\theta = \frac{mv^2}{R}$$

$$\Rightarrow T = \frac{mv^2}{R} + mg\cos\theta$$

$$\Rightarrow T = \frac{0.1\times(1.4)^2}{1} + (0.1)\times 9.8\times\left(1 - \frac{\theta^2}{2}\right)$$

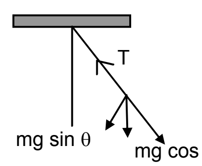
$$\Rightarrow T = 0.196 + 9.8\times\left(1 - \frac{(.2)^2}{2}\right) \qquad \left(\therefore\ \cos\theta = 1 - \frac{\theta^2}{2}\text{ for small }\theta\right)$$

$$\Rightarrow T = 0.196 + (0.98)\times(0.98) = 0.196 + 0.964 = 1.156\text{N} \approx 1.16\text{ N}$$

16. At the extreme position, velocity of the pendulum is zero.

So there is no centrifugal force.

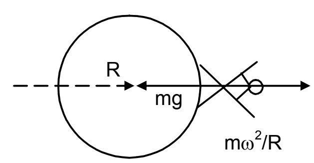
So $T = mg\cos\theta_o$



17. a) Net force on the spring balance.

$R = mg - m\omega^2 r$

So, fraction less than the true weight (3mg) is



$$= \frac{mg - (mg - m\omega^2 r)}{mg} = \frac{\omega^2}{g} = \left(\frac{2\pi}{24\times 3600}\right)^2\times\frac{6400\times 10^3}{10} = 3.5\times 10^{-3}$$

b) When the balance reading is half the true weight,

$$\frac{mg - (mg - m\omega^2 r)}{mg} = 1/2$$

$$\omega^2 r = g/2 \Rightarrow \omega = \sqrt{\frac{g}{2r}} = \sqrt{\frac{10}{2\times 6400\times 10^3}}\text{ rad/sec}$$

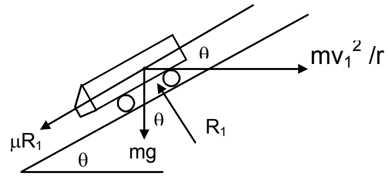
$\therefore$ Duration of the day is

$$T = \frac{2\pi}{\omega} = 2\pi\times\sqrt{\frac{2\times 6400\times 10^3}{9.8}}\text{ sec} = 2\pi\times\sqrt{\frac{64\times 10^6}{49}}\text{ sec} = \frac{2\pi\times 8000}{7\times 3600}\text{ hr} = 2\text{hr}$$

18. Given, v = 36km/hr = 10m/s, r = 20m, μ = 0.4

The road is banked with an angle,

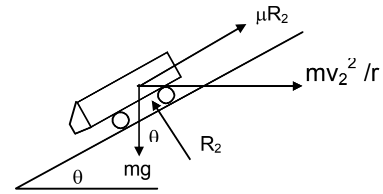
$\theta = \tan^{-1}\left(\frac{v^2}{rg}\right) = \tan^{-1}\left(\frac{100}{20 \times 10}\right) = \tan^{-1}\left(\frac{1}{2}\right)$ or $\tan\theta = 0.5$



When the car travels at max. speed so that it slips upward, μR_1 acts downward as shown in Fig.1

So, $R_1 - mg\cos\theta - \frac{mv_1^2}{r}\sin\theta = 0$ ..(i)

And $\mu R_1 + mg\sin\theta - \frac{mv_1^2}{r}\cos\theta = 0$ ..(ii)



Solving the equation we get,

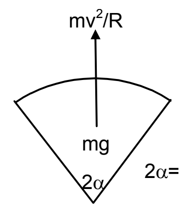
$V_1 = \sqrt{rg\frac{\tan\theta - \mu}{1 + \mu\tan\theta}} = \sqrt{20 \times 10 \times \frac{0.1}{1.2}}$ = 4.082 m/s = 14.7 km/hr

So, the possible speeds are between 14.7 km/hr and 54km/hr.

19. R = radius of the bridge

L = total length of the over bridge

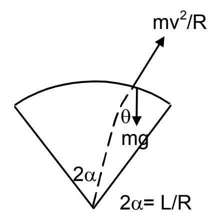
a) At the highest pt.



$mg = \frac{mv^2}{R} \Rightarrow v^2 = Rg \Rightarrow v = \sqrt{Rg}$

b) Given, $v = \frac{1}{\sqrt{2}}\sqrt{Rg}$

suppose it loses contact at B. So, at B, $mg\cos\theta = \frac{mv^2}{R}$



$\Rightarrow v^2 = Rg\cos\theta$

$\Rightarrow \left(\sqrt{\frac{Rv}{2}}\right)^2 = Rg\cos\theta \Rightarrow \frac{Rg}{2} = Rg\cos\theta \Rightarrow \cos\theta = 1/2 \Rightarrow \theta = 60° = \pi/3$

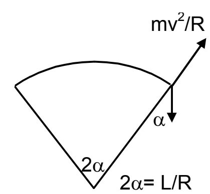
$\theta = \frac{\ell}{r} \rightarrow \ell = r\theta = \frac{\pi R}{3}$

So, it will lose contact at distance $\frac{\pi R}{3}$ from highest point

c) Let the uniform speed on the bridge be v.

The chances of losing contact is maximum at the end of the bridge for which $\alpha = \frac{L}{2R}$.

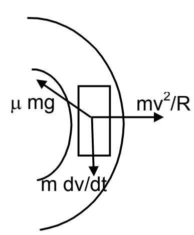
So, $\frac{mv^2}{R} = mg\cos\alpha \Rightarrow v = \sqrt{gR\cos\left(\frac{L}{2R}\right)}$

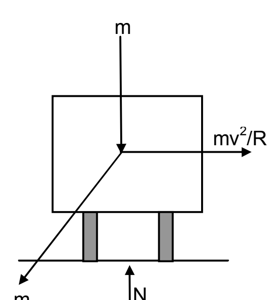


20. Since the motion is nonuniform, the acceleration has both radial & tangential component

$a_r = \frac{v^2}{r}$

$a_t = \frac{dv}{dt} = a$





Resultant magnitude = $\sqrt{\left(\frac{v^2}{r}\right)^2 + a^2}$

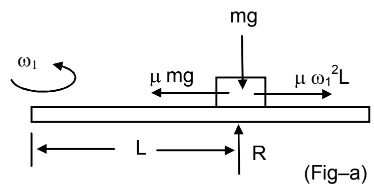
Now $\mu N = m\sqrt{\left(\frac{v^2}{r}\right)^2 + a^2} \Rightarrow \mu mg = m\sqrt{\left(\frac{v^2}{r}\right)^2 + a^2} \Rightarrow \mu^2 g^2 = \left(\frac{v^4}{r2}\right) + a^2$

$\Rightarrow v^4 = (\mu^2 g^2 - a^2)\, r^2 \Rightarrow v = [(\mu^2 g^2 - a^2)\, r^2]^{1/4}$

21. a) When the ruler makes uniform circular motion in the horizontal plane, (fig–a)

μ mg = $m\omega_1^2 L$

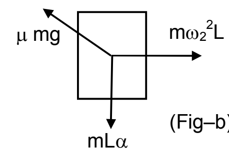
$\omega_1 = \sqrt{\frac{\mu g}{L}}$



(Fig–a)

b) When the ruler makes uniformly accelerated circular motion,(fig–b)

μ mg = $\sqrt{(m\omega_2^2 L)^2 + (mL\alpha)^2} \Rightarrow \omega_2^4 + \alpha^2 = \frac{\mu^2 g^2}{L^2} \Rightarrow \omega_2 = \left[\left(\frac{\mu g}{L}\right)^2 - \alpha^2\right]^{1/4}$



(Fig–b)

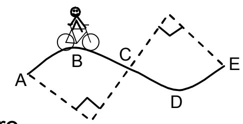
(When viewed from top)

22. Radius of the curves = 100m

Weight = 100kg

Velocity = 18km/hr = 5m/sec

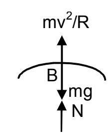
a) at B mg – $\frac{mv^2}{R}$ = N $\Rightarrow$ N = (100 × 10) – $\frac{100 \times 25}{100}$ = 1000 – 25 = 975N



At d, N = mg + $\frac{mv^2}{R}$ = 1000 + 25 = 1025 N

b) At B & D the cycle has no tendency to slide. So at B & D, frictional force is zero.

At 'C', mg sin θ = F $\Rightarrow$ F = 1000 × $\frac{1}{\sqrt{2}}$ = 707N



c) (i) Before 'C' mg cos θ – N = $\frac{mv^2}{R}$ $\Rightarrow$ N = mg cos θ – $\frac{mv^2}{R}$ = 707 – 25 = 683N

(ii) N – mg cos θ = $\frac{mv^2}{R}$ $\Rightarrow$ N = $\frac{mv^2}{R}$ + mg cos θ = 25 + 707 = 732N

d) To find out the minimum desired coeff. of friction, we have to consider a point just before C. (where N is minimum)

Now, μ N = mg sin $\theta \Rightarrow \mu$ × 682 = 707

So, μ = 1.037

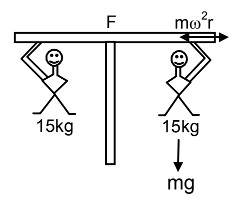
23. d = 3m $\Rightarrow$ R = 1.5m

R = distance from the centre to one of the kids

N = 20 rev per min = 20/60 = 1/3 rev per sec

$\omega = 2\pi r = 2\pi/3$

m = 15kg



$\therefore$ Frictional force F = $mr\omega^2$ = 15 × (1.5) × $\frac{(2\pi)^2}{9}$ = 5 × (0.5) × $4\pi^2$ = $10\pi^2$

$\therefore$ Frictional force on one of the kids is $10\pi^2$

24. If the bowl rotates at maximum angular speed, the block tends to slip upwards. So, the frictional force acts downward.

Here, r = R sin θ

From FBD –1

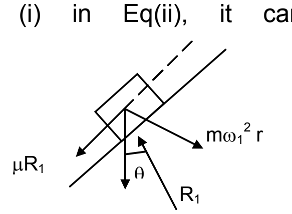
R_1 – mg cos θ – $m\omega_1^2$ (R sin θ) sin θ = 0 ..(i) [because r = R sin θ]

and μR_1 mg sin θ – $m\omega_1^2$ (R sin θ) cos θ = 0 ..(ii)

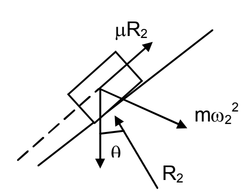
Substituting the value of R_1 from Eq (i) in Eq(ii), it can be found out that

$\omega_1 = \left[\frac{g(\sin\theta + \mu\cos\theta)}{R\sin\theta(\cos\theta - \mu\sin\theta)}\right]^{1/2}$

Again, for minimum speed, the frictional force μR_2 acts upward. From FBD–2, it can be proved that,



(FBD – 1)



(FBD – 2)

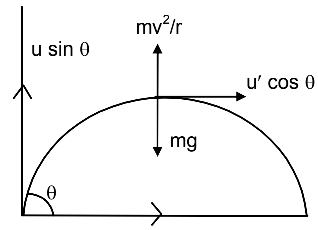
$\omega_2 = \left[\dfrac{g(\sin\theta - \mu\cos\theta)}{R\sin\theta(\cos\theta + \mu\sin\theta)}\right]^{1/2}$

$\therefore$ the range of speed is between ω_1 and ω_2

25. Particle is projected with speed 'u' at an angle θ. At the highest pt. the vertical component of velocity is '0'

So, at that point, velocity = u cos θ

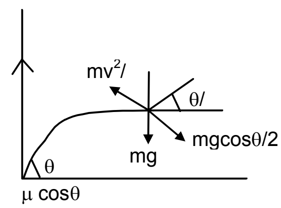
centripetal force = $m\,u^2 \cos^2\left(\dfrac{\theta}{r}\right)$



At highest pt.

$mg = \dfrac{mv^2}{r} \Rightarrow r = \dfrac{u^2\cos^2\theta}{g}$

26. Let 'u' the velocity at the pt where it makes an angle $\theta/2$ with horizontal. The horizontal component remains unchanged



So, $v \cos\theta/2 = \omega \cos\theta \Rightarrow v = \dfrac{u\cos\theta}{\cos\left(\dfrac{\theta}{2}\right)}$ ...(i)

From figure

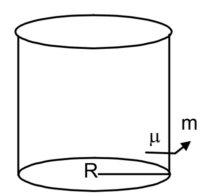
$mg\cos(\theta/2) = \dfrac{mv^2}{r} \Rightarrow r = \dfrac{v^2}{g\cos(\theta/2)}$

putting the value of 'v' from equn(i)

$r = \dfrac{u^2\cos^2\theta}{g\cos^3(\theta/2)}$

27. A block of mass 'm' moves on a horizontal circle against the wall of a cylindrical room of radius 'R' Friction coefficient between wall & the block is μ.

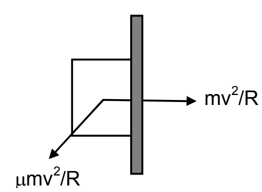
a) Normal reaction by the wall on the block is = $\dfrac{mv^2}{R}$



b) $\therefore$ Frictional force by wall = $\dfrac{\mu mv^2}{R}$

c) $\dfrac{\mu mv^2}{R}$ = ma $\Rightarrow a = -\dfrac{\mu v^2}{R}$ (Deceleration)

d) Now, $\dfrac{dv}{dt} = v\dfrac{dv}{ds} = -\dfrac{\mu v^2}{R} \Rightarrow ds = -\dfrac{R}{\mu}\dfrac{dv}{v}$



$\Rightarrow s = -\dfrac{R\mu}{}\ \ln V + c$

At s = 0, v = v_0

Therefore, $c = \dfrac{R}{\mu}\ln V_0$

so, $s = -\dfrac{R}{\mu}\ln\dfrac{v}{v_0} \Rightarrow \dfrac{v}{v_0} = e^{-\mu s/R}$

For, one rotation s = $2\pi R$, so $v = v_0 e^{-2\pi\mu}$

28. The cabin rotates with angular velocity ω & radius R

$\therefore$ The particle experiences a force $mR\omega^2$.

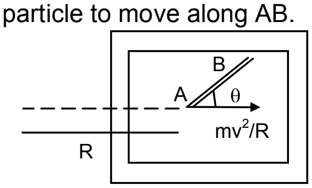
The component of $mR\omega^2$ along the groove provides the required force to the particle to move along AB.

$\therefore mR\omega^2\cos\theta = ma \Rightarrow a = R\omega^2\cos\theta$

length of groove = L

$L = ut + ½ at^2 \Rightarrow L = ½ R\omega^2\cos\theta\, t^2$

$\Rightarrow t^2 = \dfrac{2L}{R\omega^2\cos\theta} = \Rightarrow t = 1\sqrt{\dfrac{2L}{R\omega^2\cos\theta}}$



29. v = Velocity of car = 36km/hr = 10 m/s
r = Radius of circular path = 50m
m = mass of small body = 100g = 0.1kg.
μ = Friction coefficient between plate & body = 0.58
a) The normal contact force exerted by the plate on the block

$$N = \frac{mv^2}{r} = \frac{0.1\times100}{50} = 0.2N$$

b) The plate is turned so the angle between the normal to the plate & the radius of the road slowly increases

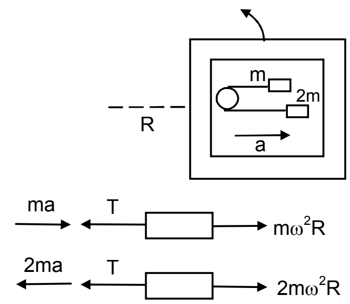
$$N = \frac{mv^2}{r}\cos\theta \quad ..(i)$$

$$\mu N = \frac{mv^2}{r}\sin\theta \quad ..(ii)$$

Putting value of N from (i)

$$\mu\frac{mv^2}{r}\cos\theta = \frac{mv^2}{r}\sin\theta \Rightarrow \mu = \tan\theta \Rightarrow \theta = \tan^{-1}\mu = \tan^{-1}(0.58) = 30°$$

30. Let the bigger mass accelerates towards right with 'a'.
From the free body diagrams,



$T - ma - m\omega^2R = 0$ …(i)
$T + 2ma - 2m\omega^2R = 0$ …(ii)
Eq (i) – Eq (ii) $\Rightarrow 3ma = m\omega^2R$

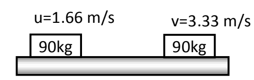
$$\Rightarrow a = \frac{m\omega^2R}{3}$$

Substituting the value of a in Equation (i), we get T = 4/3 $m\omega^2R$.

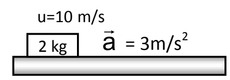
* * * *

SOLUTIONS TO CONCEPTS
CHAPTER – 8

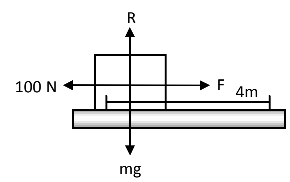
1. $M = m_c + m_b = 90kg$



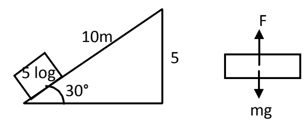
   u = 6 km/h = 1.666 m/sec
   v = 12 km/h = 3.333 m/sec
   Increase in K.E. = $\frac{1}{2} Mv^2 - \frac{1}{2} Mu^2$
   $= \frac{1}{2}\, 90 \times (3.333)^2 - \frac{1}{2} \times 90 \times (1.66)^2 = 494.5 - 124.6 = 374.8 \approx 375$ J
2. $m_b = 2$ kg.
   u = 10 m/sec
   a = 3 m/aec^2



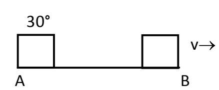
   t = 5 sec
   v = u + at = 10 + 3 I 5 = 25 m/sec.
   $\therefore$ F.K.E = $\frac{1}{2} mv^2 = \frac{1}{2} \times 2 \times 625 = 625$ J.
3. F = 100 N



   S = 4m, $\theta = 0°$
   $\omega = \vec{F}.\vec{S} = 100 \times 4 = 400$ J
4. m = 5 kg
   $\theta = 30°$
   S = 10 m
   F = mg

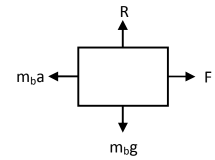


   So, work done by the force of gravity
   $\omega = mgh = 5 \times 9.8 \times 5 = 245$ J
5. F= 2.50N, S = 2.5m, m =15g = 0.015kg.

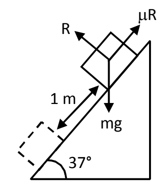


   So, $w = F \times S \Rightarrow a = \frac{F}{m} = \frac{2.5}{0.015} = \frac{500}{3} m/s^2$
   =F × S cos 0° (acting along the same line)
   = 2.5 × 2.5 = 6.25J
   Let the velocity of the body at b = U. Applying work-energy principle $\frac{1}{2} mv^2 - 0 = 6.25$
   $\Rightarrow V = \sqrt{\frac{6.25 \times 2}{0.015}} = 28.86$ m/sec.
   So, time taken to travel from A to B.
   $\Rightarrow t = \frac{v-u}{a} = \frac{28.86 \times 3}{500}$
   $\therefore$ Average power $= \frac{W}{t} = \frac{6.25 \times 500}{(28.86) \times 3} = 36.1$
6. Given
   $\vec{r}_1 = 2\hat{i} + 3\hat{j}$
   $r_2 = 3\hat{i} + 2\hat{j}$
   So, displacement vector is given by,
   $\vec{r} = \vec{r}_1 - \vec{r}_2 \Rightarrow \vec{r} = (3\hat{i} + 2\hat{j}) - (2\hat{i} + 3\hat{j}) = \hat{i} - \hat{j}$

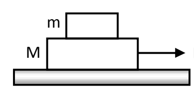
So, work done = $\vec{F} \times \vec{s}$ = 5 × 1 + 5(-1) = 0



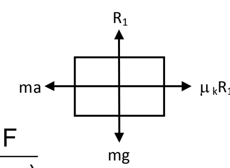
7. m_b = 2kg, s = 40m, a = 0.5m/sec^2
   So, force applied by the man on the box
   F = $m_b a$ = 2 × (0.5) = 1 N
   ω = FS = 1 × 40 = 40 J
8. Given that F= a + bx
   Where a and b are constants.
   So, work done by this force during this force during the displacement x = 0 and x = d is given by
   W = $\int_0^d F\,dx = \int_0^d (a+bx)dx$ = ax + (bx^2/2) = [a + ½ bd] d



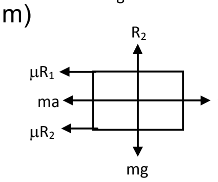
9. m_b = 250g = .250 kg
   θ = 37°, S = 1m.
   Frictional force f = μR
   mg sin θ = μ R ..(1)
   mg cos θ ..(2)
   so, work done against μR = μRS cos 0° = mg sin θ S = 0.250 × 9.8 × 0.60 × 1 = 1.5 J
10. a = $\frac{F}{2(M+m)}$ (given)



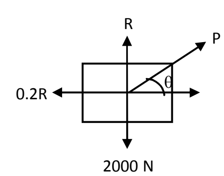
   a) from fig (1)
   ma = μ_k R_1 and R_1 = mg



   $\Rightarrow \mu = \frac{ma}{R_1} = \frac{F}{2(M+m)g}$
   b) Frictional force acting on the smaller block f = μR = $\frac{F}{2(M+m)g} \times mg = \frac{m \times F}{2(M+m)}$

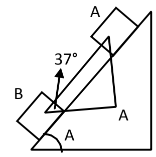


   c) Work done w = fs s = d
   $$w = \frac{mF}{2(M+m)} \times d = \frac{mFd}{2(M+m)}$$
11. Weight = 2000 N, S = 20m, μ = 0.2



   a) R + Psinθ - 2000 = 0 ..(1)
   P cosθ - 0.2 R =0 ..(2)
   From (1) and (2) P cosθ – 0.2 (2000 – P sinθ)=0
   P = $\frac{400}{\cos\theta + 0.2\sin\theta}$ ..(3)
   So, work done by the person, W = PS cosθ = $\frac{8000\cos\theta}{\cos\theta+0.2\sin\theta} = \frac{8000}{1+0.2\sin\theta} = \frac{40000}{5+\tan\theta}$
   b) For minimum magnitude of force from equn(1)
   d/dθ (cos θ + 0.2 sinθ) = 0 $\Rightarrow$ tan θ = 0.2
   putting the value in equn (3)
   W = $\frac{40000}{5+\tan\theta} = \frac{40000}{(5.2)}$ = 7690 J
12. w = 100 N, θ = 37°, s = 2m

Force F= mg sin 37° = 100 × 0.60 = 60 N



So, work done, when the force is parallel to incline.

w = Fs cos θ = 60 × 2 × cos θ = 120 J

In ΔABC AB= 2m

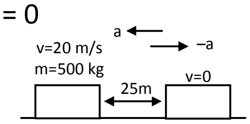
CB = 37°

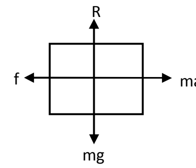
so, h = C = 1m

∴ work done when the force in horizontal direction

W = mgh = 100 × 1.2 = 120 J

13. m = 500 kg, s = 25m, u = 72km/h= 20 m/s, v = 0

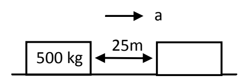


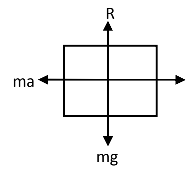


$(-a) = \frac{v^2 - u^2}{2S} \Rightarrow a = \frac{400}{50} = 8m/sec^2$

Frictional force f = ma = 500 × 8 = 4000 N

14. m = 500 kg, u = 0, v = 72 km/h = 20m/s





$a = \frac{v^2 - u^2}{2s} = \frac{400}{50} = 8m/sec^2$

force needed to accelerate the car F = ma = 500 × 8 = 4000 N

15. Given, $v = a\sqrt{x}$ (uniformly accelerated motion)

displacement s = d – 0 = d

putting x = 0, $v_1 = 0$

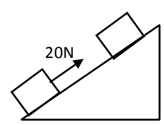
putting x = d, $v_2 = a\sqrt{d}$

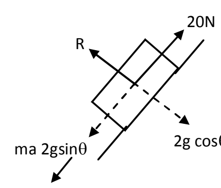
$a = \frac{{v_2}^2 - {u_2}^2}{2s} = \frac{a^2 d}{2d} = \frac{a^2}{2}$

force $f = ma = \frac{ma^2}{2}$

work done $w = FS\cos\theta = \frac{ma^2}{2} \times d = \frac{ma^2 d}{2}$

16. a) m = 2kg, θ = 37°, F = 20 N





From the free body diagram

F = (2g sin θ) + ma ⇒ a = (20 – 20 sinθ)/s = 4m/sec^2

S = ut + ½ at^2 (u = 0, t = 1s, a = 1.66)

= 2m

So, work, done w = Fs = 20 × 2 = 40 J

b) If W = 40 J

$S = \frac{W}{F} = \frac{40}{20}$

h = 2 sin 37° = 1.2 m

So, work done W = –mgh = – 20 × 1.2 = –24 J

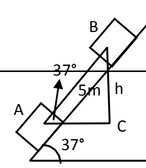
c) v = u + at = 4 × 10 = 40 m/sec

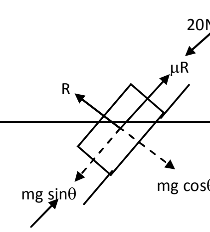
So, K.E. = ½ mv^2 = ½ × 2 × 16 = 16 J

17. m = 2kg, θ = 37°, F = 20 N, a = 10 m/sec^2

a) t = 1sec

So, s= ut + ½ at^2 = 5m





Work done by the applied force w = FS cos 0° = 20 × 5 = 100 J

b) BC (h) = 5 sin 37° = 3m

So, work done by the weight W = mgh = 2 × 10 × 3 = 60 J

c) So, frictional force f = mg sinθ

work done by the frictional forces w = fs cos0° = (mg sinθ) s = 20 × 0.60 × 5 = 60 J

18. Given, m = 25o g = 0.250kg,

u = 40 cm/sec = 0.4m/sec

μ = 0.1, v=0

Here, μ R = ma {where, a = deceleration}

$a = \frac{\mu R}{m} = \frac{\mu mg}{m} = \mu g = 0.1 \times 9.8 = 0.98 \text{ m/sec}^2$

$S = \frac{v^2 - u^2}{2a} = 0.082\text{m} = 8.2 \text{ cm}$

Again, work done against friction is given by

– w = μ RS cos θ

= 0.1 × 2.5 × 0.082 × 1 (θ = 0°) = 0.02 J

⇒ W = – 0.02 J

19. h = 50m, m = 1.8×10^5 kg/hr, P = 100 watt,

P.E. = mgh = $1.8 \times 10^5 \times 9.8 \times 50 = 882 \times 10^5$ J/hr

Because, half the potential energy is converted into electricity,

Electrical energy ½ P.E. = 441×10^5 J/hr

So, power in watt (J/sec) is given by = $\frac{441 \times 10^5}{3600}$

∴ number of 100 W lamps, that can be lit $\frac{441 \times 10^5}{3600 \times 100} = 122.5 \approx 122$

20. m = 6kg, h = 2m

P.E. at a height ‘2m’ = mgh = 6 × (9.8) × 2 = 117.6 J

P.E. at floor = 0

Loss in P.E. = 117.6 – 0 = 117. 6 J ≈ 118 J

21. h = 40m, u = 50 m/sec

Let the speed be ‘v’ when it strikes the ground.

Applying law of conservation of energy

mgh + ½ mu^2 = ½ mv^2

⇒ 10 × 40 + (1/2) × 2500 = ½ v^2 ⇒ v^2 = 3300 ⇒ v = 57.4 m/sec ≈58 m/sec

22. t = 1 min 57.56 sec = 11.56 sec, p= 400 W, s =200 m

$p = \frac{w}{t}$, Work w = pt = 460 × 117.56 J

Again, $W = FS = \frac{460 \times 117.56}{200} = 270.3 \text{ N} \approx 270 \text{ N}$

23. S = 100 m, t = 10.54 sec, m = 50 kg

The motion can be assumed to be uniform because the time taken for acceleration is minimum.

a) Speed v = S/t = 9.487 e/s

So, K.E. = ½ mv^2 = 2250 J

b) Weight = mg = 490 J

given R = mg /10 = 49 J

so, work done against resistance W_F = – RS = – 49 × 100 = – 4900 J

c) To maintain her uniform speed, she has to exert 4900 j of energy to over come friction

$P = \frac{W}{t}$ = 4900 / 10.54 = 465 W

24. h = 10 m

flow rate = (m/t) = 30 kg/min = 0.5 kg/sec

power $P = \frac{mgh}{t}$ = (0.5) × 9.8 × 10 = 49 W

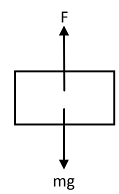
So, horse power (h.p) P/746 = 49/746 = 6.6×10^{-2}hp

25. m = 200g = 0.2kg, h = 150cm = 1.5m, v = 3m/sec, t = 1 sec

Total work done = ½ mv^2 + mgh = (1/2) × (0.2) ×9 + (0.2) × (9.8) × (1.5) = 3.84 J

h.p. used = $\frac{3.84}{746} = 5.14 \times 10^{-3}$

26. m = 200 kg, s = 12m, t = 1 min = 60 sec



So, work W = F cos θ = mgs cos0° [θ = 0°, for minimum work]

= 2000 × 10 × 12 = 240000 J

So, power $p = \frac{W}{t} = \frac{240000}{60}$ = 4000 watt

h.p = $\frac{4000}{746}$ = 5.3 hp.

27. The specification given by the company are

U = 0, m = 95 kg, P_m = 3.5 hp

V_m = 60 km/h = 50/3 m/sec t_m = 5 sec

So, the maximum acceleration that can be produced is given by,

$a = \frac{(50/3)-0}{5} = \frac{10}{3}$

So, the driving force is given by

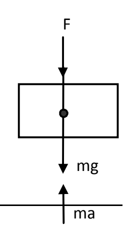
$F = ma = 95 \times \frac{10}{3} = \frac{950}{3}$ N

So, the velocity that can be attained by maximum h.p. white supplying $\frac{950}{3}$ will be

$v = \frac{p}{F} \Rightarrow v = \frac{3.5 \times 746 \times 5}{950}$ = 8.2 m/sec.

Because, the scooter can reach a maximum of 8.s m/sec while producing a force of 950/3 N, the specifications given are some what over claimed.

28. Given m = 30kg, v = 40 cm/sec = 0.4 m/sec s = 2m



From the free body diagram, the force given by the chain is,

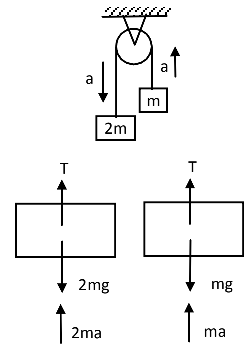
F = (ma – mg) = m(a – g) [where a = acceleration of the block]

$a = \frac{(v2\ \ u2)}{2s} = \frac{0.16}{0.4}$ = 0.04 m/sec^2

So, work done $W = Fs \cos\theta = m(a - g) s \cos\theta$

$\Rightarrow W = 30 (0.04 - 9.8) \times 2 \Rightarrow W = -585.5 \Rightarrow W = -586$ J.

So, W = – 586 J



29. Given, T = 19 N

From the freebody diagrams,

$T - 2\,mg + 2\,ma = 0$ ...(i)

$T - mg - ma = 0$ ...(ii)

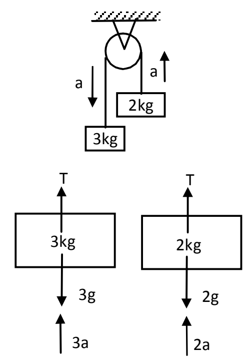
From, Equation (i) & (ii) $T = 4ma \Rightarrow a = \frac{T}{4m} \Rightarrow A = \frac{16}{4m} = \frac{4}{m}$ m/s^2.

Now, $S = ut + ½ at^2$

$\Rightarrow S = \frac{1}{2} \times \frac{4}{m} \times 1 \Rightarrow S = \frac{2}{m}$ m [because u=0]

Net mass = 2m – m = m

Decrease in P.E. = mgh $\Rightarrow$P.E. = $m \times g \times \frac{2}{m} \Rightarrow$ P.E. = 9.8 × 2 $\Rightarrow$P.E. = 19.6 J



30. Given, m_1 = 3 kg, m_2 = 2kg, t = during 4^{th} second

From the freebody diagram

$T - 3g + 3a = 0$ ..(i)

$T - 2g - 2a = 0$ ..(ii)

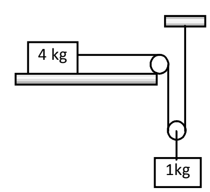
Equation (i) & (ii), we get $3g - 3a = 2g + 2a \Rightarrow a = \frac{g}{5}$ m/sec^2

Distance travelled in 4^{th} sec is given by

$$S_{4th} = \frac{a}{2}(2n-1) = \frac{\left(\frac{g}{5}\right)}{s}(2\times 4-1) = \frac{7g}{10} = \frac{7\times 9.8}{10} \text{ m}$$

Net mass 'm' = $m_1 - m_2$ = 3 – 2 = 1kg

So, decrease in P.E. = mgh = $1 \times 9.8 \times \frac{7}{10} \times 9.8 = 67.2 = 67$ J



31. m_1 = 4kg, m_2 = 1kg, V_2 = 0.3m/sec V_1 = 2 × (0.3) = 0.6 m/sec

($v_1 = 2x_2$ m this system)

h = 1m = height descent by 1kg block

s = 2 × 1 = 2m distance travelled by 4kg block

u = 0

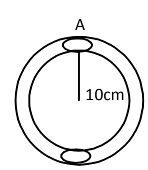
Applying change in K.E. = work done (for the system)

$[(1/2)m_1v_1^2 + (1/2)\, m_2v_m^2] - 0 = (-\mu R)S + m_2g$ [R = 4g = 40 N]

$\Rightarrow ½ \times 4 \times (0.36) \times ½ \times 1 \times (0.09) = -\mu \times 40 \times 2 + 1 \times 40 \times 1$

$\Rightarrow 0.72 + 0.045 = -80\mu + 10$

$\Rightarrow \mu = \frac{9.235}{80} = 0.12$



32. Given, m = 100g = 0.1kg, v = 5m/sec, r = 10cm

Work done by the block = total energy at A – total energy at B

$(1/2\, mv^2 + mgh) - 0$

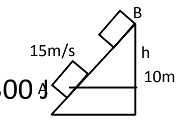
$\Rightarrow W = ½\, mv^2 + mgh - 0 = ½ \times (0.1) \times 25 + (0.1) \times 10 \times (0.2)$ [h = 2r = 0.2m]

$\Rightarrow$ W = 1.25 – 0.2 $\Rightarrow$ W = 1.45 J

So, the work done by the tube on the body is

W_t = –1.45 J

33. m = 1400kg, v = 54km/h = 15m/sec, h = 10m



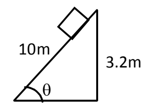
Work done = (total K.E.) – total P.E.

= 0 + ½ mv^2 – mgh = ½ × 1400 × $(15)^2$ – 1400 × 9.8 × 10 = 157500 – 137200 = 20300 J

So, work done against friction, W_t = 20300 J

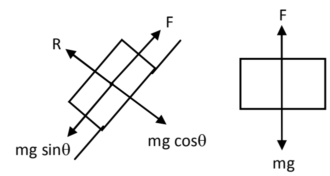
34. m = 200g = 0.2kg, s = 10m, h = 3.2m, g = 10 m/sec^2

a) Work done W = mgh = 0.2 × 10 × 3.2 = 6.4 J



b) Work done to slide the block up the incline

$w = (mg \sin\theta) = (0.2) \times 10 \times \frac{3.2}{10} \times 10 = 6.4$ J



c) Let, the velocity be v when falls on the ground vertically,

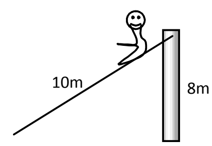
½ mv^2 – 0 = 6.4J $\Rightarrow$ v = 8 m/s

d) Let V be the velocity when reaches the ground by liding

½ mV^2 – 0 = 6.4 J $\Rightarrow$ V = 8m/sec

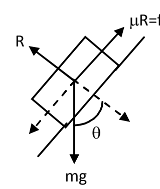
35. ℓ = 10m, h = 8m, mg = 200N

$f = 200 \times \frac{3}{10} = 60$N



a) Work done by the ladder on the boy is zero when the boy is going up because the work is done by the boy himself.

b) Work done against frictional force, W = μRS = f ℓ = (–60) × 10 = – 600 J



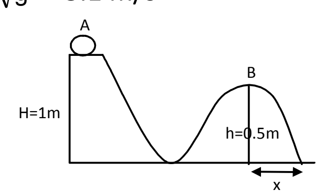
c) Work done by the forces inside the boy is

$W_b = (mg \sin\theta) \times 10 = 200 \times \frac{8}{10} \times 10 = 1600$ J

36. H = 1m, h = 0.5m

Applying law of conservation of Energy for point A & B

$mgH = ½ mv^2 + mgh \Rightarrow g = (1/2) v^2 + 0.5g \Rightarrow v^2\ 2(g – 0.59) = g \Rightarrow v = \sqrt{g} = 3.1$ m/s



After point B the body exhibits projectile motion for which

θ = 0°, v = – 0.5

So, –0.5 = (u sinθ) t - (1/2) gt^2 $\Rightarrow$ 0.5 = 4.9 t^2 $\Rightarrow$ t = 0.31 sec.

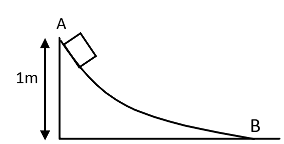
So, x = (4 cos θ) t = 3.1 × 3.1 = 1m.

So, the particle will hit the ground at a horizontal distance in from B.

37. mg = 10N, μ = 0.2, H = 1m, u = v = 0

change in P.E. = work done.

Increase in K.E.



$\Rightarrow$ w = mgh = 10 × 1 = 10 J

Again, on the horizontal surface the fictional force

F = μR = μmg = 0.2 × 10 = 2 N

So, the K.E. is used to overcome friction

$\Rightarrow S = \frac{W}{F} = \frac{10J}{2N} = 5$m

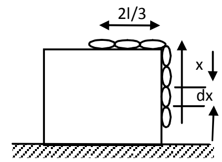
38. Let 'dx' be the length of an element at a distance × from the table

mass of 'dx' length = (m/ℓ) dx

Work done to put dx part back on the table

W = (m/ℓ) dx g(x)

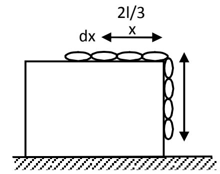
So, total work done to put ℓ/3 part back on the table



$$W = \int_0^{1/3} (m/\ell) gx\, dx \Rightarrow w = (m/\ell)\, g \left[\frac{x^2}{2}\right]_0^{\ell/3} = \frac{mg\ell^2}{18\ell} = \frac{mg\ell}{18}$$

39. Let, x length of chain is on the table at a particular instant.

So, work done by frictional force on a small element 'dx'



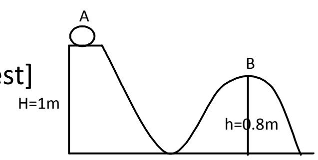
$$dW_f = \mu Rx = \mu\left(\frac{M}{L}dx\right)gx \qquad \left[\text{where } dx = \frac{M}{L}dx\right]$$

Total work don by friction,

$$W_f = \int_{2L/3}^{0} \mu \frac{M}{L} gx\ dx$$

$$\therefore W_f = \mu \frac{m}{L} g\left[\frac{x^2}{2}\right]_{2L/3}^{0} = \mu \frac{M}{L}\left[\frac{4L^2}{18}\right] = 2\mu Mg\, L/9$$

40. Given, m = 1kg, H = 1m, h = 0.8m



Here, work done by friction = change in P.E. [as the body comes to rest]

$\Rightarrow W_f = mgh - mgH$

= mg (h – H)

= 1 × 10 (0.8 – 1) = – 2J

41. m = 5kg, x = 10cm = 0.1m, v = 2m/sec,

h =? G = 10m/sec^2

$$S0,\ k = \frac{mg}{x} = \frac{50}{0.1} = 500 \text{ N/m}$$

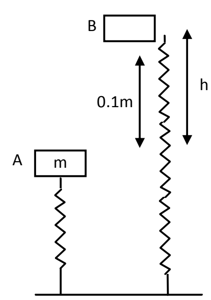
Total energy just after the blow E = ½ mv^2 + ½ kx^2 …(i)

Total energy a a height h = ½ k (h – x)^2 + mgh …(ii)

½ mv^2 + ½ kx^2 = ½ k (h – x)^2 + mgh

On, solving we can get,

H = 0.2 m = 20 cm



42. m = 250 g = 0.250 kg,

k = 100 N/m, m = 10 cm = 0.1m

g = 10 m/sec^2

Applying law of conservation of energy

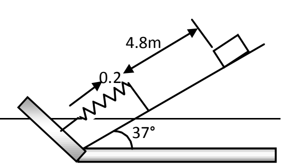
$$½\, kx^2 = mgh \Rightarrow h = \frac{1}{2}\left(\frac{kx^2}{mg}\right) = \frac{100\times(0.1)^2}{2\times 0.25\times 10} = 0.2 \text{ m} = 20 \text{ cm}$$

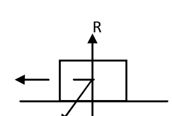
43. m = 2kg, s_1 = 4.8m, x = 20cm = 0.2m, s_2 = 1m,

sin 37° = 0.60 = 3/5, θ = 37°, cos 37° = .79 = 0.8 = 4/5

g = 10m/sec^2

Applying work – Energy principle for downward motion of the body





$0 - 0 = mg \sin 37° \times 5 - \mu R \times 5 - ½ kx^2$

$\Rightarrow 20 \times (0.60) \times 1 - \mu \times 20 \times (0.80) \times 1 + ½ k (0.2)^2 = 0$

$\Rightarrow 60 - 80\mu - 0.02k = 0 \Rightarrow 80\mu + 0.02k = 60$ ...(i)

Similarly, for the upward motion of the body the equation is

$0 - 0 = (-mg \sin 37°) \times 1 - \mu R \times 1 + ½ k (0.2)^2$

$\Rightarrow -20 \times (0.60) \times 1 - \mu \times 20 \times (0.80) \times 1 + ½ k (0.2)^2 = 0$

$\Rightarrow -12 - 16\mu + 0.02 K = 0$ ..(ii)

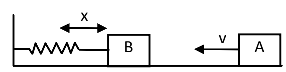
Adding equation (i) & equation (ii), we get $96 \mu = 48$

$\Rightarrow \mu = 0.5$

Now putting the value of μ in equation (i) K = 1000N/m

44. Let the velocity of the body at A be v

So, the velocity of the body at B is v/2

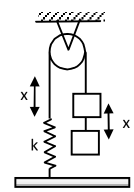


Energy at point A = Energy at point B

So, $½ mv_A^2 = ½ mv_B^2 + ½ kx^{2+}$

$\Rightarrow ½ kx^2 = ½ mv_A^2 - ½ mv_B^2 \Rightarrow kx^2 = m (v_A^{2+} - v_B^2) \Rightarrow kx^2 = m\left(v^2 - \frac{v^2}{4}\right) \Rightarrow k = \frac{3mv^2}{3x^2}$

45. Mass of the body = m



Let the elongation be x

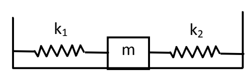
So, $½ kx^2 = mgx$

$\Rightarrow x = 2mg / k$

46. The body is displaced x towards right

Let the velocity of the body be v at its mean position

Applying law of conservation of energy

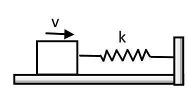


$½ mv^2 = ½ k_1x^2 + ½ k_2x^2 \Rightarrow mv^2 = x^2 (k_1 + k_2) \Rightarrow v^2 = \frac{x^2(k_1 + k_2)}{m}$

$\Rightarrow v = x\sqrt{\frac{k_1 + k_2}{m}}$

47. Let the compression be x

According to law of conservation of energy

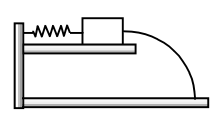


$½ mv^2 = ½ kx^2 \Rightarrow x^2 = mv^2 / k \Rightarrow x = v\sqrt{(m/k)}$

b) No. It will be in the opposite direction and magnitude will be less due to loss in spring.

48. m = 100g = 0.1kg, x = 5cm = 0.05m, k = 100N/m

when the body leaves the spring, let the velocity be v



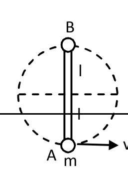
$½ mv^2 = ½ kx^2 \Rightarrow v = x\sqrt{k/m} = 0.05 \times \sqrt{\frac{100}{0.1}} = 1.58$m/sec

For the projectile motion, $\theta = 0°$, Y = –2

Now, $y = (u \sin \theta)t - ½ gt^2$

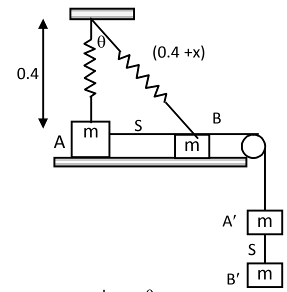
$\Rightarrow -2 = (-1/2) \times 9.8 \times t^2 \Rightarrow t = 0.63$ sec.

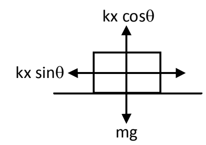
So, $x = (u \cos \theta) t \Rightarrow 1.58 \times 0.63 = 1$m



49. Let the velocity of the body at A is 'V' for minimum velocity given at A velocity of the body at point B is zero.
Applying law of conservation of energy at A & B
$\frac{1}{2} mv^2 = mg (2\ell) \Rightarrow v = \sqrt{(4g\ell)} = 2\sqrt{g\ell}$

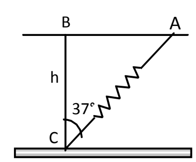
50. m = 320g = 0.32kg
k = 40N/m
h = 40cm = 0.4m
$g = 10 \text{ m/s}^2$
From the free body diagram,





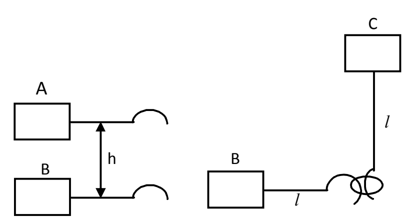
$kx \cos\theta = mg$
(when the block breaks off R = 0)
$\Rightarrow \cos\theta = mg/kx$
So, $\frac{0.4}{0.4 + x} = \frac{3.2}{40 \times x} \Rightarrow 16x = 3.2x + 1.28 \Rightarrow x = 0.1 \text{ m}$
S0, s = AB = $\sqrt{(h + x)^2 - h^2} = \sqrt{(0.5)^2 - (0.4)^2} = 0.3 \text{ m}$
Let the velocity of the body at B be v
Charge in K.E. = work done (for the system)
$(1/2\ mv^2 + \frac{1}{2} mv^2) = -1/2\ kx^2 + mgs$
$\Rightarrow (0.32) \times v^2 = -(1/2) \times 40 \times (0.1)^2 + 0.32 \times 10 \times (0.3) \Rightarrow v = 1.5 \text{ m/s}.$

51. $\theta = 37°$; l = h = natural length
Let the velocity when the spring is vertical be 'v'.



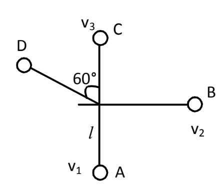
Cos 37° = BC/AC = 0.8 = 4/5
Ac = (h + x) = 5h/4 (because BC = h)
So, x = (5h/4) – h = h/4
Applying work energy principle $\frac{1}{2} kx^2 = \frac{1}{2} mv^2$
$\Rightarrow v = x\sqrt{(k/m)} = \frac{h}{4}\sqrt{\frac{k}{m}}$

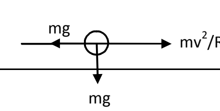
52. The minimum velocity required to cross the height point c = $\sqrt{2gl}$
Let the rod released from a height h.

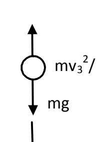


Total energy at A = total energy at B
mgh = $1/2\ mv^2$; mgh = 1/2 m (2gl)
[Because v = required velocity at B such that the block makes a complete circle. [Refer Q – 49]
So, h = l.

53. a) Let the velocity at B be v_2
$1/2\ mv_1^2 = 1/2\ mv_2^2 + mgl$
$\Rightarrow 1/2\ m (10\ gl) = 1/2\ mv_2^2 + mgl$
$v_2^2 = 8\ gl$
So, the tension in the string at horizontal position







$T = \frac{mv^2}{R} = \frac{m8gl}{l} = 8\text{ mg}$

b) Let the velocity at C be V_3

$1/2\ mv_1^2 = 1/2\ mv_3^2 + mg\ (2l)$

$\Rightarrow 1/2\ m\ (\log l) = 1/2\ mv_3^2 + 2mgl$

$\Rightarrow v_3^2 = 6\ mgl$

So, the tension in the string is given by

$T_c = \frac{mv^2}{l} - mg = \frac{6\ glm}{l}\ mg = 5\text{ mg}$

c) Let the velocity at point D be v_4

Again, $1/2\ mv_1^2 = 1/2\ mv_4^2 + mgh$

$1/2 \times m \times (10\ gl) = 1.2\ mv_4^2 + mgl\ (1 + \cos 60°)$

$\Rightarrow v_4^2 = 7\ gl$

So, the tension in the string is

$T_D = (mv^2/l) - mg \cos 60°$

$= m(7\ gl)/l - l - 0.5\ mg \Rightarrow 7\ mg - 0.5\ mg = 6.5\ mg.$

54. From the figure, $\cos \theta = AC/AB$

$\Rightarrow AC = AB \cos \theta \Rightarrow (0.5) \times (0.8) = 0.4.$

So, $CD = (0.5) - (0.4) = (0.1)$ m

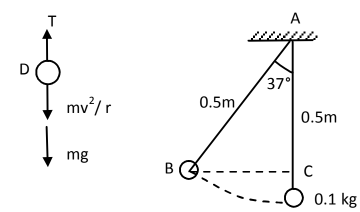
Energy at D = energy at B

$1/2\ mv^2 = mg\ (CD)$

$v^2 = 2 \times 10 \times (0.1) = 2$

So, the tension is given by,

$T = \frac{mv^2}{r} + mg = (0.1)\left(\frac{2}{0.5} + 10\right) = 1.4\text{ N}.$



55. Given, N = mg

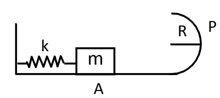
As shown in the figure, $mv^2 / R = mg$

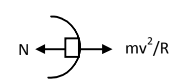
$\Rightarrow v^2 = gR$ ...(1)

Total energy at point A = energy at P

$1/2\ kx^2 = \frac{mgR + 2mgR}{2}$ [because $v^2 = gR$]

$\Rightarrow x^2 = 3mgR/k \Rightarrow x = \sqrt{(3mgR)/k}$.





56. $V = \sqrt{3gl}$

$1/2\ mv^2 - 1/2\ mu^2 = -mgh$

$v^2 = u^2 - 2g(l + l\cos\theta)$

$\Rightarrow v^2 = 3gl - 2gl\ (1 + \cos \theta)$ ...(1)

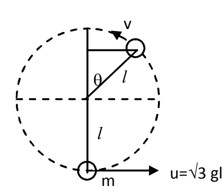
Again,

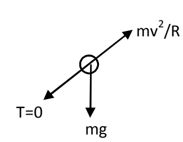
$mv^2/l = mg \cos \theta$

$v^2 = lg \cos \theta$

From equation (1) and (2), we get

$3gl - 2gl - 2gl \cos \theta = gl \cos \theta$





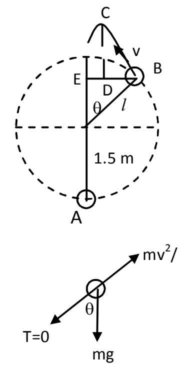
$3 \cos\theta = 1 \Rightarrow \cos\theta = 1/3$

$\theta = \cos^{-1}(1/3)$

So, angle rotated before the string becomes slack

$= 180° - \cos^{-1}(1/3) = \cos^{-1}(-1/3)$

57. $l = 1.5$ m; $u = \sqrt{57}$ m/sec.



a) $mg \cos\theta = mv^2 / l$

$v^2 = lg \cos\theta$ ...(1)

change in K.E. = work done

$1/2\, mv^2 - 1/2\, mu^2 = mgh$

$\Rightarrow v^2 - 57 = -2 \times 1.5\, g\,(1 + \cos\theta)$...(2)

$\Rightarrow v^2 = 57 - 3g(1 + \cos\theta)$

Putting the value of v from equation (1)

$15 \cos\theta = 57 - 3g\,(1 + \cos\theta) \Rightarrow 15 \cos\theta = 57 - 30 - 30 \cos\theta$

$\Rightarrow 45 \cos\theta = 27 \Rightarrow \cos\theta = 3/5.$

$\Rightarrow \theta = \cos^{-1}(3/5) = 53°$

b) $v = \sqrt{57 - 3g(1 + \cos\theta)}$ from equation (2)

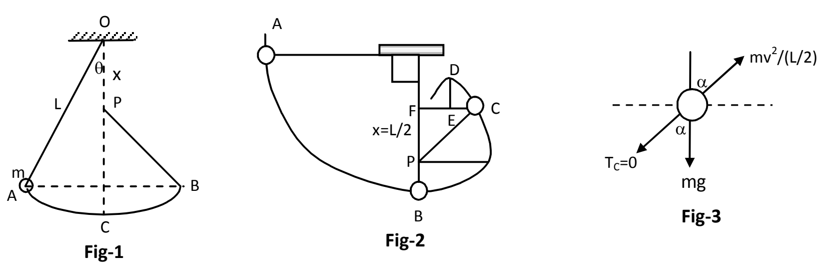
$= \sqrt{9} = 3$ m/sec.

c) As the string becomes slack at point B, the particle will start making projectile motion.

$H = OE + DC = 1.5 \cos\theta + \dfrac{u^2 \sin^2\theta}{2g}$

$= (1.5) \times (3/5) + \dfrac{9 \times (0.8)^2}{2 \times 10} = 1.2$ m.

58.



a) When the bob has an initial height less than the peg and then released from rest (figure 1), let body travels from A to B.

Since, Total energy at A = Total energy at B

$\therefore (K.E)_A = (PE)_A = (KE)_B + (PE)_B$

$\Rightarrow$ $(PE)_A = (PE)_B$ [because, $(KE)_A = (KE)_B = 0$]

So, the maximum height reached by the bob is equal to initial height.

b) When the pendulum is released with $\theta = 90°$ and $x = L/2$, (figure 2) the path of the particle is shown in the figure 2.

At point C, the string will become slack and so the particle will start making projectile motion. (Refer Q.No. 56)

$(1/2)mv_c^2 - 0 = mg\,(L/2)\,(1 - \cos\alpha)$

because, distance between A nd C in the vertical direction is L/2 (1 – cos α)

$\Rightarrow v_c^2 = gL(1 - \cos\theta)$ ..(1)

Again, form the freebody diagram (fig – 3)

$\frac{mv_c^2}{L/2} = mg\cos\alpha$ {because $T_c = 0$}

So, $V_C^2 = \frac{gL}{2}\cos\alpha$ ..(2)

From Eqn.(1) and equn (2),

$gL(1 - \cos\alpha) = \frac{gL}{2}\cos\alpha$

$\Rightarrow 1 - \cos\alpha = 1/2 \cos\alpha$

$\Rightarrow 3/2 \cos\alpha = 1 \Rightarrow \cos\alpha = 2/3$ ..(3)

To find highest position C, before the string becomes slack

$BF = \frac{L}{2} + \frac{L}{2}\cos\theta = \frac{L}{2} + \frac{L}{2} \times \frac{2}{3} = L\left(\frac{1}{2} + \frac{1}{3}\right)$

So, BF = (5L/6)

c) If the particle has to complete a vertical circle, at the point C.

$\frac{mv_c^2}{(L-x)} = mg$

$\Rightarrow v_c^2 = g(L - x)$ ..(1)

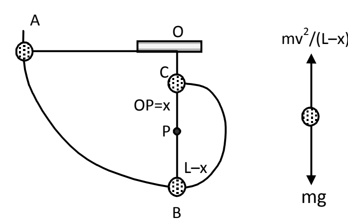
Again, applying energy principle between A and C,

$1/2\, mv_c^2 - 0 = mg\,(OC)$

$\Rightarrow 1/2\, v_c^2 = mg\,[L - 2(L - x)] = mg\,(2x - L)$

$\Rightarrow v_c^2 = 2g(2x - L)$ ..(2)

From equn. (1) and equn (2)



$g(L - x) = 2g\,(2x - L)$

$\Rightarrow L - x = 4x - 2L$

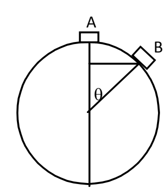
$\Rightarrow 5x = 3L$

$\therefore \frac{x}{L} = \frac{3}{5} = 0.6$

So, the rates (x/L) should be 0.6

59. Let the velocity be v when the body leaves the surface.

From the freebody diagram,

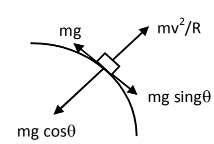


$\frac{mv^2}{R} = mg\cos\theta$ [Because normal reaction]

$v^2 = Rg\cos\theta$ ..(1)

Again, form work-energy principle,

Change in K.E. = work done



$\Rightarrow 1/2\, mv^2 - 0 = mg(R - R\cos\theta)$

$\Rightarrow v^2 = 2gR\,(1 - \cos\theta)$ ..(2)

From (1) and (2)

$Rg\cos\theta = 2gR\,(1 - \cos\theta)$

$3gR \cos\theta = 2\, gR$

$\text{Cos}\,\theta = 2/3$

$\theta = \cos^{-1}(2/3)$

60. a) When the particle is released from rest (fig–1), the centrifugal force is zero.

N force is zero = mg cos θ

$= mg \cos 30° = \frac{\sqrt{3}mg}{2}$

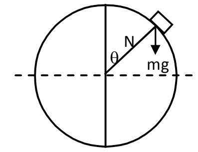


Fig-1

b) When the particle leaves contact with the surface (fig–2), N = 0.

So, $\frac{mv^2}{R}$ mg cos θ

$\Rightarrow v^2 = Rg \cos\theta$ ..(1)

Again, $\frac{1}{2} mv^2 = mgR (\cos 30° - \cos\theta)$

$\Rightarrow v^2 = 2Rg \left(\frac{\sqrt{3}}{2} - \cos\theta\right)$ ..(2)

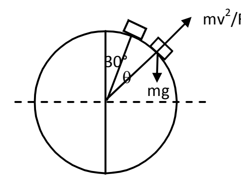


Fig-2

From equn. (1) and equn. (2)

$Rg \cos\theta = \sqrt{3}\, Rg - 2Rg \cos\theta$

$\Rightarrow 3 \cos\theta = \sqrt{3}$

$\Rightarrow \cos\theta = \frac{1}{\sqrt{3}} \Rightarrow \theta = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

So, the distance travelled by the particle before leaving contact,

$\ell = R(\theta - \pi/6)$ [because $30° = \pi/6$]

putting the value of θ, we get $\ell = 0.43R$

61. a) Radius =R

horizontal speed = v

From the free body diagram, (fig–1)

N = Normal force $= mg - \frac{mv^2}{R}$

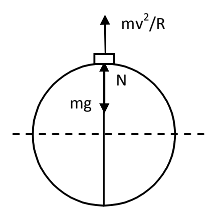


Fig-1

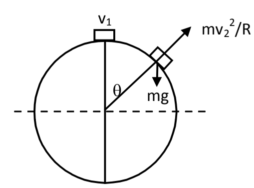


Fig-2

b) When the particle is given maximum velocity so that the centrifugal force balances the weight, the particle does not slip on the sphere.

$\frac{mv^2}{R} = mg \Rightarrow v = \sqrt{gR}$

c) If the body is given velocity v_1

$v_1 = \sqrt{gR}/2$

$v_1^2 - gR/4$

Let the velocity be v_2 when it leaves contact with the surface, (fig–2)

So, $\frac{mv^2}{R} = mg \cos\theta$

$\Rightarrow v_2^2 = Rg \cos\theta$ ..(1)

Again, $1/2\ mv_2^2 - 1/2\ mv_1^2 = mgR(1 - \cos\theta)$

$\Rightarrow v_2^2 = v_1^2 + 2gR(1 - \cos\theta)$ ..(2)

From equn. (1) and equn (2)

Rg cos θ = (Rg/4) + 2gR (1 – cos θ)

⇒ cos θ = (1/4) + 2 – 2 cos θ

⇒ 3 cos θ = 9/4

⇒ cos θ = 3/4

⇒ θ = $\cos^{-1}$ (3/4)

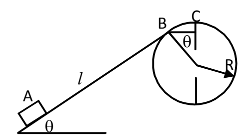
62. a) Net force on the particle between A & B, F = mg sin θ

work done to reach B, W = FS = mg sin θ ℓ

Again, work done to reach B to C = mgh = mg R (1 – cos θ)

So, Total workdone = mg[ℓsin θ + R(1 – cos θ)]

Now, change in K.E. = work done



⇒ 1/2 mv_O^2 = mg [ℓ sin θ + R (1 – cos θ)

⇒ $v_o = \sqrt{2g(R(1-\cos\theta)+\ell\sin\theta)}$

b) When the block is projected at a speed $2v_o$.

Let the velocity at C will be V_c.

Applying energy principle,

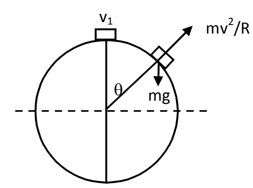
1/2 mv_c^2 - 1/2 m $(2v_o)^2$ = –mg [ℓsin θ + R(1 – cos θ)]

⇒ $v_c^2 = 4v_o – 2g$ [ℓsin θ + R(1 – cosθ)]

4.2g [ℓsin θ + R(1 – cos θ)] – 2g [ℓsin θ + R(1 – cos θ)

= 6g [ℓsin θ + R(1 – cos θ)]

So, force acting on the body,



⇒ $N = \frac{mv_c^2}{R}$ = 6mg [(ℓ/R) sin θ + 1 – cos θ]

c) Let the block loose contact after making an angle θ

$\frac{mv^2}{R}$ = mg cos θ ⇒ v^2 = Rg cos θ ..(1)

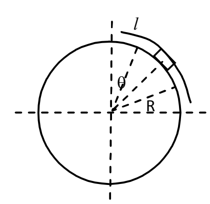
Again, 1/2 mv^2 = mg (R – R cos θ) ⇒ v^2 = 2gR (1 – cos θ) ..(2)........(?)

From (1) and (2) cos θ = 2/3 ⇒ θ = $\cos^{-1}$ (2/3)

63. Let us consider a small element which makes angle 'dθ' at the centre.

∴ dm = (m/ℓ)Rd θ

a) Gravitational potential energy of 'dm' with respect to centre of the sphere



= (dm)g R cos θ

= (mg/ℓ) Rcos θ dθ

So, Total G.P.E. = $\int_0^{\ell/r} \frac{mgR^2}{\ell}$ cos θ d θ [α = (ℓ/R)](angle subtended by the chain at the centre)……….

= $\frac{mR^2g}{\ell}$ [sin θ] (ℓ/R) = $\frac{mRg}{\ell}$ sin (ℓ/R)

b) When the chain is released from rest and slides down through an angle θ, the K.E. of the chain is given

K.E. = Change in potential energy.

$= \frac{mR^2g}{\ell} \sin(\ell/R) - m\int \frac{gR^2}{\ell} \cos\theta \, d\theta$..........?

$= \frac{mR^2g}{\ell} [\sin(\ell/R) + \sin\theta - \sin\{\theta + (\ell/R)\}]$

c) Since, K.E. = 1/2 $mv^2 = \frac{mR^2g}{\ell} [\sin(\ell/R) + \sin\theta - \sin\{\theta + (\ell/R)\}]$

Taking derivative of both sides with respect to 't'

$(1/2) \times 2v \times \frac{dv}{dt} = \frac{R^2g}{\ell} [\cos\theta \times \frac{d\theta}{dt} - \cos(\theta + \ell/R) \frac{d\theta}{dt}]$

$\therefore (R \frac{d\theta}{dt}) \frac{dv}{dt} = \frac{R^2g}{\ell} \times \frac{d\theta}{dt} [\cos\theta - \cos(\theta + (\ell/R))]$

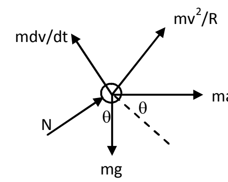
When the chain starts sliding down, $\theta = 0$.

So, $\frac{dv}{dt} = \frac{Rg}{\ell} [1 - \cos(\ell/R)]$

64. Let the sphere move towards left with an acceleration 'a

Let m = mass of the particle

The particle 'm' will also experience the inertia due to acceleration 'a' as it is on the sphere. It will also experience the tangential inertia force (m (dv/dt)) and centrifugal force (mv^2/R).



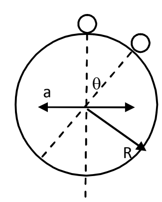
$m\frac{dv}{dt} = ma\cos\theta + mg\sin\theta \Rightarrow mv \frac{dv}{dt} = ma\cos\theta \left(R\frac{d\theta}{dt}\right) + mg\sin\theta \left(R\frac{d\theta}{dt}\right)$

Because, $v = R\frac{d\theta}{dt}$

$\Rightarrow v\,dv = aR\cos\theta\, d\theta + gR\sin\theta\, d\theta$

Integrating both sides we get,

$\frac{v^2}{2} = aR\sin\theta - gR\cos\theta + C$



Given that, at $\theta = 0$, $v = 0$, So, $C = gR$

So, $\frac{v^2}{2} = aR\sin\theta - gR\cos\theta + gR$

$\therefore v^2 = 2R(a\sin\theta + g - g\cos\theta) \Rightarrow v = [2R(a\sin\theta + g - g\cos\theta)]^{1/2}$

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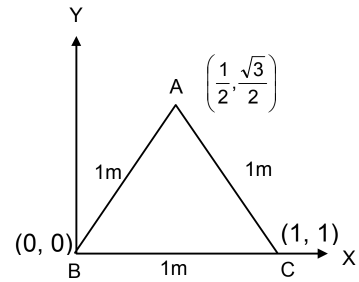
SOLUTIONS TO CONCEPTS
CHAPTER 9

1. $m_1 = 1\text{kg}$, $m_2 = 2\text{kg}$, $m_3 = 3\text{kg}$,
$x_1 = 0$, $x_2 = 1$, $x_3 = 1/2$
$y_1 = 0$, $y_2 = 0$, $y_3 = \sqrt{3}/2$
The position of centre of mass is

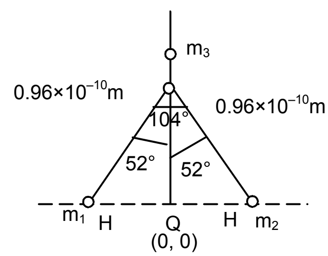


$$\text{C.M} = \left(\frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3}, \frac{m_1y_1 + m_2y_2 + m_3y_3}{m_1 + m_2 + m_3}\right)$$

$$= \left(\frac{(1\times 0) + (2\times 1) + (3\times 1/2)}{1+2+3}, \frac{(1\times 0) + (2\times 0) + (3\times(\sqrt{3}/2))}{1+2+3}\right)$$

$$= \left(\frac{7}{12}, \frac{3\sqrt{3}}{12}\right) \text{ from the point B.}$$

2. Let θ be the origin of the system
In the above figure
$m_1 = 1\text{gm}$, $x_1 = -(0.96\times10^{-10})\sin 52°$ $y_1 = 0$
$m_2 = 1\text{gm}$, $x_2 = -(0.96\times10^{-10})\sin 52°$ $y_2 = 0$
$x_3 = 0$ $y_3 = (0.96 \times 10^{-10}) \cos 52°$
The position of centre of mass

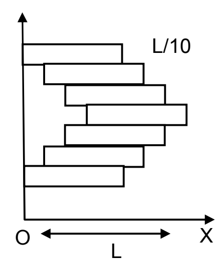


$$\left(\frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3}, \frac{m_1y_1 + m_2y_2 + m_3y_3}{m_1 + m_2 + m_3}\right)$$

$$= \left(\frac{-(0.96\times10^{-10})\times\sin 52 + (0.96\times10^{-10})\sin 52 + 16\times 0}{1+1+16}, \frac{0+0+16y_3}{18}\right)$$

$$= \left(0,\ (8/9)0.96\times10^{-10}\cos 52^\circ\right)$$

3. Let 'O' (0,0) be the origin of the system.
Each brick is mass 'M' & length 'L'.
Each brick is displaced w.r.t. one in contact by 'L/10'
∴The X coordinate of the centre of mass

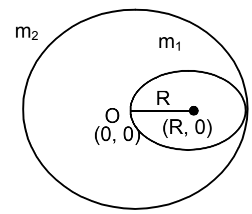


$$\overline{X}_{cm} = \frac{m\left(\frac{L}{2}\right) + m\left(\frac{L}{2} + \frac{L}{10}\right) + m\left(\frac{L}{2} + \frac{2L}{10}\right) + m\left(\frac{L}{2} + \frac{3L}{10}\right) + m\left(\frac{L}{2} + \frac{3L}{10} - \frac{L}{10}\right) + m\left(\frac{L}{2} + \frac{L}{10}\right) + m\left(\frac{L}{2}\right)}{7m}$$

$$= \frac{\frac{L}{2} + \frac{L}{2} + \frac{L}{10} + \frac{L}{2} + \frac{L}{5} + \frac{L}{2} + \frac{3L}{10} + \frac{L}{2} + \frac{L}{5} + \frac{L}{2} + \frac{L}{10} + \frac{L}{2}}{7}$$

$$= \frac{\frac{7L}{2} + \frac{5L}{10} + \frac{2L}{5}}{7} = \frac{35L + 5L + 4L}{10\times 7} = \frac{44L}{70} = \frac{11}{35}L$$

4. Let the centre of the bigger disc be the origin.
2R = Radius of bigger disc
R = Radius of smaller disc
$m_1 = \pi R^2 \times T \times \rho$
$m_2 = \pi(2R)^2 \text{ I } T \times \rho$
where T = Thickness of the two discs
ρ = Density of the two discs
∴ The position of the centre of mass



$\left(\frac{m_1x_1+m_2x_2}{m_1+m_2}, \frac{m_1y_1+m_2y_2}{m_1+m_2}\right)$

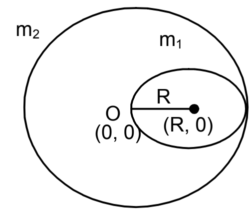
$x_1 = R \qquad y_1 = 0$

$x_2 = 0 \qquad y_2 = 0$

$\left(\frac{\pi R^2 T\rho R + 0}{\pi R^2 T\rho + \pi(2R)^2 T\rho}, \frac{0}{m_1+m_2}\right) = \left(\frac{\pi R^2 T\rho R}{5\pi R^2 T\rho}, 0\right) = \left(\frac{R}{5}, 0\right)$

At R/5 from the centre of bigger disc towards the centre of smaller disc.

5. Let '0' be the origin of the system.



R = radius of the smaller disc

2R = radius of the bigger disc

The smaller disc is cut out from the bigger disc

As from the figure

$m_1 = \pi R^2 T\rho \qquad x_1 = R \qquad y_1 = 0$

$m_2 = \pi(2R)^2 T\rho \qquad x_2 = 0 \qquad y_2 = 0$

The position of C.M. = $\left(\frac{-\pi R^2 T\rho R + 0}{-\pi R^2 T\rho + \pi(2R)^2 T\rho R}, \frac{0+0}{m_1+m_2}\right)$

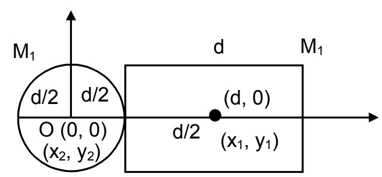
$= \left(\frac{-\pi R^2 T\rho R}{3\pi R^2 T\rho}, 0\right) = \left(-\frac{R}{3}, 0\right)$

C.M. is at R/3 from the centre of bigger disc away from centre of the hole.

6. Let m be the mass per unit area.

∴ Mass of the square plate = $M_1 = d^2 m$

Mass of the circular disc = $M_2 = \frac{\pi d^2}{4} m$



Let the centre of the circular disc be the origin of the system.

∴ Position of centre of mass

$= \left(\frac{d^2 md + \pi(d^2/4)m \times 0}{d^2 m + \pi(d^2/4)m}, \frac{0+0}{M_1+M_2}\right) = \left(\frac{d^3 m}{d^2 m\left(1+\frac{\pi}{4}\right)}, 0\right) = \left(\frac{4d}{\pi+4}, 0\right)$

The new centre of mass is $\left(\frac{4d}{\pi+4}\right)$ right of the centre of circular disc.

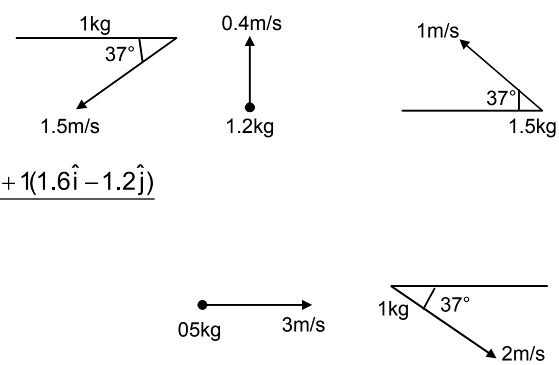
7. $m_1 = 1\text{kg.} \qquad \vec{v}_1 = -1.5 \cos 37\ \hat{i} - 1.55 \sin 37\ \hat{j} = -1.2\ \hat{i} - 0.9\ \hat{j}$

$m_2 = 1.2\text{kg.} \qquad \vec{v}_2 = 0.4\ \hat{j}$

$m_3 = 1.5\text{kg} \qquad \vec{v}_3 = -0.8\ \hat{i} + 0.6\ \hat{j}$

$m_4 = 0.5\text{kg} \qquad \vec{v}_4 = 3\ \hat{i}$

$m_5 = 1\text{kg} \qquad \vec{v}_5 = 1.6\ \hat{i} - 1.2\ \hat{j}$



So, $\vec{v}_c = \frac{m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + m_4\vec{v}_4 + m_5\vec{v}_5}{m_1+m_2+m_3+m_4+m_5}$

$= \frac{1(-1.2\hat{i} - 0.9\hat{j}) + 1.2(0.4\hat{j}) + 1.5(-0.8\hat{i} + 0.6\hat{j}) + 0.5(3\hat{i}) + 1(1.6\hat{i} - 1.2\hat{j})}{5.2}$

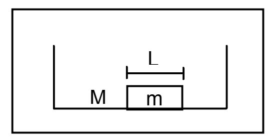
$= \frac{-1.2\hat{i} - 0.9\hat{j} + 4.8\hat{j} - 1.2\hat{i} + .90\hat{j} + 1.5\hat{i} + 1.6\hat{i} - 1.2\hat{j}}{5.2}$

$= \frac{0.7\hat{i}}{5.2} - \frac{0.72\hat{j}}{5.2}$

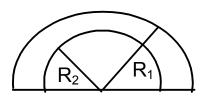
8. Two masses m_1 & m_2 are placed on the X-axis
m_1 = 10 kg, m_2 = 20kg.
The first mass is displaced by a distance of 2 cm
$\therefore \overline{X}_{cm} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2} = \frac{10 \times 2 + 20x_2}{30}$
$\Rightarrow 0 = \frac{20 + 20x_2}{30} \Rightarrow 20 + 20x_2 = 0$
$\Rightarrow 20 = -20x_2 \Rightarrow x_2 = -1.$
$\therefore$ The 2nd mass should be displaced by a distance 1cm towards left so as to kept the position of centre of mass unchanged.

9. Two masses m_1 & m_2 are kept in a vertical line
m_1 = 10kg, m_2 = 30kg
The first block is raised through a height of 7 cm.
The centre of mass is raised by 1 cm.
$\therefore 1 = \frac{m_1y_1 + m_2y_2}{m_1 + m_2} = \frac{10 \times 7 + 30y_2}{40}$
$\Rightarrow 1 = \frac{70 + 30y_2}{40} \Rightarrow 70 + 30y_2 = 40 \Rightarrow 30y_2 = -30 \Rightarrow y_2 = -1.$
The 30 kg body should be displaced 1cm downward inorder to raise the centre of mass through 1 cm.

10. As the hall is gravity free, after the ice melts, it would tend to acquire a spherical shape. But, there is no external force acting on the system. So, the centre of mass of the system would not move.

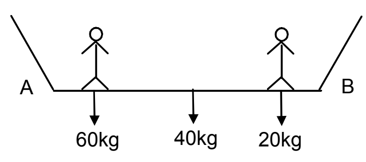


11. The centre of mass of the blate will be on the symmetrical axis.



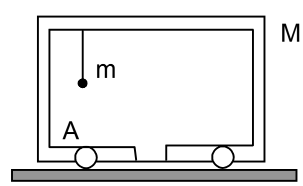
$$\Rightarrow \overline{y}_{cm} = \frac{\left(\frac{\pi R_2^2}{2}\right)\left(\frac{4R_2}{3\pi}\right) - \left(\frac{\pi R_1^2}{2}\right)\left(\frac{4R_1}{3\pi}\right)}{\frac{\pi R_2^2}{2} - \frac{\pi R_1^2}{2}}$$
$$= \frac{(2/3)R_2^3 - (2/3)R_1^3}{\pi/2(R_2^2 - R_1^2)} = \frac{4}{3\pi}\frac{(R_2 - R_1)(R_2^2 + R_1^2 + R_1R_2)}{(R_2 - R_1)(R_2 + R_1)}$$
$$= \frac{4}{3\pi}\frac{(R_2^2 + R_1^2 + R_1R_2)}{R_1 + R_2}$$ above the centre.

12. m_1 = 60kg, m_2 = 40kg , m_3 = 50kg,
Let A be the origin of the system.
Initially Mr. Verma & Mr. Mathur are at extreme position of the boat.
$\therefore$ The centre of mass will be at a distance
$= \frac{60 \times 0 + 40 \times 2 + 50 \times 4}{150} = \frac{280}{150}$ = 1.87m from 'A'



When they come to the mid point of the boat the CM lies at 2m from 'A'.
$\therefore$ The shift in CM = 2 – 1.87 = 0.13m towards right.
But as there is no external force in longitudinal direction their CM would not shift.
So, the boat moves 0.13m or 13 cm towards right.

13. Let the bob fall at A,. The mass of bob = m.
The mass of cart = M.
Initially their centre of mass will be at
$\frac{m \times L + M \times 0}{M + m} = \left(\frac{m}{M + m}\right)L$
Distance from P
When, the bob falls in the slot the CM is at a distance 'O' from P.



Shift in CM = 0 – $\frac{mL}{M+m}$ = $-\frac{mL}{M+m}$ towards left

= $\frac{mL}{M+m}$ towards right.

But there is no external force in horizontal direction.

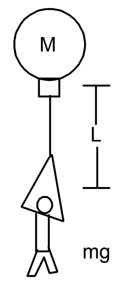
So the cart displaces a distance $\frac{mL}{M+m}$ towards right.

14. Initially the monkey & balloon are at rest.

So the CM is at 'P'

When the monkey descends through a distance 'L'

The CM will shift



$t_o = \frac{m \times L + M \times 0}{M+m} = \frac{mL}{M+m}$ from P

So, the balloon descends through a distance $\frac{mL}{M+m}$

15. Let the mass of the to particles be m_1 & m_2 respectively

m_1 = 1kg, m_2 = 4kg

$\therefore$ According to question

$\frac{1}{2} m_1 v_1^2 = \frac{1}{2} m_2 v_2^2$

$\Rightarrow \frac{m_1}{m_2} = \frac{v_2^2}{v_1^2} \Rightarrow \frac{v_2}{v_1} = \sqrt{\frac{m_1}{m_2}} \Rightarrow \frac{v_1}{v_2} = \sqrt{\frac{m_2}{m_1}}$

Now, $\frac{m_1 v_1}{m_2 v_2} = \frac{m_1}{m_2} \times \sqrt{\frac{m_2}{m_1}} = \frac{\sqrt{m_1}}{\sqrt{m_2}} = \frac{\sqrt{1}}{\sqrt{4}}$ = 1/2

$\Rightarrow \frac{m_1 v_1}{m_2 v_2}$ = 1 : 2

16. As uranium 238 nucleus emits a α-particle with a speed of 1.4 × 10^7m/sec. Let v_2 be the speed of the residual nucleus thorium 234.

$\therefore m_1 v_1 = m_2 v_2$

$\Rightarrow 4 \times 1.4 \times 10^7 = 234 \times v_2$

$\Rightarrow v_2 = \frac{4 \times 1.4 \times 10^7}{234}$ = 2.4 × 10^5 m/sec.

17. $m_1 v_1 = m_2 v_2$

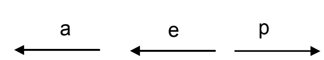
$\Rightarrow 50 \times 1.8 = 6 \times 10^{24} \times v_2$

$\Rightarrow v_2 = \frac{50 \times 1.8}{6 \times 10^{24}}$ = 1.5 × 10^{-23} m/sec

so, the earth will recoil at a speed of 1.5 × 10^{-23}m/sec.

18. Mass of proton = 1.67 × 10^{-27}

Let 'V_p' be the velocity of proton



Given momentum of electron = 1.4 × 10^{-26}kg m/sec

Given momentum of antineutrino = 6.4 × 10^{-27} kg m/sec

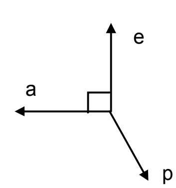
a) The electron & the antineutrino are ejected in the same direction. As the total momentum is conserved the proton should be ejected in the opposite direction.

$1.67 \times 10^{-27} \times V_p = 1.4 \times 10^{-26} + 6.4 \times 10^{-27} = 20.4 \times 10^{-27}$

$\Rightarrow V_p$ = (20.4 /1.67) = 12.2 m/sec in the opposite direction.

b) The electron & antineutrino are ejected $\perp^r$ to each other.

Total momentum of electron and antineutrino,

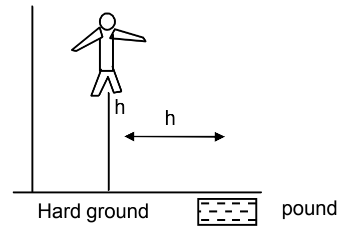


$= \sqrt{(14)^2 + (6.4)^2} \times 10^{-27}$ kg m/s = 15.4 × 10^{-27} kg m/s

Since, 1.67 × 10^{-27} V_p = 15.4 × 10^{-27} kg m/s

So V_p = 9.2 m/s

19. Mass of man = M, Initial velocity = 0
Mass of bad = m
Let the throws the bag towards left with a velocity v towards left. So, there is no external force in the horizontal direction.
The momentum will be conserved. Let he goes right with a velocity



$mv = MV \Rightarrow V = \frac{mv}{M} \Rightarrow v = \frac{MV}{m}$ ..(i)

Let the total time he will take to reach ground = $\sqrt{2H/g}$ = t_1

Let the total time he will take to reach the height h = t_2 = $\sqrt{2(H-h)/g}$

Then the time of his flying = $t_1 - t_2$ = $\sqrt{2H/g} - \sqrt{2(H-h)/g} = \sqrt{2/g}\left(\sqrt{H} - \sqrt{H-h}\right)$

Within this time he reaches the ground in the pond covering a horizontal distance x

$\Rightarrow x = V \times t \Rightarrow V = x / t$

$\therefore v = \frac{M}{m}\frac{x}{t} = \frac{M}{m} \times \frac{\sqrt{g}}{\sqrt{2}\left(\sqrt{H} - \sqrt{H-h}\right)}$

As there is no external force in horizontal direction, the x-coordinate of CM will remain at that position.

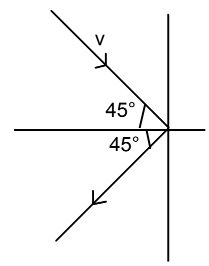
$\Rightarrow 0 = \frac{M \times (x) + m \times x_1}{M+m} \Rightarrow x_1 = -\frac{M}{m}x$

$\therefore$ The bag will reach the bottom at a distance (M/m) x towards left of the line it falls.

20. Mass = 50g = 0.05kg

$v = 2\cos 45°\,\hat{i} - 2\sin 45°\,\hat{j}$

$v_1 = -2\cos 45°\,\hat{i} - 2\sin 45°\,\hat{j}$



a) change in momentum = $m\vec{v} - m\vec{v}_1$

$= 0.05\,(2\cos 45°\,\hat{i} - 2\sin 45°\,\hat{j}) - 0.05\,(-2\cos 45°\,\hat{i} - 2\sin 45°\,\hat{j})$

$= 0.1\cos 45°\,\hat{i} - 0.1\sin 45°\,\hat{j} + 0.1\cos 45°\,\hat{i} + 0.1\sin 45°\,\hat{j}$

$= 0.2\cos 45°\,\hat{i}$

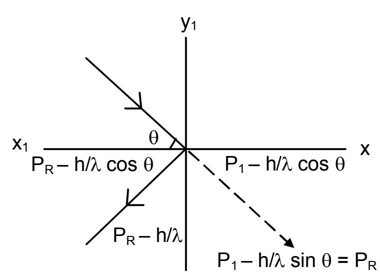
$\therefore$ magnitude = $\sqrt{\left(\frac{0.2}{\sqrt{2}}\right)^2} = \frac{0.2}{\sqrt{2}} = 0.14$ kg m/s

c) The change in magnitude of the momentum of the ball

$-\left|\vec{P}_i\right| - \left|\vec{P}_f\right| = 2 \times 0.5 - 2 \times 0.5 = 0.$

21. $\vec{P}_{incidence} = (h/\lambda)\cos\theta\,\hat{i} - (h/\lambda)\sin\theta\,\hat{j}$

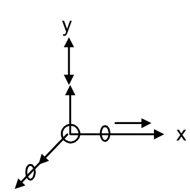
$P_{Reflected} = -(h/\lambda)\cos\theta\,\hat{i} - (h/\lambda)\sin\theta\,\hat{j}$



The change in momentum will be only in the x-axis direction. i.e.

$\left|\Delta P\right| = (h/\lambda)\cos\theta - ((h/\lambda)\cos\theta) = (2h/\lambda)\cos\theta$

22. As the block is exploded only due to its internal energy. So net external force during this process is 0. So the centre mass will not change.
Let the body while exploded was at the origin of the co-ordinate system.
If the two bodies of equal mass is moving at a speed of 10m/s in + x & +y axis direction respectively,



$\sqrt{10^2 + 10^2 + 2 10.10\cos 90^o}$ = $10\sqrt{2}$ m/s 45° w.r.t. + x axis

If the centre mass is at rest, then the third mass which have equal mass with other two, will move in the opposite direction (i.e. 135° w.r.t. + x- axis) of the resultant at the same velocity.

23. Since the spaceship is removed from any material object & totally isolated from surrounding, the missions by astronauts couldn't slip away from the spaceship. So the total mass of the spaceship remain unchanged and also its velocity.

24. d = 1cm, v = 20 m/s, u = 0, ρ = 900 kg/m^3 = 0.9gm/cm^3
volume = $(4/3)\pi r^3 = (4/3)\,\pi\,(0.5)^3 = 0.5238\text{cm}^3$
$\therefore$ mass = vρ = 0.5238 × 0.9 = 0.4714258gm
$\therefore$ mass of 2000 hailstone = 2000 × 0.4714 = 947.857
$\therefore$ Rate of change in momentum per unit area = 947.857 × 2000 = 19N/m^3
$\therefore$ Total force exerted = 19 × 100 = 1900 N.

25. A ball of mass m is dropped onto a floor from a certain height let 'h'.
$\therefore v_1 = \sqrt{2gh}$, $v_1 = 0$, $v_2 = -\sqrt{2gh}$ & $v_2 = 0$
$\therefore$ Rate of change of velocity :-
$$F = \frac{m \times 2\sqrt{2gh}}{t}$$
$\therefore v = \sqrt{2gh}$, s = h, v = 0
$\Rightarrow$ v = u + at
$$\Rightarrow \sqrt{2gh} = g\,t \Rightarrow t = \sqrt{\frac{2h}{g}}$$
$\therefore$ Total time $2\sqrt{\frac{2h}{t}}$
$$\therefore F = \frac{m \times 2\sqrt{2gh}}{2\sqrt{\frac{2h}{g}}} = mg$$

26. A railroad car of mass M is at rest on frictionless rails when a man of mass m starts moving on the car towards the engine. The car recoils with a speed v backward on the rails.
Let the mass is moving with a velocity x w.r.t. the engine.
$\therefore$The velocity of the mass w.r.t earth is (x – v) towards right
V_{cm} = 0 (Initially at rest)
$\therefore$ 0 = –Mv + m(x – v)
$$\Rightarrow Mv = m(x - v) \Rightarrow mx = Mv + mv \Rightarrow x = \left(\frac{M+m}{m}\right)v \Rightarrow x = \left(1+\frac{M}{m}\right)v$$

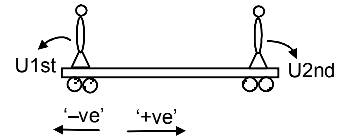
27. A gun is mounted on a railroad car. The mass of the car, the gun, the shells and the operator is 50m where m is the mass of one shell. The muzzle velocity of the shells is 200m/s.
Initial, V_{cm} = 0.
$$\therefore 0 = 49\,m \times V + m \times 200 \Rightarrow V = \frac{-200}{49}\text{ m/s}$$
$\therefore \frac{200}{49}$ m/s towards left.
When another shell is fired, then the velocity of the car, with respect to the platform is,
$\Rightarrow V` = \frac{200}{49}$ m/s towards left.
When another shell is fired, then the velocity of the car, with respect to the platform is,
$\Rightarrow v` = \frac{200}{48}$ m/s towards left
$\therefore$ Velocity of the car w.r.t the earth is $\left(\frac{200}{49}+\frac{200}{48}\right)$ m/s towards left.

28. Two persons each of mass m are standing at the two extremes of a railroad car of mass m resting on a smooth track.
Case – I
Let the velocity of the railroad car w.r.t the earth is V after the jump of the left man.
$\therefore$ 0 = – mu + (M + m) V

$\Rightarrow V = \frac{mu}{M+m}$ towards right

Case – II

When the man on the right jumps, the velocity of it w.r.t the car is u.



$\therefore$ 0 = mu – Mv'

$\Rightarrow$ **$v' = \frac{mu}{M}$**

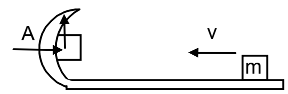
(V′ is the change is velocity of the platform when platform itself is taken as reference assuming the car to be at rest)

$\therefore$ So, net velocity towards left (i.e. the velocity of the car w.r.t. the earth)

$= \frac{mv}{M} - \frac{mv}{M+m} = \frac{mMu + m^2v - Mmu}{M(M+m)} = \frac{m^2v}{M(M+m)}$

29. A small block of mass m which is started with a velocity V on the horizontal part of the bigger block of mass M placed on a horizontal floor.

Since the small body of mass m is started with a velocity V in the horizontal direction, so the total initial momentum at the initial position in the horizontal direction will remain same as the total final momentum at the point A on the bigger block in the horizontal direction.



From L.C.K. m:

mv + M×O = (m + M) v $\Rightarrow v' = \frac{mv}{M+m}$

30. Mass of the boggli = 200kg, V_B = 10 km/hour.

$\therefore$ Mass of the boy = 2.5kg & V_{Boy} = 4km/hour.

If we take the boy & boggle as a system then total momentum before the process of sitting will remain constant after the process of sitting.

$\therefore m_b V_b = m_{boy}V_{boy} = (m_b + m_{boy})\ v$

$\Rightarrow$ 200 × 10 + 25 × 4 = (200 +25) × v

$\Rightarrow v = \frac{2100}{225} = \frac{28}{3}$ = 9.3 m/sec

31. Mass of the ball = m_1 = 0.5kg, velocity of the ball = 5m/s

Mass of the another ball m_2 = 1kg

Let it's velocity = v′ m/s

Using law of conservation of momentum,

0.5 × 5 + 1 × v′ = 0 $\Rightarrow$ v′ = – 2.5

$\therefore$ Velocity of second ball is 2.5 m/s opposite to the direction of motion of 1^{st} ball.

32. Mass of the man = m_1 = 60kg

Speed of the man = v_1 = 10m/s

Mass of the skater = m_2 = 40kg

let its velocity = v′

$\therefore$ 60 × 10 + 0 = 100 × v′ $\Rightarrow$ v′ = 6m/s

loss in K.E. = $(1/2)60 \times (10)^2 - (1/2) \times 100 \times 36$ = 1200 J

33. Using law of conservation of momentum.

$m_1u_1 + m_2u_2 = m_1v(t) + m_2v'$

Where v′ = speed of 2^{nd} particle during collision.

$\Rightarrow m_1u_1 + m_2u_2 = m_1u_1 + m_1 + (t/\Delta t)(v_1 - u_1) + m_2v'$

$\Rightarrow \frac{m_2u_2}{m^2} - \frac{m_1}{m2}\frac{t}{\Delta t}(v_1 - u_1)v'$

$\therefore v' = u_2 - \frac{m_1}{m_2}\frac{t}{\Delta t}(v_1 - u)$

34. Mass of the bullet = m and speed = v

Mass of the ball = M

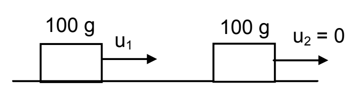
m′ = frictional mass from the ball.

Using law of conservation of momentum,
$mv + 0 = (m' + m) v' + (M - m') v_1$
where v' = final velocity of the bullet + frictional mass
$\Rightarrow v' = \frac{mv - (M + m')V_1}{m + m'}$

35. Mass of 1st ball = m and speed = v
Mass of 2nd ball = m
Let final velocities of 1st and 2nd ball are v_1 and v_2 respectively
Using law of conservation of momentum,
$m(v_1 + v_2) = mv.$
$\Rightarrow v_1 + v_2 = v$ …(1)
Also
$v_1 - v_2 = ev$ …(2)
Given that final K.E. = ¾ Initial K.E.
$\Rightarrow \frac{1}{2} mv_1^2 + \frac{1}{2} mv_2^2 = \frac{3}{4} \times \frac{1}{2} mv^2$
$\Rightarrow v_1^2 + v_2^2 = \frac{3}{4} v^2$
$\Rightarrow \frac{(v_1 + v_2)^2 + (v_1 - v_2)^2}{2} = \frac{3}{4} v^2$
$\Rightarrow \frac{(1 + e^2)v^2}{2} = \frac{3}{4} v^2 \Rightarrow 1 + e^2 = \frac{3}{2} \Rightarrow e^2 = \frac{1}{2} \Rightarrow e = \frac{1}{\sqrt{2}}$

36. Mass of block = 2kg and speed = 2m/s
Mass of 2nd block = 2kg.
Let final velocity of 2nd block = v
using law of conservation of momentum.
$2 \times 2 = (2 + 2) v \Rightarrow v = 1$m/s
∴ Loss in K.E. in inelastic collision
$= (1/2) \times 2 \times (2)^2 v - (1/2)(2 + 2) \times (1)^2 = 4 - 2 = 2$ J
b) Actual loss = $\frac{\text{Maximum loss}}{2} = 1$J
$(1/2) \times 2 \times 2^2 - (1/2) 2 \times v_1^2 + (1/2) \times 2 \times v_2^2 = 1$
$\Rightarrow 4 - (v_1^2 + v_2^2) = 1$
$\Rightarrow 4 - \frac{(1 + e^2) \times 4}{2} = 1$
$\Rightarrow 2(1 + e^2) = 3 \Rightarrow 1 + e^2 = \frac{3}{2} \Rightarrow e^2 = \frac{1}{2} \Rightarrow e = \frac{1}{\sqrt{2}}$

37. Final K.E. = 0.2J
Initial K.E. = $\frac{1}{2} mV_1^2 + 0 = \frac{1}{2} \times 0.1 u^2 = 0.05 u^2$
$mv_1 = mv_2' = mu$
Where v_1 and v_2 are final velocities of 1st and 2nd block respectively.



$\Rightarrow v_1 + v_2 = u$ …(1)
$(v_1 - v_2) + \ell (a_1 - u_2) = 0 \Rightarrow \ell a = v_2 - v_1$ ..(2)
$u_2 = 0, \quad u_1 = u.$
Adding Eq.(1) and Eq.(2)
$2v_2 = (1 + \ell)u \Rightarrow v_2 = (u/2)(1 + \ell)$
$\therefore v_1 = u - \frac{u}{2} - \frac{u}{2}\ell$
$v_1 = \frac{u}{2}(1 - \ell)$
Given $(1/2)mv_1^2 + (1/2)mv_2^2 = 0.2$
$\Rightarrow v_1^2 + v_2^2 = 4$

$\Rightarrow \frac{u^2}{4}(1-\ell)^2 + \frac{u^2}{4}(1+\ell)^2 = 4$ $\Rightarrow \frac{u^2}{2}(1+\ell^2) = 4$ $\Rightarrow u^2 = \frac{8}{1+\ell^2}$

For maximum value of u, denominator should be minimum,

$\Rightarrow \ell = 0$.

$\Rightarrow u^2 = 8 \Rightarrow u = 2\sqrt{2}$ m/s

For minimum value of u, denominator should be maximum,

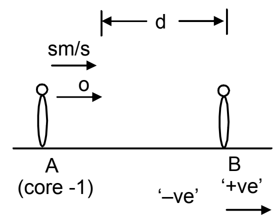
$\Rightarrow \ell = 1$

$u^2 = 4 \Rightarrow u = 2$ m/s

38. Two friends A & B (each 40kg) are sitting on a frictionless platform some distance d apart A rolls a ball of mass 4kg on the platform towards B, which B catches. Then B rolls the ball towards A and A catches it. The ball keeps on moving back & forth between A and B. The ball has a fixed velocity 5m/s.

a) Case – I :– Total momentum of the man A & the ball will remain constant

$\therefore 0 = 4 \times 5 - 40 \times v$ $\Rightarrow v = 0.5$ m/s towards left



b) Case – II :– When B catches the ball, the momentum between the B & the ball will remain constant.

$\Rightarrow 4 \times 5 = 44v \Rightarrow v = (20/44)$ m/s

Case – III :– When B throws the ball, then applying L.C.L.M

$\Rightarrow 44 \times (20/44) = -4 \times 5 + 40 \times v$ $\Rightarrow v = 1$m/s (towards right)

Case – IV :– When a Catches the ball, the applying L.C.L.M.

$\Rightarrow -4 \times 5 + (-0.5) \times 40 = -44v$ $\Rightarrow v = \frac{10}{11}$ m/s towards left.

c) Case – V :– When A throws the ball, then applying L.C.L.M.

$\Rightarrow 44 \times (10/11) = 4 \times 5 - 40 \times V$ $\Rightarrow V = 60/40 = 3/2$ m/s towards left.

Case – VI :– When B receives the ball, then applying L.C.L.M

$\Rightarrow 40 \times 1 + 4 \times 5 = 44 \times v$ $\Rightarrow v = 60/44$ m/s towards right.

Case – VII :– When B throws the ball, then applying L.C.L.M.

$\Rightarrow 44 \times (66/44) = -4 \times 5 + 40 \times V$ $\Rightarrow V = 80/40 = 2$ m/s towards right.

Case – VIII :– When A catches the ball, then applying L.C.L.M

$\Rightarrow -4 \times 5 - 40 \times (3/2) = -44v$ $\Rightarrow v = (80/44) = (20/11)$ m/s towards left.

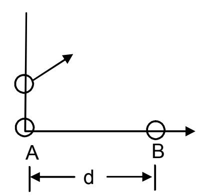
Similarly after 5 round trips

The velocity of A will be (50/11) & velocity of B will be 5 m/s.

d) Since after 6 round trip, the velocity of A is 60/11 i.e.

> 5m/s. So, it can't catch the ball. So it can only roll the ball six.

e) Let the ball & the body A at the initial position be at origin.



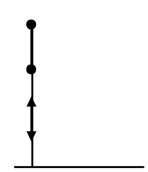
$\therefore X_C = \frac{40 \times 0 + 4 \times 0 + 40 \times d}{40 + 40 + 4} = \frac{10}{11}d$

39. $u = \sqrt{2gh}$ = velocity on the ground when ball approaches the ground.

$\Rightarrow u = \sqrt{2 \times 9.8 \times 2}$

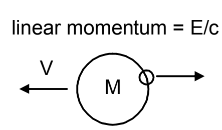
v = velocity of ball when it separates from the ground.

$\vec{v} + \ell\vec{u} = 0$



$\Rightarrow \ell\vec{u} = -\vec{v} \Rightarrow \ell = \frac{\sqrt{2 \times 9.8 \times 1.5}}{\sqrt{2 \times 9.8 \times 2}} = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$

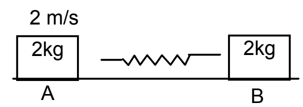
40. K.E. of Nucleus = $(1/2)mv^2 = (1/2)\,m\left(\frac{E}{mc}\right)^2 = \frac{E^2}{2mc^2}$



Energy limited by Gamma photon = E.

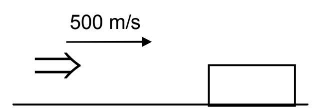
Decrease in internal energy = $E + \frac{E^2}{2mc^2}$

41. Mass of each block M_A and M_B = 2kg.
Initial velocity of the 1[st] block, (V) = 1m/s
V_A = 1 m/s, V_B = 0m/s
Spring constant of the spring = 100 N/m.
The block A strikes the spring with a velocity 1m/s/
After the collision, it's velocity decreases continuously and at a instant the whole system (Block A + the compound spring + Block B) move together with a common velocity.
Let that velocity be V.
Using conservation of energy, $(1/2)\ M_A V_A^2 + (1/2) M_B V_B^2 = (1/2) M_A v^2 + (1/2) M_B v^2 + (1/2) kx^2$.
$(1/2) \times 2(1)^2 + 0 = (1/2) \times 2\times v^2 + (1/2) \times 2 \times v^2 + (1/2)\ x^2 \times 100$
(Where x = max. compression of spring)
$\Rightarrow 1 = 2v^2 + 50x^2$ …(1)
As there is no external force in the horizontal direction, the momentum should be conserved.
$\Rightarrow M_A V_A + M_B V_B = (M_A + M_B)V$.
$\Rightarrow 2 \times 1 = 4 \times v$
$\Rightarrow V = (1/2)$ m/s. …(2)



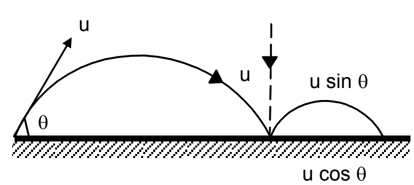
Putting in eq.(1)
1 = 2 × (1/4) + 50x+2+
$\Rightarrow (1/2) = 50x^2$
$\Rightarrow x^2 = 1/100 m^2$
$\Rightarrow x = (1/10)m = 0.1m = 10cm$.

42. Mass of bullet m = 0.02kg.
Initial velocity of bullet V_1 = 500m/s
Mass of block, M = 10kg.
Initial velocity of block u_2 = 0.
Final velocity of bullet = 100 m/s = v.



Let the final velocity of block when the bullet emerges out, if block = v′.
$mv_1 + Mu_2 = mv + Mv'$
$\Rightarrow 0.02 \times 500 = 0.02 \times 100 + 10 \times v'$
$\Rightarrow v' = 0.8$m/s
After moving a distance 0.2 m it stops.
$\Rightarrow$ change in K.E. = Work done
$\Rightarrow 0 - (1/2) \times 10\times (0.8)^2 = -\mu \times 10 \times 10 \times 0.2 \Rightarrow \mu = 0.16$

43. The projected velocity = u.
The angle of projection = θ.
When the projectile hits the ground for the 1[st] time, the velocity would be the same i.e. u.
Here the component of velocity parallel to ground, u cos θ should remain constant. But the vertical component of the projectile undergoes a change after the collision.



$\Rightarrow e = \dfrac{u \sin\theta}{v} \Rightarrow v = eu \sin \theta$.
Now for the 2[nd] projectile motion,
U = velocity of projection = $\sqrt{(u\cos\theta)^2 + (eu\sin\theta)^2}$
and Angle of projection = $\alpha = \tan^{-1}\left(\dfrac{eu\sin\theta}{a\cos\theta}\right) = \tan^{-1}(e \tan \theta)$
or $\tan \alpha = e \tan \theta$ …(2)
Because, $y = x \tan \alpha - \dfrac{gx^2 \sec^2 \alpha}{2u^2}$ …(3)
Here, y = 0, $\tan \alpha = e \tan \theta$, $\sec^2 \alpha = 1 + e^2 \tan^2 \theta$
And $u^2 = u^2 \cos^2 \theta + e^2 \sin^2 \theta$
Putting the above values in the equation (3),

$x\, e \tan\theta = \dfrac{gx^2(1+e^2\tan^2\theta)}{2u^2(\cos^2\theta + e^2\sin^2\theta)}$

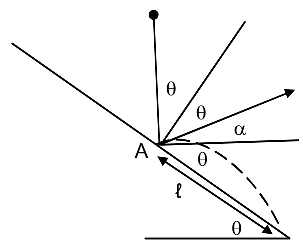
$\Rightarrow x = \dfrac{2eu^2\tan\theta(\cos^2\theta + e^2\sin^2\theta)}{g(1+e^2\tan^2\theta)}$

$\Rightarrow x = \dfrac{2eu^2\tan\theta - \cos^2\theta}{g} = \dfrac{eu^2\sin 2\theta}{g}$

$\Rightarrow$ So, from the starting point O, it will fall at a distance

$= \dfrac{u^2\sin 2\theta}{g} + \dfrac{eu^2\sin 2\theta}{g} = \dfrac{u^2\sin 2\theta}{g}(1+e)$

44. Angle inclination of the plane = θ
M the body falls through a height of h,
The striking velocity of the projectile with the indined plane v = $\sqrt{2gh}$
Now, the projectile makes on angle (90° – 2θ)
Velocity of projection = u = $\sqrt{2gh}$



Let AB = L.
So, x = $\ell\cos\theta$, y = $-\ell\sin\theta$
From equation of trajectory,

$y = x\tan\alpha - \dfrac{gx^2\sec^2\alpha}{2u^2}$

$-\ell\sin\theta = \ell\cos\theta \cdot \tan(90° - 2\theta) - \dfrac{g\times\ell^2\cos^2\theta\sec^2(90° - 2\theta)}{2\times 2gh}$

$\Rightarrow -\ell\sin\theta = \ell\cos\theta \cdot \cot 2\theta - \dfrac{g\ell^2\cos^2\theta\,\mathrm{cosec}^2 2\theta}{4gh}$

So, $\dfrac{\ell\cos^2\theta\,\mathrm{cosec}^2 2\theta}{4h} = \sin\theta + \cos\theta\cot 2\theta$

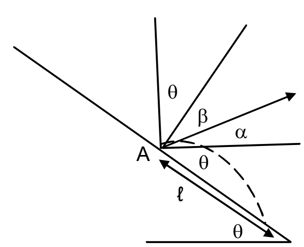
$\Rightarrow \ell = \dfrac{4h}{\cos^2\theta\,\mathrm{cosec}^2 2\theta}(\sin\theta + \cos\theta\cot 2\theta) = \dfrac{4h\times\sin^2 2\theta}{\cos^2\theta}\left(\sin\theta + \cos\theta\times\dfrac{\cos 2\theta}{\sin 2\theta}\right)$

$= \dfrac{4h\times 4\sin^2\theta\cos^2\theta}{\cos^2\theta}\left(\dfrac{\sin\theta\times\sin 2\theta + \cos\theta\cos 2\theta}{\sin 2\theta}\right) = 16\,h\sin^2\theta \times \dfrac{\cos\theta}{2\sin\theta\cos\theta} = 8h\sin\theta$

45. h = 5m, θ = 45°, e = (3/4)
Here the velocity with which it would strike = v = $\sqrt{2g\times 5}$ = 10m/sec
After collision, let it make an angle β with horizontal. The horizontal component of velocity 10 cos 45° will remain unchanged and the velocity in the perpendicular direction to the plane after wllisine.

$\Rightarrow V_y = e \times 10\sin 45°$

$= (3/4)\times 10\times\dfrac{1}{\sqrt{2}} = (3.75)\sqrt{2}$ m/sec



$V_x = 10\cos 45° = 5\sqrt{2}$ m/sec

So, $u = \sqrt{V_x^2 + V_y^2} = \sqrt{50 + 28.125} = \sqrt{78.125} = 8.83$ m/sec

Angle of reflection from the wall $\beta = \tan^{-1}\left(\dfrac{3.75\sqrt{2}}{5\sqrt{2}}\right) = \tan^{-1}\left(\dfrac{3}{4}\right) = 37°$

$\Rightarrow$ Angle of projection $\alpha = 90 - (\theta + \beta) = 90 - (45° + 37°) = 8°$
Let the distance where it falls = L
$\Rightarrow$ x = L cos θ, y = – L sin θ
Angle of projection (α) = –8°

Using equation of trajectory, $y = x \tan \alpha - \frac{gx^2 \sec^2 \alpha}{2u^2}$

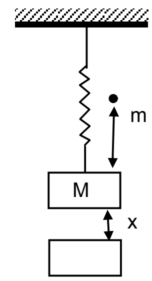
$\Rightarrow - \ell \sin \theta = \ell \cos \theta \times \tan 8° - \frac{g}{2} \times \frac{\ell \cos^2\theta \sec^2 8°}{u^2}$

$\Rightarrow - \sin 45° = \cos 45° - \tan 8° - \frac{10 \cos^2 45° \sec 8°}{(8.83)^2} (\ell)$

Solving the above equation we get,

$\ell = 18.5$ m.

46. Mass of block



Block of the particle = m = 120gm = 0.12kg.

In the equilibrium condition, the spring is stretched by a distance x = 1.00 cm = 0.01m.

$\Rightarrow 0.2 \times g = K. x.$

$\Rightarrow 2 = K \times 0.01 \Rightarrow K = 200$ N/m.

The velocity with which the particle m will strike M is given by u

$= \sqrt{2 \times 10 \times 0.45} = \sqrt{9} = 3$ m/sec.

So, after the collision, the velocity of the particle and the block is

$V = \frac{0.12 \times 3}{0.32} = \frac{9}{8}$ m/sec.

Let the spring be stretched through an extra deflection of δ.

$0 - (1/2) \times 0.32 \times (81/64) = 0.32 \times 10 \times \delta - (1/2 \times 200 \times (\delta + 0.1)^2 - (1/2) \times 200 \times (0.01)^2$

Solving the above equation we get

$\delta = 0.045 = 4.5$cm

47. Mass of bullet = 25g = 0.025kg.

Mass of pendulum = 5kg.

The vertical displacement h = 10cm = 0.1m

Let it strike the pendulum with a velocity u.

Let the final velocity be v.

$\Rightarrow mu = (M + m)v.$

$\Rightarrow v = \frac{m}{(M + m)} u = \frac{0.025}{5.025} \times u = \frac{u}{201}$

Using conservation of energy.

$0 - (1/2) (M + m). V^2 = - (M + m) g \times h \Rightarrow \frac{u^2}{(201)^2} = 2 \times 10 \times 0.1 = 2$

$\Rightarrow u = 201 \times \sqrt{2} = 280$ m/sec.

48. Mass of bullet = M = 20gm = 0.02kg.

Mass of wooden block M = 500gm = 0.5kg

Velocity of the bullet with which it strikes u = 300 m/sec.

Let the bullet emerges out with velocity V and the velocity of block = V′

As per law of conservation of momentum.

$mu = Mv' + mv$ ….(1)

Again applying work – energy principle for the block after the collision,

$0 - (1/2) M \times V'^2 = - Mgh$ (where h = 0.2m)

$\Rightarrow V'^2 = 2gh$

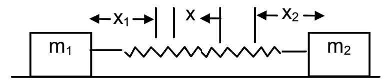
$V' = \sqrt{2gh} = \sqrt{20 \times 0.2} = 2$m/sec

Substituting the value of V′ in the equation (1), we get\

$0.02 \times 300 = 0.5 \times 2 + 0.2 \times v$

$\Rightarrow V = \frac{6.1}{0.02} = 250$m/sec.

49. Mass of the two blocks are m_1 , m_2.
Initially the spring is stretched by x_0
Spring constant K.
For the blocks to come to rest again,
Let the distance travelled by m_1 & m_2
Be x_1 and x_2 towards right and left respectively.
As o external forc acts in horizontal direction,

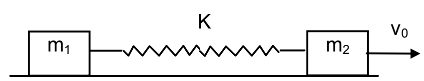


$m_1x_1 = m_2x_2$ …(1)
Again, the energy would be conserved in the spring.
$\Rightarrow (1/2)\ k \times x^2 = (1/2)\ k\ (x_1 + x_2 - x_0)^2$
$\Rightarrow x_o = x_1 + x_2 - x_0$
$\Rightarrow x_1 + x_2 = 2x_0$ …(2)
$\Rightarrow x_1 = 2x_0 - x_2$ similarly $x_1 = \left(\dfrac{2m_2}{m_1 + m_2}\right)x_0$

$\Rightarrow m_1(2x_0 - x_2) = m_2x_2$ $\quad \Rightarrow 2m_1x_0 - m_1x_2 = m_2x_2$ $\quad \Rightarrow x_2 = \left(\dfrac{2m_1}{m_1 + m_2}\right)x_0$

50. a) $\therefore$ Velocity of centre of mass = $\dfrac{m_2 \times v_0 + m_1 \times 0}{m_1 + m_2} = \dfrac{m_2v_0}{m_1 + m_2}$

b) The spring will attain maximum elongation when both velocity of two blocks will attain the velocity of centre of mass.
d) $x \rightarrow$ maximum elongation of spring.
Change of kinetic energy = Potential stored in spring.

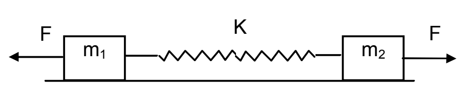


$\Rightarrow (1/2)\ m_2\ v_0^2 - (1/2)\ (m_1 + m_2)\ \left(\dfrac{m_2v_0}{m_1 + m_2}\right)^2 = (1/2)\ kx^2$

$\Rightarrow m_2\ v_0^2\left(1 - \dfrac{m_2}{m_1 + m_2}\right) = kx^2$ $\quad \Rightarrow x = \left(\dfrac{m_1m_2}{m_1 + m_2}\right)^{1/2} \times v_0$

51. If both the blocks are pulled by some force, they suddenly move with some acceleration and instantaneously stop at same position where the elongation of spring is maximum.
$\therefore$ Let x_1, $x_2 \rightarrow$ extension by block m_1 and m_2
Total work done = $Fx_1 + Fx_2$ …(1)
$\therefore$ Increase the potential energy of spring = $(1/2)\ K\ (x_1 + x_2)^2$ …(2)
Equating (1) and (2)
$F(x_1 + x_2) = (1/2)\ K\ (x_1 + x_2)^2 \Rightarrow (x_1 + x_2) = \dfrac{2F}{K}$
Since the net external force on the two blocks is zero thus same force act on opposite direction.
$\therefore m_1x_1 = m_2x_2$ …(3)
And $(x_1 + x_2) = \dfrac{2F}{K}$

$\therefore x_2 = \dfrac{m_1}{m_2} \times 1$



Substituting $\dfrac{m_1}{m_2} \times 1 + x_1 = \dfrac{2F}{K}$

$\Rightarrow x_1\left(1 + \dfrac{m_1}{m_2}\right) = \dfrac{2F}{K}$ $\quad \Rightarrow x_1 = \dfrac{2F}{K}\dfrac{m_2}{m_1 + m_2}$

Similarly $x_2 = \dfrac{2F}{K}\dfrac{m_1}{m_1 + m_2}$

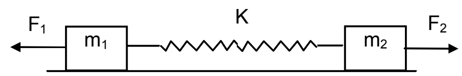
52. Acceleration of mass $m_1 = \frac{F_1 - F_2}{m_1 + m_2}$

Similarly Acceleration of mass $m_2 = \frac{F_2 - F_1}{m_1 + m_2}$

Due to F_1 and F_2 block of mass m_1 and m_2 will experience different acceleration and experience an inertia force.

$\therefore$ Net force on $m_1 = F_1 - m_1 a$

$= F_1 - m_1 \times \frac{F_1 - F_2}{m_1 + m_2} = \frac{m_1F_1 + m_2F_1 - m_1F_1 + F_2m_1}{m_1 + m_2} = \frac{m_2F_1 + m_1F_2}{m_1 + m_2}$



Similarly Net force on $m_2 = F_2 - m_2 a$

$= F_2 - m_2 \times \frac{F_2 - F_1}{m_1 + m_2} = \frac{m_1F_2 + m_2F_2 - m_2F_2 + F_1m_2}{m_1 + m_2} = \frac{m_1F_2 + m_2F_2}{m_1 + m_2}$

$\therefore$ If m_1 displaces by a distance x_1 and x_2 by m_2 the maximum extension of the spring is $x_1 + m_2$.

$\therefore$ Work done by the blocks = energy stored in the spring.,

$\Rightarrow \frac{m_2F_1 + m_1F_2}{m_1 + m_2} \times x_1 + \frac{m_2F_1 + m_1F_2}{m_1 + m_2} \times x_2 = (1/2)\,K\,(x_1 + x_2)^2$

$\Rightarrow x_1 + x_2 = \frac{2}{K}\frac{m_2F_1 + m_1F_2}{m_1 + m_2}$

53. Mass of the man (M_m) is 50 kg.

Mass of the pillow (M_p) is 5 kg.

When the pillow is pushed by the man, the pillow will go down while the man goes up. It becomes the external force on the system which is zero.

$\Rightarrow$ acceleration of centre of mass is zero

$\Rightarrow$ velocity of centre of mass is constant

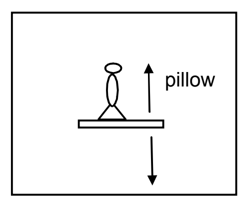
$\therefore$As the initial velocity of the system is zero.

$\therefore M_m \times V_m = M_p \times V_p$ …(1)

Given the velocity of pillow is 80 ft/s.

Which is relative velocity of pillow w.r.t. man.

$\vec{V}_{p/m} = \vec{V}_p - \vec{V}_m = V_p - (-V_m) = V_p + V_m \Rightarrow V_p = V_{p/m} - V_m$



Putting in equation (1)

$M_m \times V_m = M_p\,(V_{p/m} - V_m)$

$\Rightarrow 50 \times V_m = 5 \times (8 - V_m)$

$\Rightarrow 10 \times V_m = 8 - V_m \Rightarrow V_m = \frac{8}{11} = 0.727\text{m/s}$

$\therefore$ Absolute velocity of pillow = 8 – 0.727 = 7.2 ft/sec.

$\therefore$ Time taken to reach the floor $= \frac{S}{v} = \frac{8}{7.2} = 1.1$ sec.

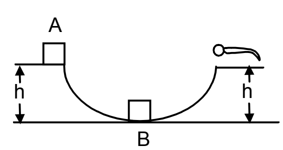
As the mass of wall >>> then pillow

The velocity of block before the collision = velocity after the collision.

$\Rightarrow$ Times of ascent = 1.11 sec.

$\therefore$ Total time taken = 1.11 + 1.11 = 2.22 sec.

54. Let the velocity of A = u_1.



Let the final velocity when reaching at B becomes collision = v_1.

$\therefore (1/2)\,mv_1^2 - (1/2)mu_1^2 = mgh$

$\Rightarrow v_1^2 - u_1^2 = 2\,gh \qquad \Rightarrow v_1 = \sqrt{2gh - u_1^2}$ …(1)

When the block B reached at the upper man's head, the velocity of B is just zero.

For B, block

$\therefore (1/2) \times 2m \times 0^2 - (1/2) \times 2m \times v^2 = mgh \qquad \Rightarrow v = \sqrt{2gh}$

$\therefore$ Before collision velocity of $u_A = v_1$, $u_B = 0$.

After collision velocity of v_A = v (say) $v_B = \sqrt{2gh}$

Since it is an elastic collision the momentum and K.E. should be coserved.

$\therefore$ m × v_1 + 2m × 0 = m × v + 2m × $\sqrt{2gh}$

$\Rightarrow v_1 - v = 2\sqrt{2gh}$

Also, (1/2) × m × v_1^2 + (1/2) I 2m × 0^2 = (1/2) × m × v^2 + (1/2) ×2m × $\left(\sqrt{2gh}\right)^2$

$\Rightarrow v_1^2 - v^2 = 2 \times \sqrt{2gh} \times \sqrt{2gh}$ …(2)

Dividing (1) by (2)

$\frac{(v_1+v)(v_1-v)}{(v_1+v)} = \frac{2\times\sqrt{2gh}\times\sqrt{2gh}}{2\times\sqrt{2gh}} \Rightarrow v_1 + v = \sqrt{2gh}$ …(3)

Adding (1) and (3)

$2v_1 = 3\sqrt{2gh} \Rightarrow v_1 = \left(\frac{3}{2}\right)\sqrt{2gh}$

But $v_1 = \sqrt{2gh+u^2} = \left(\frac{3}{2}\right)\sqrt{2gh}$

$\Rightarrow 2gh + u^2 = \frac{9}{4}\times 2gh$

$\Rightarrow u = 2.5\sqrt{2gh}$

So the block will travel with a velocity greater than $2.5\sqrt{2gh}$ so awake the man by B.

55. Mass of block = 490 gm.

Mass of bullet = 10 gm.

Since the bullet embedded inside the block, it is an plastic collision.

Initial velocity of bullet $v_1 = 50\sqrt{7}$ m/s.

Velocity of the block is v_2 = 0.

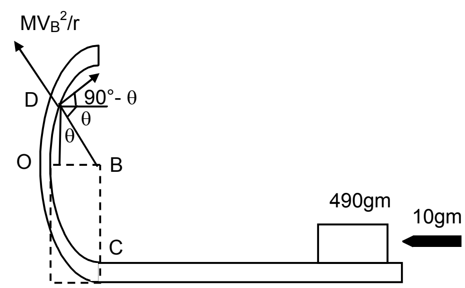
Let Final velocity of both = v.

$\therefore 10 \times 10^{-3} \times 50 \times \sqrt{7} + 10^{-3} \times 190$ I $0 = (490 + 10) \times 10^{-3} \times V_A$

$\Rightarrow V_A = \sqrt{7}$ m/s.

When the block losses the contact at 'D' the component mg will act on it.

$\frac{m(V_B)^2}{r}$ = mg sin θ $\Rightarrow (V_B)^2$ = gr sin θ …(1)



Puttin work energy principle

(1/2) m × $(V_B)^2$ – (1/2) × m × $(V_A)^2$ = – mg (0.2 + 0.2 sin θ)

$\Rightarrow$ (1/2) × gr sin θ – (1/2) × $\left(\sqrt{7}\right)^2$ = – mg (0.2 + 0.2 sin θ)

$\Rightarrow$ 3.5 – (1/2) × 9.8 × 0.2 × sin θ = 9.8 × 0.2 (1 + sin θ)

$\Rightarrow$ 3.5 – 0.98 sin θ = 1.96 + 1.96 sin θ

$\Rightarrow$ sin θ = (1/2) $\Rightarrow$ θ = 30°

$\therefore$ Angle of projection = 90° - 30° = 60°.

$\therefore$ time of reaching the ground = $\sqrt{\frac{2h}{g}}$

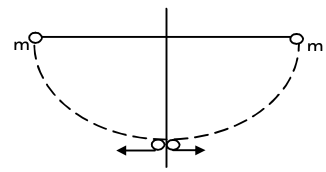
$= \sqrt{\frac{2\times(0.2+0.2\times\sin 30°)}{9.8}}$ = 0.247 sec.

$\therefore$ Distance travelled in horizontal direction.

s = V cos θ × t = $\sqrt{gr\sin\theta}\times t = \sqrt{9.8\times 2\times(1/2)}\times 0.247$ = 0.196m

$\therefore$ Total distance = (0.2 – 0.2 cos 30°) + 0.196 = 0.22m.

56. Let the velocity of m reaching at lower end = V_1
From work energy principle.
$\therefore$ (1/2) × m × V_1^2 – (1/2) × m × 0^2 = mg ℓ
$\Rightarrow v_1 = \sqrt{2g\ell}$.
Similarly velocity of heavy block will be $v_2 = \sqrt{2gh}$.
$\therefore$ $v_1 = V_2$ = u(say)
Let the final velocity of m and 2m v_1 and v_2 respectively.
According to law of conservation of momentum.
m × x_1 + 2m × V_2 = $mv_1 + 2mv_2$
$\Rightarrow$ m × u – 2 m u = $mv_1 + 2mv_2$
$\Rightarrow v_1 + 2v_2 = -u$ …(1)
Again, $v_1 - v_2 = -(V_1 - V_2)$
$\Rightarrow v_1 - v_2 = -[u - (-v)] = -2V$ …(2)
Subtracting.



$$3v_2 = u \Rightarrow v_2 = \frac{u}{3} = \frac{\sqrt{2g\ell}}{3}$$

Substituting in (2)

$$v_1 - v_2 = -2u \Rightarrow v_1 = -2u + v_2 = -2u + \frac{u}{3} = -\frac{5}{3}u = -\frac{5}{3}\times\sqrt{2g\ell} = -\frac{\sqrt{50g\ell}}{3}$$

b) Putting the work energy principle
(1/2) × 2m × 0^2 – (1/2) × 2m × $(v_2)^2$ = – 2m × g × h
[h $\rightarrow$ height gone by heavy ball]

$$\Rightarrow (1/2)\ \frac{2g}{9} = \ell \times h \qquad \Rightarrow h = \frac{\ell}{9}$$

Similarly, (1/2) × m × 0^2 – (1/2) × m × v_1^2 = m × g × h_2
[height reached by small ball]

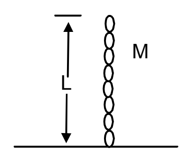
$$\Rightarrow (1/2) \times \frac{50g\ell}{9} = g \times h_2 \qquad \Rightarrow h_2 = \frac{25\ell}{9}$$

Someh_2 is more than 2ℓ, the velocity at height point will not be zero. And the ‘m’ will rise by a distance 2ℓ.

57. Let us consider a small element at a distance ‘x’ from the floor of length ‘dy’ .

$$\text{So, } dm = \frac{M}{L}dx$$

So, the velocity with which the element will strike the floor is, $v = \sqrt{2gx}$
$\therefore$ So, the momentum transferred to the floor is,



$$M = (dm)v = \frac{M}{L}\times dx \times \sqrt{2gx}$$ [because the element comes to rest]

So, the force exerted on the floor change in momentum is given by,

$$F_1 = \frac{dM}{dt} = \frac{M}{L}\times\frac{dx}{dt}\times\sqrt{2gx}$$

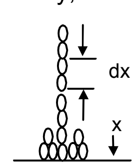
Because, $v = \frac{dx}{dt} = \sqrt{2gx}$ (for the chain element)

$$F_1 = \frac{M}{L}\times\sqrt{2gx}\times\sqrt{2gx} = \frac{M}{L}\times 2gx = \frac{2Mgx}{L}$$

Again, the force exerted due to ‘x’ length of the chain on the floor due to its own weight is given by,

$$W = \frac{M}{L}(x)\times g = \frac{Mgx}{L}$$

So, the total forced exerted is given by,

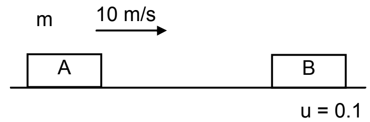


$$F = F_1 + W = \frac{2Mgx}{L} + \frac{Mgx}{L} = \frac{3Mgx}{L}$$

58. V_1 = 10 m/s V_2 = 0

V_1, v_2 → velocity of ACB after collision.

a) If the edlision is perfectly elastic.



$mV_1 + mV_2 = mv_1 + mv_2$

$\Rightarrow 10 + 0 = v_1 + v_2$

$\Rightarrow v_1 + v_2 = 10$ …(1)

Again, $v_1 – v_2 = – (u_1 – v_2) = – (10 – 0) = –10$ …(2)

Subtracting (2) from (1)

$2v_2 = 20 \Rightarrow v_2 = 10$ m/s.

The deacceleration of B = μg

Putting work energy principle

$\therefore (1/2) \times m \times 0^2 – (1/2) \times m \times v_2^2 = – m \times a \times h$

$\Rightarrow – (1/2) \times 10^2 = - \mu g \times h$ $\Rightarrow h = \frac{100}{2 \times 0.1 \times 10} = 50m$

b) If the collision perfectly in elastic.

$m \times u_1 + m \times u_2 = (m + m) \times v$

$\Rightarrow m \times 10 + m \times 0 = 2m \times v$ $\Rightarrow v = \frac{10}{2} = 5$ m/s.

The two blocks will move together sticking to each other.

$\therefore$ Putting work energy principle.

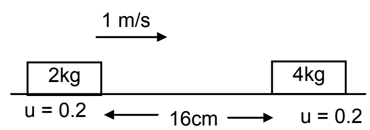
$(1/2) \times 2m \times 0^2 – (1/2) \times 2m \times v^2 = 2m \times \mu g \times s$

$\Rightarrow \frac{5^2}{0.1 \times 10 \times 2} = s$ $\Rightarrow s = 12.5$ m.

59. Let velocity of 2kg block on reaching the 4kg block before collision $=u_1$.

Given, $V_2 = 0$ (velocity of 4kg block).

$\therefore$ From work energy principle,



$(1/2)\, m \times u_1^2 – (1/2)\, m \times 1^2 = – m \times ug \times s$

$\Rightarrow \frac{u_1^2 - 1}{2} = – 2 \times 5$ $\Rightarrow – 16 = \frac{u_1^2 - 1}{4}$

$\Rightarrow 64 \times 10^{–2} = u_1^2 – 1$ $\Rightarrow u_1 = 6$m/s

Since it is a perfectly elastic collision.

Let V_1, V_2 → velocity of 2kg & 4kg block after collision.

$m_1V_1 + m_2V_2 = m_1v_1 + m_2v_2$

$\Rightarrow 2 \times 0.6 + 4 \times 0 = 2v_1 + 4 v_2$ $\Rightarrow v_1 + 2v_2 = 0.6$ …(1)

Again, $V_1 – V_2 = – (u_1 – u_2) = – (0.6 – 0) = –0.6$ …(2)

Subtracting (2) from (1)

$3v_2 = 1.2$ $\Rightarrow v_2 = 0.4$ m/s.

$\therefore v_1 = – 0.6 + 0.4 = – 0.2$ m/s

$\therefore$ Putting work energy principle for 1st 2kg block when come to rest.

$(1/2) \times 2 \times 0^2 – (1/2) \times 2 \times (0.2)^2 = – 2 \times 0.2 \times 10 \times s$

$\Rightarrow (1/2) \times 2 \times 0.2 \times 0.2 = 2 \times 0.2 \times 10 \times s$ $\Rightarrow S_1 = 1$cm.

Putting work energy principle for 4kg block.

$(1/2) \times 4 \times 0^2 – (1/2) \times 4 \times (0.4)^2 = – 4 \times 0.2 \times 10 \times s$

$\Rightarrow 2 \times 0.4 \times 0.4 = 4 \times 0.2 \times 10 \times s$ $\Rightarrow S_2 = 4$ cm.

Distance between 2kg & 4kg block = $S_1 + S_2 = 1 + 4 = 5$ cm.

60. The block 'm' will slide down the inclined plane of mass M with acceleration a_1 g sin α (relative) to the inclined plane.

The horizontal component of a_1 will be, $a_x = g \sin \alpha \cos \alpha$, for which the block M will accelerate towards left. Let, the acceleration be a_2.

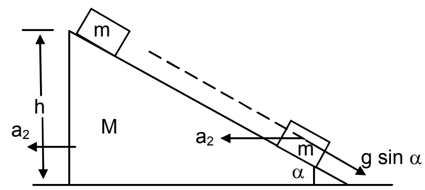
According to the concept of centre of mass, (in the horizontal direction external force is zero).

$ma_x = (M + m)\, a_2$

$\Rightarrow a_2 = \dfrac{ma_x}{M+m} = \dfrac{mg\sin\alpha\cos\alpha}{M+m}$ …(1)

So, the absolute (Resultant) acceleration of 'm' on the block 'M' along the direction of the incline will be,

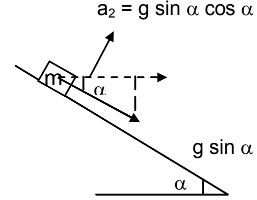
$a = g \sin\alpha - a_2 \cos\alpha$



$= g\sin\alpha - \dfrac{mg\sin\alpha\cos^2\alpha}{M+m} = g\sin\alpha\left[1-\dfrac{m\cos^2\alpha}{M+m}\right]$

$= g\sin\alpha\left[\dfrac{M+m-m\cos^2\alpha}{M+m}\right]$

So, $a = g\sin\alpha\left[\dfrac{M+m\sin^2\alpha}{M+m}\right]$ …(2)



Let, the time taken by the block 'm' to reach the bottom end be 't'.

Now, $S = ut + (1/2)\, at^2$

$\Rightarrow \dfrac{h}{\sin\alpha} = (1/2)\, at^2 \qquad \Rightarrow t = \sqrt{\dfrac{2}{a\sin\alpha}}$

So, the velocity of the bigger block after time 't' will be.

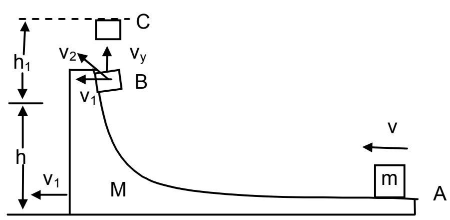
$V_m = u + a_2 t = \dfrac{mg\sin\alpha\cos\alpha}{M+m}\sqrt{\dfrac{2h}{a\sin\alpha}} = \sqrt{\dfrac{2m^2g^2h\sin^2\alpha\cos^2\alpha}{(M+m)^2 a\sin\alpha}}$

Now, subtracting the value of a from equation (2) we get,

$V_M = \left[\dfrac{2m^2g^2h\sin^2\alpha\cos^2\alpha}{(M+m)^2\sin\alpha}\times\dfrac{(M+m)}{g\sin\alpha(M+m\sin^2\alpha)}\right]^{1/2}$

or $V_M = \left[\dfrac{2m^2g^2h\cos^2\alpha}{(M+m)(M+m\sin^2\alpha)}\right]^{1/2}$

61.



The mass 'm' is given a velocity 'v' over the larger mass M.

a) When the smaller block is travelling on the vertical part, let the velocity of the bigger block be v_1 towards left.

From law of conservation of momentum, (in the horizontal direction)

$mv = (M + m)\, v_1$

$\Rightarrow v_1 = \dfrac{mv}{M+m}$

b) When the smaller block breaks off, let its resultant velocity is v_2.

From law of conservation of energy,

$(1/2)\, mv^2 = (1/2)\, Mv_1^2 + (1/2)\, mv_2^2 + mgh$

$\Rightarrow v_2^2 = v^2 - \dfrac{M}{m}v_1^2 - 2gh$ ..(1)

$\Rightarrow v_2^2 = v^2\left[1-\dfrac{M}{m}\times\dfrac{m^2}{(M+m)^2}\right] - 2gh$

$\Rightarrow v_2 = \left[\dfrac{(m^2+Mm+m^2)}{(M+m)^2}v^2 - 2gh\right]^{1/2}$

e) Now, the vertical component of the velocity v_2 of mass 'm' is given by,

$v_y^2 = v_2^2 - v_1^2$

$$= \frac{(M^2 + Mm + m^2)}{(M+m)^2} v^2 - 2gh - \frac{m^2v^2}{(M+m)^2}$$

$[\therefore v_1 = \frac{mv}{M+v}]$

$$\Rightarrow v_y^2 = \frac{M^2 + Mm + m^2 - m^2}{(M+m)^2} v^2 - 2gh$$

$$\Rightarrow v_y^2 = \frac{Mv^2}{(M+m)} - 2gh \quad \ldots(2)$$

To find the maximum height (from the ground), let us assume the body rises to a height 'h', over and above 'h'.

Now, $(1/2)mv_y^2 = mgh_1 \Rightarrow h_1 = \frac{v_y^2}{2g}$ …(3)

So, Total height $= h + h_1 = h + \frac{v_y^2}{2g} = h + \frac{mv^2}{(M+m)2g} - h$

[from equation (2) and (3)]

$$\Rightarrow H = \frac{mv^2}{(M+m)2g}$$

d) Because, the smaller mass has also got a horizontal component of velocity 'v_1' at the time it breaks off from 'M' (which has a velocity v_1), the block 'm' will again land on the block 'M' (bigger one).

Let us find out the time of flight of block 'm' after it breaks off.

During the upward motion (BC),

$0 = v_y - gt_1$

$$\Rightarrow t_1 = \frac{v_y}{g} = \frac{1}{g}\left[\frac{Mv^2}{(M+m)} - 2gh\right]^{1/2} \quad \ldots(4) \text{ [from equation (2)]}$$

So, the time for which the smaller block was in its flight is given by,

$$T = 2t_1 = \frac{2}{g}\left[\frac{Mv^2 - 2(M+m)gh}{(M+m)}\right]^{1/2}$$

So, the distance travelled by the bigger block during this time is,

$$S = v_1T = \frac{mv}{M+m} \times \frac{2}{g}\frac{[Mv^2 - 2(M+m)gh]^{1/2}}{(M+m)^{1/2}}$$

$$\text{or } S = \frac{2mv[Mv^2 - 2(M+m)gh]^{1/2}}{g(M+m)^{3/2}}$$

62. Given h < < < R.

$G_{mass} = 6$ I 10^{24} kg.

$M_b = 3 \times 10^{24}$ kg.

Let $V_e \rightarrow$ Velocity of earth

$V_b \rightarrow$ velocity of the block.

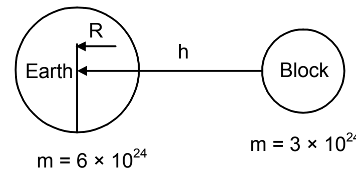
The two blocks are attracted by gravitational force of attraction. The gravitation potential energy stored will be the K.E. of two blocks.

$$\overline{G}^{pim}\left[\frac{1}{R+(h/2)} - \frac{1}{R+h}\right] = (1/2)\, m_e \times v_e^2 + (1/2)\, m_b \times v_b^2$$

Again as the an internal force acts.

$M_eV_e = m_bV_b \qquad \Rightarrow V_e = \frac{m_bV_b}{M_e} \quad \ldots(2)$

Putting in equation (1)



$G_{me} \times m_b \left[\frac{2}{2R+h} - \frac{1}{R+h}\right]$

$= (1/2) \times M_e \times \frac{{m_b}^2 {v_b}^2}{{M_e}^2} \times {v_e}^2 + (1/2)\, M_b \times {V_b}^2$

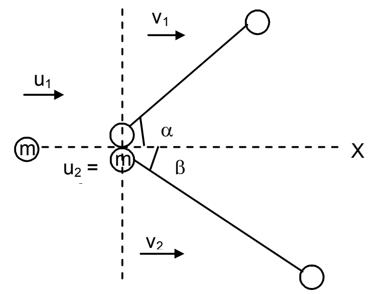
$= (1/2) \times m_b \times {V_b}^2 \left(\frac{M_b}{M_e} + 1\right)$

$\Rightarrow GM\left[\frac{2R+2h-2R-h}{(2R+h)(R+h)}\right] = (1/2) \times {V_b}^2 \times \left(\frac{3\times10^{24}}{6\times10^{24}} + 1\right)$ $\Rightarrow \left[\frac{GM\times h}{2R^2 + 3Rh + h^2}\right] = (1/2) \times {V_b}^2 \times (3/2)$

As h < < < R, if can be neglected

$\Rightarrow \frac{GM\times h}{2R^2} = (1/2) \times {V_b}^2 \times (3/2)$ $\Rightarrow V_b = \sqrt{\frac{2gh}{3}}$

63. Since it is not an head on collision, the two bodies move in different dimensions. Let V_1, $V_2 \rightarrow$ velocities of the bodies vector collision. Since, the collision is elastic. Applying law of conservation of momentum on X-direction.



$mu_1 + mxo = mv_1 \cos\alpha + mv_2 \cos\beta$

$\Rightarrow v_1 \cos a + v_2 \cos b = u_1$ …(1)

Putting law of conservation of momentum in y direction.

$0 = mv_1 \sin\alpha - mv_2 \sin\beta$

$\Rightarrow v_1 \sin\alpha = v_2 \sin\beta$ …(2)

Again $½\, m\, {u_1}^2 + 0 = ½\, m\, {v_1}^2 + ½\, m\, x\, {v_2}^2$

$\Rightarrow {u_1}^2 = {v_1}^2 + {v_2}^2$ …(3)

Squaring equation(1)

${u_1}^2 = {v_1}^2 \cos^2\alpha + {v_2}^2 \cos^2\beta + 2\, v_1 v_2 \cos\alpha \cos\beta$

Equating (1) & (3)

${v_1}^2 + {v_2}^2 = {v_1}^2 \cos^2\alpha + {v_2}^2 \cos^2\beta + 2\, v_1 v_2 \cos\alpha \cos\beta$

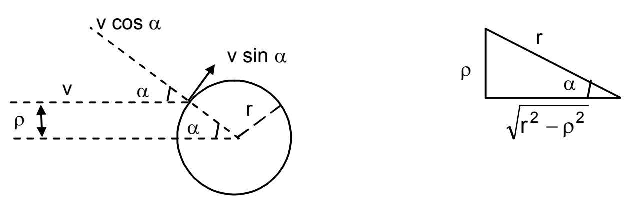
$\Rightarrow {v_1}^2 \sin^2\alpha + {v_2}^2 \sin^2\beta = 2\, v_1 v_2 \cos\alpha \cos\beta$

$\Rightarrow 2{v_1}^2 \sin^2\alpha = 2 \times v_1 \times \frac{v_1 \sin\alpha}{\sin\beta} \times \cos\alpha \cos\beta$

$\Rightarrow \sin\alpha \sin\beta = \cos\alpha \cos\beta$ $\Rightarrow \cos\alpha \cos\beta - \sin\alpha \sin\beta = 0$

$\Rightarrow \cos(\alpha + \beta) = 0 = \cos 90°$ $\Rightarrow (\alpha + \beta) = 90°$

64.



Let the mass of both the particle and the spherical body be 'm'. The particle velocity 'v' has two components, v cos α normal to the sphere and v sin α tangential to the sphere.

After the collision, they will exchange their velocities. So, the spherical body will have a velocity v cos α and the particle will not have any component of velocity in this direction.

[The collision will due to the component v cos α in the normal direction. But, the tangential velocity, of the particle v sin α will be unaffected]

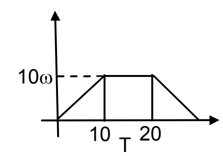
So, velocity of the sphere = $v \cos\alpha = \frac{v}{r}\sqrt{r^2 - \rho^2}$ [from (fig-2)]

And velocity of the particle = $v \sin\alpha = \frac{v\rho}{r}$

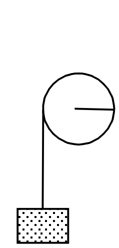
* * * * *

SOLUTIONS TO CONCEPTS
CHAPTER – 10

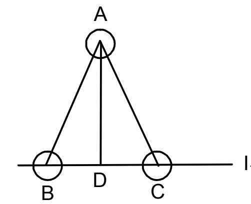
1. $\omega_0 = 0$; $\rho = 100$ rev/s ; $\omega = 2\pi$; $\rho = 200\ \pi$ rad/s
   $\Rightarrow \omega = \omega_0 = \alpha t$
   $\Rightarrow \omega = \alpha t$
   $\Rightarrow \alpha = (200\ \pi)/4 = 50\ \pi$ rad /s^2 or 25 rev/s^2
   $\therefore \theta = \omega_0 t + 1/2\ \alpha t^2 = 8 \times 50\ \pi = 400\ \pi$ rad
   $\therefore \alpha = 50\ \pi$ rad/s^2 or 25 rev/s^s
   $\theta = 400\ \pi$ rad.
2. $\theta = 100\ \pi$; t = 5 sec
   $\theta = 1/2\ \alpha t^2 \Rightarrow 100\pi = 1/2\ \alpha\ 25$
   $\Rightarrow \alpha = 8\pi \times 5 = 40\ \pi$ rad/s = 20 rev/s
   $\therefore \alpha = 8\pi$ rad/s^2 = 4 rev/s^2
   $\omega = 40\pi$ rad/s^2 = 20 rev/s^2.
3. Area under the curve will decide the total angle rotated
   $\therefore$ maximum angular velocity = 4 × 10 = 40 rad/s
   Therefore, area under the curve = 1/2 × 10 × 40 + 40 × 10 + 1/2 × 40 × 10
   = 800 rad
   $\therefore$ Total angle rotated = 800 rad.



4. $\alpha = 1$ rad/s^2, $\omega_0 = 5$ rad/s; $\omega = 15$ rad/s
   $\therefore w = w_0 + \alpha t$
   $\Rightarrow t = (\omega - \omega_0)/\alpha = (15 - 5)/1 = 10$ sec
   Also, $\theta = \omega_0 t + 1/2\ \alpha t^2$
   = 5 ×10 + 1/2 × 1 × 100 = 100 rad.
5. $\theta = 5$ rev, $\alpha = 2$ rev/s^2, $\omega_0 = 0$; ω = ?
   $\omega^2 = (2\ \alpha\ \theta)$
   $\Rightarrow \omega = \sqrt{2\times 2\times 5} = 2\sqrt{5}$ rev/s.
   or $\theta = 10\pi$ rad, $\alpha = 4\pi$ rad/s^2, $\omega_0 = 0$, ω = ?
   $\omega = \sqrt{2\alpha\theta} = 2 \times 4\pi \times 10\pi$
   $= 4\pi\sqrt{5}$ rad/s = $2\sqrt{5}$ rev/s.
6. A disc of radius = 10 cm = 0.1 m
   Angular velocity = 20 rad/s
   $\therefore$ Linear velocity on the rim = ωr = 20 × 0.1 = 2 m/s
   $\therefore$ Linear velocity at the middle of radius = $\omega r/2$ = 20 × (0.1)/2 = 1 m/s.
7. t = 1 sec, r = 1 cm = 0.01 m
   $\alpha = 4$ rd/s^2
   Therefore $\omega = \alpha t = 4$ rad/s
   Therefore radial acceleration,
   $A_n = \omega^2 r = 0.16$ m/s^2 = 16 cm/s^2
   Therefore tangential acceleration, $a_r = \alpha r = 0.04$ m/s^2 = 4 cm/s^2.
8. The Block is moving the rim of the pulley
   The pulley is moving at a $\omega = 10$ rad/s
   Therefore the radius of the pulley = 20 cm
   Therefore linear velocity on the rim = tangential velocity = $r\omega$
   = 20 × 20 = 200 cm/s = 2 m/s.



9. Therefore, the ⊥ distance from the axis (AD) = $\sqrt{3}/2 \times 10 = 5\sqrt{3}$ cm.



Therefore moment of inertia about the axis BC will be

I = mr^2 = 200 K $(5\sqrt{3})^2$ = 200 × 25 × 3

= 15000 gm – cm^2 = 1.5 × 10^{-3} kg – m^2.

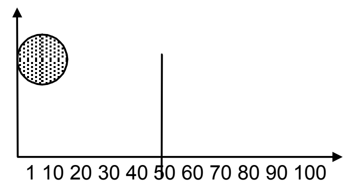
b) The axis of rotation let pass through A and ⊥ to the plane of triangle

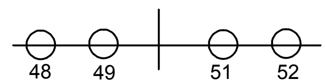
Therefore the torque will be produced by mass B and C

Therefore net moment of inertia = I = mr^2 + mr^2

= 2 × 200 ×10^2 = 40000 gm-cm^2 = 4 ×10^{-3} kg-m^2.

10. Masses of 1 gm, 2 gm ……100 gm are kept at the marks 1 cm, 2 cm, ……1000 cm on he x axis respectively. A perpendicular axis is passed at the 50th particle.





Therefore on the L.H.S. side of the axis there will be 49 particles and on the R.H.S. side there are 50 particles.

Consider the two particles at the position 49 cm and 51 cm.

Moment inertial due to these two particle will be =

$$49 \times 1^2 + 51 + 1^2 = 100 \text{ gm-cm}^2$$

Similarly if we consider 48th and 52nd term we will get 100 ×2^2 gm-cm^2

Therefore we will get 49 such set and one lone particle at 100 cm.

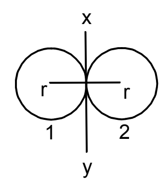
Therefore total moment of inertia =

100 {$1^2 + 2^2 + 3^2 + \ldots + 49^2$} + 100$(50)^2$.

= 100 × (50 × 51 × 101)/6 = 4292500 gm-cm^2

= 0.429 kg-m^2 = 0.43 kg-m^2.

11. The two bodies of mass m and radius r are moving along the common tangent.



Therefore moment of inertia of the first body about XY tangent.

= mr^2 + 2/5 mr^2

– Moment of inertia of the second body XY tangent = mr^2 + 2/5 mr^2 = 7/5 mr^2

Therefore, net moment of inertia = 7/5 mr^2 + 7/5 mr^2 = 14/5 mr^2 units.

12. Length of the rod = 1 m, mass of the rod = 0.5 kg

Let at a distance d from the center the rod is moving

Applying parallel axis theorem :

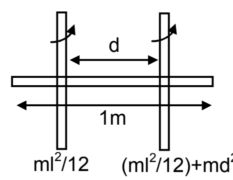
The moment of inertial about that point

⇒ (mL^2 / 12) + md^2 = 0.10

⇒ (0.5 × 1^2)/12 + 0.5 × d^2 = 0.10

⇒ d^2 = 0.2 – 0.082 = 0.118

⇒ d = 0.342 m from the centre.



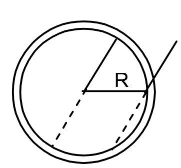
13. Moment of inertia at the centre and perpendicular to the plane of the ring.

So, about a point on the rim of the ring and the axis ⊥ to the plane of the ring, the moment of inertia

= mR^2 + mR^2 = 2mR^2 (parallel axis theorem)

⇒ mK^2 = 2mR^2 (K = radius of the gyration)

⇒ K = $\sqrt{2R^2} = \sqrt{2}$ R.



14. The moment of inertia about the center and ⊥ to the plane of the disc of radius r and mass m is = mr^2.

According to the question the radius of gyration of the disc about a point = radius of the disc.

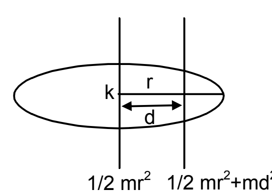
Therefore mk^2 = ½ mr^2 + md^2

(K = radius of gyration about acceleration point, d = distance of that point from the centre)

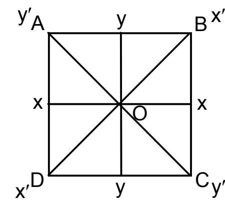
⇒ K^2 = r^2/2 + d^2

⇒ r^2 = r^2/2 + d^2 (∴ K = r)

⇒ r^2/2 = d^2 ⇒ d = r / $\sqrt{2}$.



15. Let a small cross sectional area is at a distance x from xx axis.
Therefore mass of that small section = $m/a^2 \times ax\,dx$
Therefore moment of inertia about xx axis



$$= I_{xx} = 2\int_0^{a/2}(m/a^2)\times(adx)\times x^2 = (2\times(m/a)(x^3/3)]_0^{a/2}$$

$= ma^2 / 12$

Therefore $I_{xx} = I_{xx} + I_{yy}$

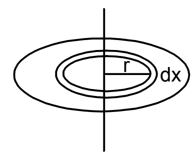
$= 2 \times *ma^2/12) = ma^2/6$

Since the two diagonals are $\perp$ to each other

Therefore $I_{zz} = I_{x'x'} + I_{y'y'}$

$\Rightarrow ma^2/6 = 2 \times I_{x'x'}$ (because $I_{x'x'} = I_{y'y'}$) $\Rightarrow I_{x'x'} = ma^2/12$

16. The surface density of a circular disc of radius a depends upon the distance from the centre as P(r) = A + Br
Therefore the mass of the ring of radius r will be



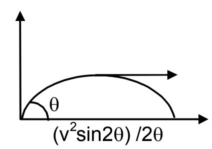
$\theta = (A + Br) \times 2\pi r\,dr \times r^2$

Therefore moment of inertia about the centre will be

$$= \int_0^a (A+Br)2\pi r\times dr = \int_0^a 2\pi Ar^3 dr + \int_0^a 2\pi Br^4 dr$$

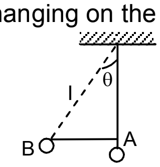
$= 2\pi A\,(r^4/4) + 2\pi\,B(r^5/5)]_0^a = 2\pi a^4\,[(A/4) + (Ba/5)]$.

17. At the highest point total force acting on the particle id its weight acting downward.
Range of the particle = $u^2 \sin 2\pi / g$



Therefore force is at a $\perp$ distance, $\Rightarrow$ (total range)/2 = $(v^2 \sin 2\theta)/2g$
(From the initial point)
Therefore $\tau = F \times r$ (θ = angle of projection)
$= mg \times v^2 \sin 2\theta/2g$ (v = initial velocity)
$= mv^2 \sin 2\theta / 2 = mv^2 \sin\theta \cos\theta$.

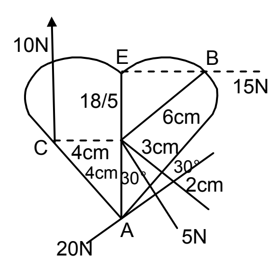
18. A simple of pendulum of length l is suspended from a rigid support. A bob of weight W is hanging on the other point.



When the bob is at an angle θ with the vertical, then total torque acting on the point of suspension = i = F × r
$\Rightarrow W\,r \sin\theta = W\,l \sin\theta$
At the lowest point of suspension the torque will be zero as the force acting on the body passes through the point of suspension.

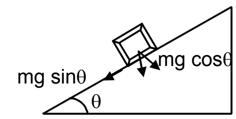
19. A force of 6 N acting at an angle of 30° is just able to loosen the wrench at a distance 8 cm from it.
Therefore total torque acting at A about the point 0
= 6 sin 30° × (8/100)
Therefore total torque required at B about the point 0
= F × 16/100 $\Rightarrow$ F × 16/100 = 6 sin 30° × 8/100
$\Rightarrow$ F = (8 × 3) / 16 = 1.5 N.

20. Torque about a point = Total force × perpendicular distance from the point to that force.
Let anticlockwise torque = + ve
And clockwise acting torque = –ve
Force acting at the point B is 15 N



Therefore torque at O due to this force
$= 15 \times 6 \times 10^{-2} \times \sin 37°$
$= 15 \times 6 \times 10^{-2} \times 3/5 = 0.54$ N-m (anticlock wise)
Force acting at the point C is 10 N
Therefore, torque at O due to this force
$= 10 \times 4 \times 10^{-2} = 0.4$ N-m (clockwise)
Force acting at the point A is 20 N
Therefore, Torque at O due to this force = $20 \times 4 \times 10^{-2} \times \sin 30°$
$= 20 \times 4 \times 10^{-2} \times 1/2 = 0.4$ N-m (anticlockwise)
Therefore resultant torque acting at ‘O’ = 0.54 – 0.4 + 0.4 = 0.54 N-m.

21. The force mg acting on the body has two components mg sin θ and mg cos θ and the body will exert a normal reaction. Let R =

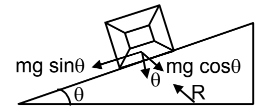


Since R and mg cos θ pass through the centre of the cube, there will be no torque due to R and mg cos θ. The only torque will be produced by mg sin θ.

$\therefore$ i = F × r (r = a/2) (a = ages of the cube)

$\Rightarrow$ i = mg sin θ × a/2

= 1/2 mg a sin θ.



22. A rod of mass m and length L, lying horizontally, is free to rotate about a vertical axis passing through its centre.

A force F is acting perpendicular to the rod at a distance L/4 from the centre.

Therefore torque about the centre due to this force

i_i = F × r = FL/4.

This torque will produce a angular acceleration α.

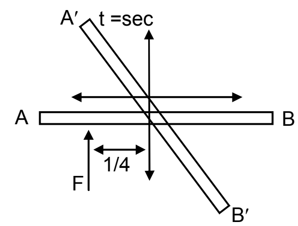
Therefore $\tau_c = I_c \times \alpha$

$\Rightarrow i_c = (mL^2 / 12) \times \alpha$ (I_c of a rod = $mL^2 / 12$)

$\Rightarrow$ F i/4 = $(mL^2 / 12) \times \alpha \Rightarrow \alpha$ = 3F/ml

Therefore $\theta = 1/2\ \alpha t^2$ (initially at rest)

$\Rightarrow \theta = 1/2 \times (3F / ml)t^2 = (3F/2ml)t^2$.



23. A square plate of mass 120 gm and edge 5 cm rotates about one of the edge.

Let take a small area of the square of width dx and length a which is at a distance x from the axis of rotation.

Therefore mass of that small area

$m/a^2 \times a\,dx$ (m = mass of the square ; a = side of the plate)

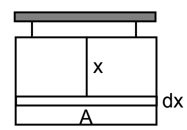
$$I = \int_0^a (m/a^2) \times ax^2 dx = (m/a)(x^3/3)]_0^a$$

$= ma^2/3$

Therefore torque produced = I × α = $(ma^2/3) \times \alpha$

$= \{(120 \times 10^{-3} \times 5^2 \times 10^{-4})/3\}\ 0.2$

$= 0.2 \times 10^{-4} = 2 \times 10^{-5}$ N-m.



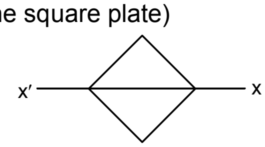
24. Moment of inertial of a square plate about its diagonal is $ma^2/12$ (m = mass of the square plate)

a = edges of the square

Therefore torque produced = $(ma^2/12) \times \alpha$

$= \{(120 \times 10^{-3} \times 5^2 \times 10^{-4})/12 \times 0.2$

$= 0.5 \times 10^{-5}$ N-m.



25. A flywheel of moment of inertia 5 kg m is rotated at a speed of 60 rad/s. The flywheel comes to rest due to the friction at the axle after 5 minutes.

Therefore, the angular deceleration produced due to frictional force = $\omega = \omega_0 + \alpha t$

$\Rightarrow \omega_0 = -\alpha t$ (ω = 0+

$\Rightarrow \alpha = -(60/5 \times 60) = -1/5$ rad/s^2.

a) Therefore total workdone in stopping the wheel by frictional force

W = 1/2 iω^2 = 1/2 × 5 × (60 × 60) = 9000 Joule = 9 KJ.

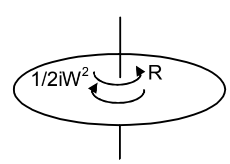
b) Therefore torque produced by the frictional force (R) is

I_R = I × α = 5 × (–1/5) = IN – m opposite to the rotation of wheel.

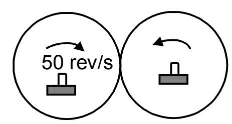
c) Angular velocity after 4 minutes

$\Rightarrow \omega = \omega_0 + \alpha t$ = 60 – 240/5 = 12 rad/s

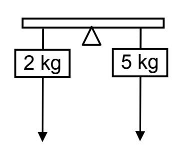
Therefore angular momentum about the centre = 1 × ω = 5 × 12 = 60 kg-m^2/s.



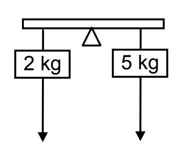
26. The earth's angular speed decreases by 0.0016 rad/day in 100 years.
 Therefore the torque produced by the ocean water in decreasing earth's angular velocity
 $\tau = I\alpha$
 $= 2/5\ mr^2 \times (\omega - \omega_0)/t$
 $= 2/6 \times 6 \times 10^{24} \times 64^2 \times 10^{10} \times [0.0016 /(26400^2 \times 100 \times 365)]$ (1 year = 365 days= 365 × 56400 sec)
 $= 5.678 \times 10^{20}$ N-m.
27. A wheel rotating at a speed of 600 rpm.
 ω_0 = 600 rpm = 10 revolutions per second.
 T = 10 sec. (In 10 sec. it comes to rest)
 $\omega = 0$
 Therefore $\omega_0 = -\alpha t$
 $\Rightarrow \alpha = -10/10 = -1$ rev/s^2
 $\Rightarrow \omega = \omega_0 + \alpha t = 10 - 1 \times 5 = 5$ rev/s.
 Therefore angular deacceleration = 1 rev/s^2 and angular velocity of after 5 sec is 5 rev/s.
28. ω = 100 rev/min = 5/8 rev/s = $10\pi/3$ rad/s
 θ = 10 rev = 20 π rad, r = 0.2 m
 After 10 revolutions the wheel will come to rest by a tangential force
 Therefore the angular deacceleration produced by the force = $\alpha = \omega^2/2\theta$
 Therefore the torque by which the wheel will come to an rest = $I_{cm} \times \alpha$
 $\Rightarrow F \times r = I_{cm} \times \alpha \rightarrow F \times 0.2 = 1/2\ mr^2 \times [(10\pi/3)^2 / (2 \times 20\pi)]$
 $\Rightarrow F = 1/2 \times 10 \times 0.2 \times 100\ \pi^2 / (9 \times 2 \times 20\pi)$
 $= 5\pi / 18 = 15.7/18 = 0.87$ N.
29. A cylinder is moving with an angular velocity 50 rev/s brought in contact with another identical cylinder in rest. The first and second cylinder has common acceleration and deacceleration as 1 rad/s^2 respectively.
 Let after t sec their angular velocity will be same 'ω'.
 For the first cylinder $\omega = 50 - \alpha t$
 $\Rightarrow t = (\omega - 50)/-1$
 And for the 2nd cylinder $\omega = \alpha_2 t$
 $\Rightarrow t = \omega/I$
 So, $\omega = (\omega - 50)/-1$
 $\Rightarrow 2\omega = 50 \Rightarrow \omega = 25$ rev/s.
 $\Rightarrow t = 25/1$ sec = 25 sec.



30. Initial angular velocity = 20 rad/s
 Therefore $\alpha = 2$ rad/s^2
 $\Rightarrow t_1 = \omega/\alpha_1 = 20/2 = 10$ sec
 Therefore 10 sec it will come to rest.
 Since the same torque is continues to act on the body it will produce same angular acceleration and since the initial kinetic energy = the kinetic energy at a instant.
 So initial angular velocity = angular velocity at that instant
 Therefore time require to come to that angular velocity,
 $t_2 = \omega_2/\alpha_2 = 20/2 = 10$ sec
 therefore time required = $t_1 + t_2$ = 20 sec.
31. $I_{net} = I_{net} \times \alpha$
 $\Rightarrow F_1r_1 - F_2r_2 = (m_1r_1^2 + m_2r_2^2) \times \alpha - 2 \times 10 \times 0.5$
 $\Rightarrow 5 \times 10 \times 0.5 = (5 \times (1/2)^2 + 2 \times (1/2)^2) \times \alpha$
 $\Rightarrow 15 = 7/4\ \alpha$
 $\Rightarrow \alpha = 60/7 = 8.57$ rad/s^2.



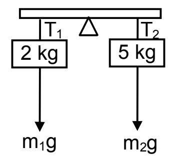
32. In this problem the rod has a mass 1 kg
 a) $\tau_{net} = I_{net} \times \alpha$
 $\Rightarrow 5 \times 10 \times 10.5 - 2 \times 10 \times 0.5$
 $= (5 \times (1/2)^2 + 2 \times (1/2)^2 + 1/12) \times \alpha$



$\Rightarrow 15 = (1.75 + 0.084)\ \alpha$

$\Rightarrow \alpha = 1500/(175 + 8.4) = 1500/183.4 = 8.1\ rad/s^2$ (g = 10)

$= 8.01\ rad/s^2$ (if g = 9.8)



b) $T_1 - m_1g = m_1a$

$\Rightarrow T_1 = m_1a + m_1g = 2(a + g)$

$= 2(\alpha r + g) = 2(8 \times 0.5 + 9.8)$

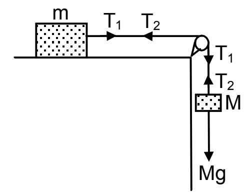
= 27.6 N on the first body.

In the second body

$\Rightarrow m_2g - T_2 = m_2a \Rightarrow T_2 = m_2g - m_2a$

$\Rightarrow T_2 = 5(g - a) = 5(9.8 - 8 \times 0.5) = 29$ N.

33. According to the question



$Mg - T_1 = Ma$ …(1)

$T_2 = ma$ …(2)

$(T_1 - T_2) = 1\ a/r^2$ …(3) [because $a = r\alpha$]…**[T.r =I(a/r)]**

If we add the equation 1 and 2 we will get

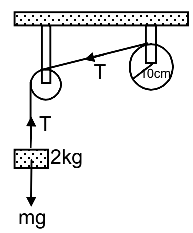
$Mg + (T_2 - T_1) = Ma + ma$ …(4)

$\Rightarrow Mg - Ia/r^2 = Ma + ma$

$\Rightarrow (M + m + I/r^2)a = Mg$

$\Rightarrow a = Mg/(M + m + I/r^2)$

34. $I = 0.20\ kg\text{-}m^2$ (Bigger pulley)



r = 10 cm = 0.1 m, smaller pulley is light

mass of the block, m = 2 kg

therefore $mg - T = ma$ …(1)

$\Rightarrow T = Ia/r^2$ …(2)

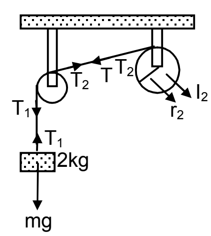
$\Rightarrow mg = (m + I/r^2)a \Rightarrow (2 \times 9.8) / [2 + (0.2/0.01)] = a$

$= 19.6 / 22 = 0.89\ m/s^2$

Therefore, acceleration of the block = $0.89\ m/s^2$.

35. m = 2 kg, $i_1 = 0.10\ kg\text{-}m^2$, r_1 = 5 cm = 0.05 m

$i_2 = 0.20\ kg\text{-}m^2$, r_2 = 10 cm = 0.1 m



Therefore $mg - T_1 = ma$ …(1)

$(T_1 - T_2)r_1 = I_1\alpha$ …(2)

$T_2r_2 = I_2\alpha$ …(3)

Substituting the value of T_2 in the equation (2), we get

$\Rightarrow (t_1 - I_2\ \alpha/r_1)r_2 = I_1\alpha$

$\Rightarrow (T_1 - I_2\ a/r_1^2) = I_1a/r_2^2$

$\Rightarrow T_1 = [(I_1/r_1^2) + I_2/r_2^2)]a$

Substituting the value of T_1 in the equation (1), we get

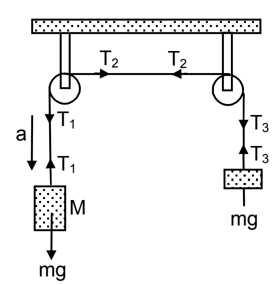
$\Rightarrow mg - [(I_1/r_1^2) + I_2/r_2^2)]a = ma$

$$\Rightarrow \frac{mg}{[(I_1/r_1^2) + (I_2/r_2^2)] + m} = a$$

$$\Rightarrow a = \frac{2 \times 9.8}{(0.1/0.0025) + (0.2/0.01) + 2} = 0.316\ m/s^2$$

$$\Rightarrow T_2 = I_2a/r_2^2 = \frac{0.20 \times 0.316}{0.01} = 6.32\ N.$$

36. According to the question



$Mg - T_1 = Ma$ …(1)

$(T_2 - T_1)R = Ia/R \Rightarrow (T_2 - T_1) = Ia/R^2$ …(2)

$(T_2 - T_3)R = Ia/R^2$ …(3)

$\Rightarrow T_3 - mg = ma$ …(4)

By adding equation (2) and (3) we will get,

$\Rightarrow (T_1 - T_3) = 2\ Ia/R^2$ …(5)

By adding equation (1) and (4) we will get

$-mg + Mg + (T_3 - T_1) = Ma + ma$ …(6)

Substituting the value for $T_3 - T_1$ we will get

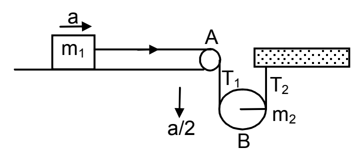
$\Rightarrow Mg - mg = Ma + ma + 2Ia/R^2$

$\Rightarrow a = \dfrac{(M-m)G}{(M+m+2I/R^2)}$

37. A is light pulley and B is the descending pulley having I = 0.20 kg – m^2 and r = 0.2 m

Mass of the block = 1 kg

According to the equation



$T_1 = m_1 a$ …(1)

$(T_2 - T_1)r = I\alpha$ …(2)

$m_2 g - m_2\, a/2 = T_1 + T_2$ …(3)

$T_2 - T_1 = Ia/2R^2 = 5a/2$ and $T_1 = a$ (because $\alpha = a/2R$)

$\Rightarrow T_2 = 7/2\ a$

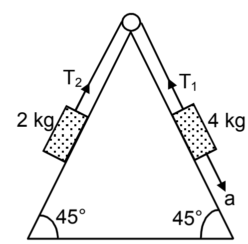
$\Rightarrow m_2 g = m_2 a/2 + 7/2\ a + a$

$\Rightarrow 2I / r^2 g = 2I/r^2\ a/2 + 9/2\ a$ $(1/2\ mr^2 = I)$

$\Rightarrow 98 = 5a + 4.5\ a$

$\Rightarrow a = 98/9.5 = 10.3\ ms^2$

38. $m_1 g \sin\theta - T_1 = m_1 a$ …(1)



$(T_1 - T_2) = Ia/r^2$ …(2)

$T_2 - m_2 g \sin\theta = m_2 a$ …(3)

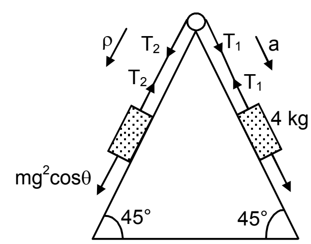
Adding the equations (1) and (3) we will get

$m_1 g \sin\theta + (T_2 - T_1) - m_2 g \sin\theta = (m_1 + m_2)a$

$\Rightarrow (m_1 - m_2)g \sin\theta = (m_1 + m_2 + 1/r^2)a$

$\Rightarrow a = \dfrac{(m_1 - m_2)g\sin\theta}{(m_1 + m_2 + 1/r^2)} = 0.248 = 0.25\ ms^{-2}$.

39. $m_1 = 4$ kg, $m_2 = 2$ kg



Frictional co-efficient between 2 kg block and surface = 0.5

R = 10 cm = 0.1 m

I = 0.5 kg – m^2

$m_1 g \sin\theta - T_1 = m_1 a$ …(1)

$T_2 - (m_2 g \sin\theta + \mu\, m_2 g \cos\theta) = m_2 a$ …(2)

$(T_1 - T_2) = Ia/r^2$

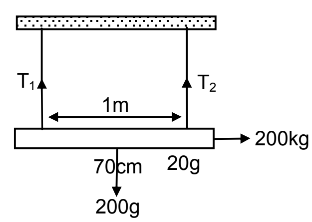
Adding equation (1) and (2) we will get

$m_1 g \sin\theta - (m_2 g \sin\theta + \mu m_2 g \cos\theta) + (T_2 - T_1) = m_1 a + m_2 a$

$\Rightarrow 4 \times 9.8 \times (1/\sqrt{2}) - \{(2 \times 9.8 \times (1/\sqrt{2}) + 0.5 \times 2 \times 9.8 \times (1/\sqrt{2})\} = (4 + 2 + 0.5/0.01)a$

$\Rightarrow 27.80 - (13.90 + 6.95) = 65\ a \Rightarrow a = 0.125\ ms^{-2}$.

40. According to the question



$m_1 = 200$ g, l = 1 m, $m_2 = 20$ g

Therefore, $(T_1 \times r_1) - (T_2 \times r_2) - (m_1 f \times r_3 g) = 0$

$\Rightarrow T_1 \times 0.7 - T_2 \times 0.3 - 2 \times 0.2 \times g = 0$

$\Rightarrow 7T_1 - 3T_2 = 3.92$ …(1)

$T_1 + T_2 = 0.2 \times 9.8 + 0.02 \times 9.8 = 2.156$ …(2)

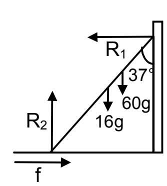
From the equation (1) and (2) we will get

$10\ T_1 = 10.3$

$\Rightarrow T_1 = 1.038$ N = 1.04 N

Therefore $T_2 = 2.156 - 1.038 = 1.118 = 1.12$ N.

41. $R_1 = \mu R_2$, $R_2 = 16g + 60\ g = 745$ N



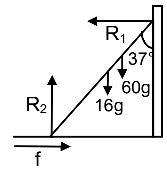
$R_1 \times 10 \cos 37° = 16g \times 5 \sin 37° + 60\ g \times 8 \times \sin 37°$

$\Rightarrow 8R_1 = 48g + 288\ g$

$\Rightarrow R_1 = 336g/8 = 412\ N = f$

Therefore $\mu = R_1 / R_2 = 412/745 = 0.553$.

42. $\mu = 0.54$, $R_2 = 16g + mg$; $R_1 = \mu R_2$



$\Rightarrow R_1 \times 10 \cos 37° = 16g \times 5 \sin 37° + mg \times 8 \times \sin 37°$

$\Rightarrow 8R_1 = 48g + 24/5\ mg$

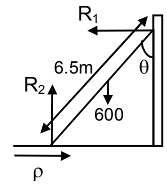
$\Rightarrow R_2 = \dfrac{48g + 24/5\ mg}{8 \times 0.54}$

$\Rightarrow 16g + mg = \dfrac{24.0g + 24mg}{5 \times 8 \times 0.54} \Rightarrow 16 + m = \dfrac{240 + 24m}{40 \times 0.54}$

$\Rightarrow m = 44$ kg.

43. m = 60 kg, ladder length = 6.5 m, height of the wall = 6 m

Therefore torque due to the weight of the body



a) $\tau = 600 \times 6.5 / 2 \sin \theta = i$

$\Rightarrow \tau = 600 \times 6.5 / 2 \times \sqrt{[1-(6/6.5)^2]}$

$\Rightarrow \tau = 735$ N-m.

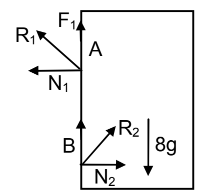
b) $R_2 = mg = 60 \times 9.8$

$R_1 = \mu R_2 \Rightarrow 6.5\ R_1 \cos \theta = 60g \sin \theta \times 6.5/2$

$\Rightarrow R_1 = 60\ g \tan \theta = 60\ g \times (2.5/12)$ [because $\tan \theta = 2.5/6$]

$\Rightarrow R_1 = (25/2)\ g = 122.5$ N.

44. According to the question



$8g = F_1 + F_2$; $N_1 = N_2$

Since, $R_1 = R_2$

Therefore $F_1 = F_2$

$\Rightarrow 2F_1 = 8\ g \Rightarrow F_1 = 40$

Let us take torque about the point B, we will get $N_1 \times 4 = 8\ g \times 0.75$.

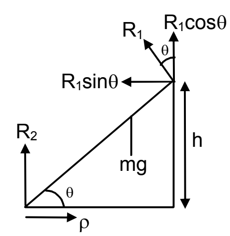
$\Rightarrow N_1 = (80 \times 3) / (4 \times 4) = 15$ N

Therefore $\sqrt{(F_1^2 + N_1^2} = R_1 = \sqrt{40^2 + 15^2} = 42.72 = 43$ N.

45. Rod has a length = L

It makes an angle θ with the floor

The vertical wall has a height = h



$R_2 = mg - R_1 \cos \theta$ …(1)

$R_1 \sin \theta = \mu R_2$ …(2)

$R_1 \cos \theta \times (h/\tan \theta) + R_1 \sin \theta \times h = mg \times 1/2 \cos \theta$

$\Rightarrow R_1 (\cos^2 \theta / \sin \theta)h + R_1 \sin \theta\ h = mg \times 1/2 \cos \theta$

$\Rightarrow R_1 = \dfrac{mg \times L/2 \cos\theta}{\{(\cos^2\theta/\sin\theta)h + \sin\theta h\}}$

$\Rightarrow R_1 \cos \theta = \dfrac{mgL/2\cos^2\theta\sin\theta}{\{(\cos^2\theta/\sin\theta)h + \sin\theta h\}}$

$\Rightarrow \mu = R_1 \sin \theta / R_2 = \dfrac{mg\ L/2\ \cos\theta.\sin\theta}{\{(\cos^2\theta/\sin\theta)h + \sin\theta h)\}mg - mg\ 1/2\ \cos^2\theta}$

$\Rightarrow \mu = \dfrac{L/2\ \cos\theta.\sin\theta \times 2\sin\theta}{2(\cos^2\theta h + \sin^2\theta h) - L\cos^2\theta\sin\theta}$

$\Rightarrow \mu = \dfrac{L\cos\theta\sin^2\theta}{2h - L\cos^2\theta\sin\theta}$.

46. A uniform rod of mass 300 grams and length 50 cm rotates with an uniform angular velocity = 2 rad/s about an axis perpendicular to the rod through an end.

a) $L = I\omega$

I at the end $= mL^2/3 = (0.3 \times 0.5^2)/3 = 0.025$ kg-m^2

$= 0.025 \times 2 = 0.05$ kg – m^2/s

b) Speed of the centre of the rod

$V = \omega r = w \times (50/2) = 50$ cm/s $= 0.5$ m/s.

c) Its kinetic energy $= 1/2\ I\omega^2 = (1/2) \times 0.025 \times 2^2 = 0.05$ Joule.

47. I = 0.10 N-m; a = 10 cm = 0.1 m; m = 2 kg

Therefore $(ma^2/12) \times \alpha = 0.10$ N-m

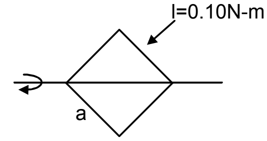
$\Rightarrow \alpha = 60$ rad/s

Therefore $\omega = \omega_0 + \alpha t$

$\Rightarrow \omega = 60 \times 5 = 300$ rad/s

Therefore angular momentum = $I\omega = (0.10 / 60) \times 300 = 0.50$ kg-m^2/s

And 0 kinetic energy = $1/2\ I\omega^2 = 1/2 \times (0.10 / 60) \times 300^2 = 75$ Joules.



48. Angular momentum of the earth about its axis is

$= 2/5\ mr^2 \times (2\pi / 85400)$ (because, $I = 2/5\ mr^2$)

Angular momentum of the earth about sun's axis

$= mR^2 \times (2\pi / 86400 \times 365)$ (because, $I = mR^2$)

Therefore, ratio of the angular momentum = $\dfrac{2/5mr^2 \times (2\pi/86400)}{mR^2 \times 2\pi/(86400 \times 365)}$

$\Rightarrow (2r^2 \times 365) / 5R^2$

$\Rightarrow (2.990 \times 10^{10}) / (1.125 \times 10^{17}) = 2.65 \times 10^{-7}$.

49. Angular momentum due to the mass m_1 at the centre of system is $= m_1\ r^{12}$.

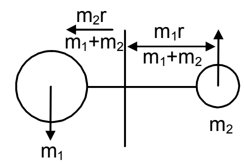
$$= m1\left(\frac{m_2}{m_1 + m_2}\right)^2 \omega = \frac{m_1 m_2^2 r^2}{(m_1 + m_2)^2}\omega \quad \ldots(1)$$

Similarly the angular momentum due to the mass m_2 at the centre of system is $m_2\ r^{112}\omega$

$$= m_2\left(\frac{m_1 r}{m_1 m_2}\right)^2 \omega = \frac{m_2 m_1^2}{(m_1 + m_2)^2}\omega \quad \ldots(2)$$

Therefore net angular momentum = $\dfrac{m_1 m_2^2 r^2 \omega}{(m_1 + m_2)^2} + \dfrac{m_2 m_1^2 r^2 \omega}{(m_1 + m_2)^2}$

$\Rightarrow \dfrac{m_1 m_2 (m_1 + m_2) r^2 \omega}{(m_1 + m_2)^2} = \dfrac{m_1 m_2}{(m_1 + m_2)} r^2 \omega = \mu r^2 \omega$ (proved)



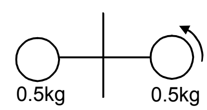
50. $\tau = I\alpha$

$\Rightarrow F \times r = (mr^2 + mr^2)\alpha \Rightarrow 5 \times 0.25 = 2mr^2 \times \alpha$

$\Rightarrow \alpha = \dfrac{1.25}{2 \times 0.5 \times 0.025 \times 0.25} = 20$

$\omega_0 = 10$ rad/s, t = 0.10 sec, $\omega = \omega_0 + \alpha t$

$\Rightarrow \omega = 10 + 010 \times 230 = 10 + 2 = 12$ rad/s.



51. A wheel has

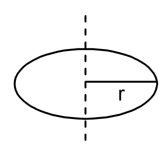
I = 0.500 Kg-m^2, r = 0.2 m, $\omega = 20$ rad/s

Stationary particle = 0.2 kg

Therefore $I_1\omega_1 = I_2\omega_2$ (since external torque = 0)

$\Rightarrow 0.5 \times 10 = (0.5 + 0.2 \times 0.2^2)\omega_2$

$\Rightarrow 10/0.508 = \omega_2 = 19.69 = 19.7$ rad/s



52. $I_1 = 6$ kg-m^2, $\omega_1 = 2$ rad/s , $I_2 = 5$ kg-m^2

Since external torque = 0

Therefore $I_1\omega_1 = I_2\omega_2$

$\Rightarrow \omega_2 = (6 \times 2) / 5 = 2.4$ rad/s

53. $\omega_1 = 120$ rpm $= 120 \times (2\pi / 60) = 4\pi$ rad /s.

$I_1 = 6$ kg – m^2, $I_2 = 2$ kgm^2

Since two balls are inside the system

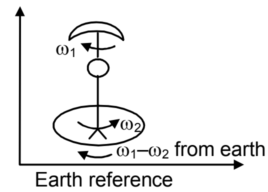
Therefore, total external torque = 0

Therefore, $I_1\omega_1 = I_2\omega_2$

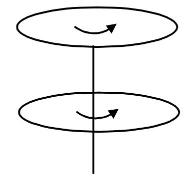
$\Rightarrow 6 \times 4\pi = 2\omega_2$

$\Rightarrow \omega_2 = 12\ \pi$ rad/s = 6 rev/s = 360 rev/minute.

54. $I_1 = 2 \times 10^{-3}$ kg-m^2 ; $I_2 = 3 \times 10^{-3}$ kg-m^2 ; $\omega_1 = 2$ rad/s
From the earth reference the umbrella has a angular velocity $(\omega_1 - \omega_2)$
And the angular velocity of the man will be ω_2
Therefore $I_1(\omega_1 - \omega_2) = I_2\omega_2$
$\Rightarrow 2 \times 10^{-3} (2 - \omega_2) = 3 \times 10^{-3} \times \omega_2$
$\Rightarrow 5\omega_2 = 4 \Rightarrow \omega_2 = 0.8$ rad/s.



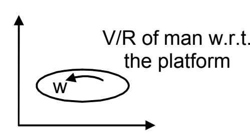
55. Wheel (1) has
$I_1 = 0.10$ kg-m^2, $\omega_1 = 160$ rev/min
Wheel (2) has
$I_2 = ?$; $\omega_2 = 300$ rev/min
Given that after they are coupled, $\omega = 200$ rev/min
Therefore if we take the two wheels to bean isolated system
Total external torque = 0
Therefore, $I_1\omega_1 + I_1\omega_2 = (I_1 + I_1)\omega$
$\Rightarrow 0.10 \times 160 + I_2 \times 300 = (0.10 + I_2) \times 200$
$\Rightarrow 5I_2 = 1 - 0.8 \Rightarrow I_2 = 0.04$ kg-m^2.



56. A kid of mass M stands at the edge of a platform of radius R which has a moment of inertia I. A ball of m thrown to him and horizontal velocity of the ball v when he catches it.
Therefore if we take the total bodies as a system
Therefore $mvR = \{I + (M + m)R^2\}\omega$
(The moment of inertia of the kid and ball about the axis = $(M + m)R^2$)
$$\Rightarrow \omega = \frac{mvR}{1+(M+m)R^2}.$$

57. Initial angular momentum = Final angular momentum
(the total external torque = 0)
Initial angular momentum = mvR (m = mass of the ball, v = velocity of the ball, R = radius of platform)
Therefore angular momentum = $I\omega + MR^2\omega$
Therefore $mVR = I\omega + MR^2\,\omega$
$$\Rightarrow \omega = \frac{mVR}{(1+MR^2)}.$$

58. From a inertial frame of reference when we see the (man wheel) system, we can find that the wheel moving at a speed of ω and the man with $(\omega + V/R)$ after the man has started walking.
(ω' = angular velocity after walking, ω = angular velocity of the wheel before walking.
Since $\Sigma I = 0$
Extended torque = 0
Therefore $(1 + MR^2)\omega = I\omega' + mR^2(\omega' + V/R)$
$\Rightarrow (I + mR^2)\omega + I\omega' + mR^2\omega' + mVR$
$$\Rightarrow \omega' = \omega - \frac{mVR}{(1+mR^2)}.$$



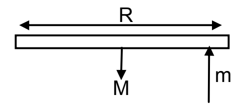
59. A uniform rod of mass m length ℓ is struck at an end by a force F. ⊥ to the rod for a short time t
a) Speed of the centre of mass
$$mv = Ft \Rightarrow v = \frac{Ft}{m}$$
b) The angular speed of the rod about the centre of mass
$\ell\omega - r \times p$
$\Rightarrow (m\ell^2 / 12) \times \omega = (1/2) \times mv$
$\Rightarrow m\ell^2 / 12 \times \omega = (1/2)\ \ell\omega^2$
$\Rightarrow \omega = 6Ft / m\ell$
c) K.E. = $(1/2)\ mv^2 + (1/2)\ \ell\omega^2$
$= (1/2) \times m(Ft / m)^2\ (1/2)\ \ell\omega^2$
$= (1/2) \times m \times (F^2t^2/m^2) + (1/2) \times (m\ell^2/12)(36 \times (F^2t^2/m^2\ell^2))$

$= F^2 t^2 / 2m + 3/2 (F^2t^2) / m = 2 F^2 t^2 / m$

d) Angular momentum about the centre of mass :-

$L = mvr = m \times Ft / m \times (1/2) = F \ell t / 2$

60. Let the mass of the particle = m & the mass of the rod = M



Let the particle strikes the rod with a velocity V.

If we take the two body to be a system,

Therefore the net external torque & net external force = 0

Therefore Applying laws of conservation of linear momentum

$MV' = mV$ (V′ = velocity of the rod after striking)

$\Rightarrow V' / V = m / M$

Again applying laws of conservation of angular momentum

$\Rightarrow \frac{mVR}{2} = \ell\omega$

$\Rightarrow \frac{mVR}{2} = \frac{MR^2}{12} \times \frac{\pi}{2t} \Rightarrow t = \frac{MR\pi}{m12 \times V}$

Therefore distance travelled :-

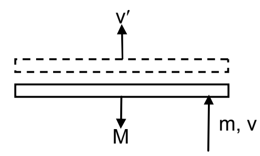
$V' t = V' \left(\frac{MR\pi}{m12\pi}\right) = \frac{m}{M} \times \frac{M}{m} \times \frac{R\pi}{12} = \frac{R\pi}{12}$

61. a) If we take the two bodies as a system therefore total external force = 0

Applying L.C.L.M :-

$mV = (M + m) v'$

$\Rightarrow v' = \frac{mv}{M+m}$



b) Let the velocity of the particle w.r.t. the centre of mass = V′

$\Rightarrow v' = \frac{m \times 0 + Mv}{M+m} \Rightarrow v' = \frac{Mv}{M+m}$

c) If the body moves towards the rod with a velocity of v, i.e. the rod is moving with a velocity – v towards the particle.

Therefore the velocity of the rod w.r.t. the centre of mass = V^-

$\Rightarrow V^- = \frac{M \times O = m \times v}{M+m} = \frac{-mv}{M+m}$

d) The distance of the centre of mass from the particle

$= \frac{M \times l/2 + m \times O}{(M+m)} = \frac{M \times l/2}{(M+m)}$

Therefore angular momentum of the particle before the collision

$= I\,\omega = Mr^2 \text{ cm } \omega$

$= m\{m\, l/2) / (M + m)\}^2 \times V/ (l/2)$

$= (mM^2vl) / 2(M + m)$

Distance of the centre of mass from the centre of mass of the rod =

$R^1_{cm} = \frac{M \times 0 + m \times (l/2)}{(M+m)} = \frac{(ml/2)}{(M+m)}$

Therefore angular momentum of the rod about the centre of mass

$= MV_{cm} R^1_{cm}$

$= M \times \{(-mv) / (M + m)\} \{(ml/2) / (M + m)\}$

$= \left|\frac{-Mm^2lv}{2(M+m)^2}\right| = \frac{Mm^2lv}{2(M+m)^2}$ (If we consider the magnitude only)

e) Moment of inertia of the system = M.I. due to rod + M.I. due to particle

$= \frac{Ml^2}{12} + \frac{M(ml/2)^2}{(M+m)^2} + \frac{m(Ml/s)^2}{(M+m)^2}$

$= \frac{Ml^2(M+4m)}{12(M+m)}$.

f) Velocity of the centre of mass $V_m = \frac{M \times 0 + mV}{(M+m)} = \frac{mV}{(M+m)}$

(Velocity of centre of mass of the system before the collision = Velocity of centre of mass of the system after the collision)

(Because External force = 0)

Angular velocity of the system about the centre of mass,

$P_{cm} = I_{cm}\,\omega$

$\Rightarrow M\vec{V}_M \times \vec{r}_m + m\vec{v}_m \times \vec{r}_m = I_{cm}\omega$

$\Rightarrow M \times \frac{mv}{(M+m)} \times \frac{ml}{2(M+m)} + m \times \frac{Mv}{(M+m)} \times \frac{Ml}{2(M+m)} = \frac{Ml^2(M+4m)}{12(M+m)} \times \omega$

$\Rightarrow \frac{Mm^2vl + mM^2vl}{2(M+m)^2} = \frac{Ml^2(M+4m)}{12(M+m)} \times \omega$

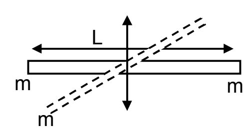
$\Rightarrow \frac{Mm/(M+m)}{2(M+m)^2} = \frac{Ml^2(M+m)}{12(M+m)} \times \omega$

$\Rightarrow \frac{6mv}{(M+4m)l} = \omega$

62. Since external torque = 0

Therefore $I_1\omega_1 = I_2\,\omega_2$

$I_1 = \frac{ml^2}{4} + \frac{ml^2}{4} = \frac{ml^2}{2}$



$\omega_1 = \omega$

$I_2 = \frac{2ml^2}{4} + \frac{ml^2}{4} = \frac{3ml^2}{4}$

Therefore $\omega_2 = \frac{I_1\omega_1}{I_2} = \frac{\left(\frac{ml^2}{2}\right) \times \omega}{\frac{3ml^2}{4}} = \frac{2\omega}{3}$

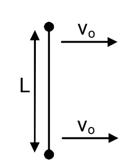
63. Two balls A & B, each of mass m are joined rigidly to the ends of a light of rod of length L. The system moves in a velocity v_0 in a direction $\perp$ to the rod. A particle P of mass m kept at rest on the surface sticks to the ball A as the ball collides with it.

a) The light rod will exert a force on the ball B only along its length. So collision will not affect its velocity.

B has a velocity = v_0

If we consider the three bodies to be a system

Applying L.C.L.M.



Therefore $mv_0 = 2mv' \Rightarrow v' = \frac{v_0}{2}$

Therefore A has velocity = $\frac{v_0}{2}$

b) if we consider the three bodies to be a system

Therefore, net external force = 0

Therefore $V_{cm} = \frac{m \times v_0 + 2m\left(\frac{v_0}{2}\right)}{m+2m} = \frac{mv_0 + mv_0}{3m} = \frac{2v_0}{3}$ (along the initial velocity as before collision)

c) The velocity of (A + P) w.r.t. the centre of mass = $\frac{2v_0}{3} - \frac{v_0}{2} = \frac{v_0}{6}$ &

The velocity of B w.r.t. the centre of mass $v_0 - \frac{2v_0}{3} = \frac{v_0}{3}$

[Only magnitude has been taken]

Distance of the (A + P) from centre of mass = l/3 & for B it is 2 l/3.

Therefore $P_{cm} = I_{cm} \times \omega$

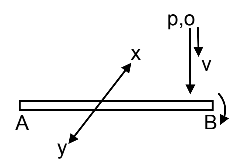
$\Rightarrow 2m \times \frac{v_0}{6} \times \frac{1}{3} + m \times \frac{v_0}{3} \times \frac{2l}{3} = 2m\left(\frac{1}{3}\right)^2 + m\left(\frac{2l}{3}\right)^2 \times \omega$

$\Rightarrow \frac{6mv_0 l}{18} = \frac{6ml}{9} \times \omega \Rightarrow \omega = \frac{v_0}{2l}$

64. The system is kept rest in the horizontal position and a particle P falls from a height h and collides with B and sticks to it.

Therefore, the velocity of the particle 'ρ' before collision = $\sqrt{2gh}$

If we consider the two bodies P and B to be a system. Net external torque and force = 0



Therefore, $m\sqrt{2gh} = 2m \times v$

$\Rightarrow v' = \sqrt{(2gh)/2}$

Therefore angular momentum of the rod just after the collision

$\Rightarrow 2m (v' \times r) = 2m \times \sqrt{(2gh)/2} \times l/2 \Rightarrow ml\sqrt{(2gh)/2}$

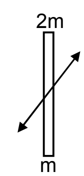
$\omega = \frac{L}{I} = \frac{ml\sqrt{2gh}}{2(ml^2/4 + 2ml^2/4)} = \frac{2\sqrt{gh}}{3l} = \frac{\sqrt{8gh}}{3l}$

b) When the mass 2m will at the top most position and the mass m at the lowest point, they will automatically rotate. In this position the total gain in potential energy = 2 mg × (l/2) – mg (l/2) = mg(l/2)

Therefore $\Rightarrow$ mg l/2 = l/2 $I\omega^2$

$\Rightarrow$ mg l/2 = $(1/2 \times 3ml^2)$ / 4 × $(8gh / 9gl^2)$

$\Rightarrow$ h = 3l/2.



65. According to the question

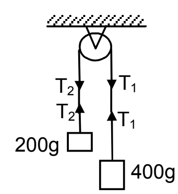
$0.4g - T_1 = 0.4\ a$ …(1)

$T_2 - 0.2g = 0.2\ a$ …(2)

$(T_1 - T_2)r = Ia/r$ …(3)

From equation 1, 2 and 3

$\Rightarrow a = \frac{(0.4 - 0.2)g}{(0.4 + 0.2 + 1.6/0.4)} = g/5$



Therefore (b) V = $\sqrt{2ah} = \sqrt{(2 \times gl^5 \times 0.5)}$

$\Rightarrow \sqrt{(g/5)} = \sqrt{(9.8/5)}$ = 1.4 m/s.

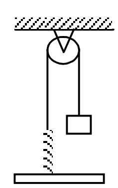
a) Total kinetic energy of the system

= $1/2\ m_1V^2 + 1/2\ m_2V^2 + 1/2\ 18^2$

= $(1/2 \times 0.4 \times 1.4^2) + (1/2 \times 0.2 \times 1.4^2) + (1/2 \times (1.6/4) \times 1.4^2)$ = 0.98 Joule.

66. $I = 0.2$ kg-m^2, r = 0.2 m, K = 50 N/m,

m = 1 kg, g = 10 ms^2, h = 0.1 m



Therefore applying laws of conservation of energy

$mgh = 1/2\ mv^2 + 1/2\ kx^2$

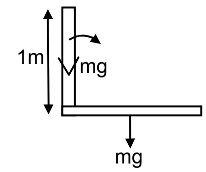
$\Rightarrow 1 = 1/2 \times 1 \times V^2 + 1/2 \times 0.2 \times V^2 /0.04 + (1/2) \times 50 \times 0.01$ (x = h)

$\Rightarrow 1 = 0.5\ v^2 + 2.5\ v^2 + 1/4$

$\Rightarrow 3v^2 = 3/4$

$\Rightarrow$ v = 1/2 = 0.5 m/s

67. Let the mass of the rod = m



Therefore applying laws of conservation of energy

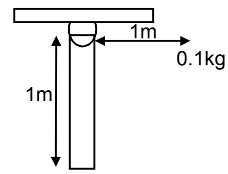
$1/2\ I\omega^2 = mg\ l/2$

$\Rightarrow 1/2 \times M\ l^2/3 \times \omega^2 = mg\ 1/2$

$\Rightarrow \omega^2 = 3g\ /\ l$

$\Rightarrow \omega = \sqrt{3g/l} = 5.42$ rad/s.

68. $1/2\ I\omega^2 - 0 = 0.1 \times 10 \times 1$



$\Rightarrow \omega = \sqrt{20}$

For collision

$0.1 \times 1^2 \times \sqrt{20} + 0 = [(0.24/3)\times 1^2 + (0.1)^2\ 1^2]\omega'$

$\Rightarrow \omega' = \sqrt{20}\ /[10.(0.18)]$

$\Rightarrow 0 - 1/2\ \omega'^2 = -m_1 g\ l\ (1 - \cos\theta) - m_2 g\ l/2\ (1 - \cos\theta)$

$= 0.1 \times 10\ (1 - \cos\theta) = 0.24 \times 10 \times 0.5\ (1 - \cos\theta)$

$\Rightarrow 1/2 \times 0.18 \times (20/3.24) = 2.2(1 - \cos\theta)$

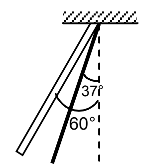
$\Rightarrow (1 - \cos\theta) = 1/(2.2 \times 1.8)$

$\Rightarrow 1 - \cos\theta = 0.252$

$\Rightarrow \cos\theta = 1 - 0.252 = 0.748$

$\Rightarrow \omega = \cos^{-1}(0.748) = 41°$.

69. Let l = length of the rod, and m = mass of the rod.



Applying energy principle

$(1/2)\ I\omega^2 - O = mg\ (1/2)\ (\cos 37° - \cos 60°)$

$$\Rightarrow \frac{1}{2} \times \frac{ml^2}{3}\ \omega^2 = mg \times \frac{1}{2}\left(\frac{4}{5} - \frac{1}{2}\right)t$$

$$\Rightarrow \omega^2 = \frac{9g}{10\ l} = 0.9\left(\frac{g}{l}\right)$$

Again $\left(\frac{ml2}{3}\right)\alpha = mg\left(\frac{1}{2}\right)\sin 37° = mgl \times \frac{3}{5}$

$\therefore \alpha = 0.9\left(\frac{g}{l}\right)$ = angular acceleration.

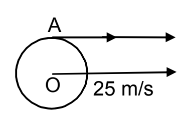
So, to find out the force on the particle at the tip of the rod

F_i = centrifugal force = (dm) ω^2 l = 0.9 (dm) g

F_t = tangential force = (dm) α l = 0.9 (dm) g

So, total force F = $\sqrt{(F_i^2 + F_t^2)} = 0.9\sqrt{2}$ (dm) g

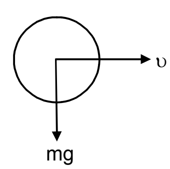
70. A cylinder rolls in a horizontal plane having centre velocity 25 m/s.



At its age the velocity is due to its rotation as well as due to its leniar motion & this two velocities are same and acts in the same direction ($v = r\ \omega$)

Therefore Net velocity at A = 25 m/s + 25 m/s = 50 m/s

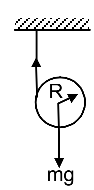
71. A sphere having mass m rolls on a plane surface. Let its radius R. Its centre moves with a velocity v



Therefore Kinetic energy = $(1/2)\ I\omega^2 + (1/2)\ mv^2$

$$= \frac{1}{2} \times \frac{2}{5} mR^2 \times \frac{v^2}{R^2} + \frac{1}{2} mv^2 = \frac{2}{10} mv^2 + \frac{1}{2} mv^2 = \frac{2+5}{10} mv^2 = \frac{7}{10} mv^2$$

72. Let the radius of the disc = R



Therefore according to the question & figure

$Mg - T = ma$ …(1)

& the torque about the centre

$= T \times R = I \times \alpha$

$\Rightarrow TR = (1/2)\ mR^2 \times a/R$

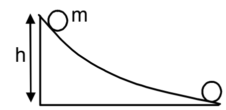
$\Rightarrow$ T = (1/2) ma

Putting this value in the equation (1) we get

$\Rightarrow$ mg – (1/2) ma = ma

$\Rightarrow$ mg = 3/2 ma $\Rightarrow$ a = 2g/3

73. A small spherical ball is released from a point at a height on a rough track & the sphere does not slip. Therefore potential energy it has gained w.r.t the surface will be converted to angular kinetic energy about the centre & linear kinetic energy.



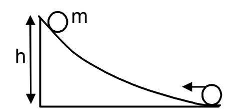
Therefore mgh = (1/2) $I\omega^2$ + (1/2) mv^2

$\Rightarrow mgh = \frac{1}{2}\times\frac{2}{5}\ mR^2\ \omega^2\ +\ \frac{1}{2}mv^2$

$\Rightarrow gh = \frac{1}{5}v^2\ +\ \frac{1}{2}v^2$

$\Rightarrow v^2 = \frac{10}{7}gh \Rightarrow v = \sqrt{\frac{10}{7}gh}$

74. A disc is set rolling with a velocity V from right to left. Let it has attained a height h.



Therefore (1/2) mV^2 + (1/2) $I\omega^2$ = mgh

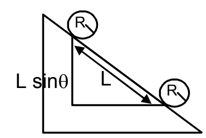
$\Rightarrow$ (1/2) mV^2 + (1/2) × (1/2) $mR^2\ \omega^2$ =mgh

$\Rightarrow$ (1/2) V^2 + 1/4 V^2 = gh $\Rightarrow$ (3/4) V^2 = gh

$\Rightarrow h = \frac{3}{4}\times\frac{V^2}{g}$

75. A sphere is rolling in inclined plane with inclination θ

Therefore according to the principle



Mgl sin θ = (1/2) $I\omega^2$ + (1/2) mv^2

$\Rightarrow$ mgl sin θ = 1/5 mv^2 + (1/2) mv^2

Gl sin θ = 7/10 ω^2

$\Rightarrow v = \sqrt{\frac{10}{7}gl\sin\theta}$

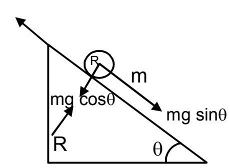
76. A hollow sphere is released from a top of an inclined plane of inclination θ.

To prevent sliding, the body will make only perfect rolling. In this condition,

mg sin θ – f = ma ...(1)

& torque about the centre

$f \times R = \frac{2}{3}mR^2\ \times\ \frac{a}{R}$



$\Rightarrow f = \frac{2}{3}ma$ ...(2)

Putting this value in equation (1) we get

$\Rightarrow mg\sin\theta - \frac{2}{3}ma = ma \Rightarrow a = \frac{3}{5}\ g\sin\theta$

$\Rightarrow mg\sin\theta - f = \frac{3}{5}\ mg\sin\theta \Rightarrow f = \frac{2}{5}mg\sin\theta$

$\Rightarrow \mu\ mg\cos\theta = \frac{2}{5}\ mg\sin\theta \Rightarrow \mu = \frac{2}{5}\ \tan\theta$

b) $\frac{1}{5}\ \tan\theta\ (mg\cos\theta)\ R = \frac{2}{3}mR^2\ \alpha$

$\Rightarrow\ \alpha = \frac{3}{10}\times\frac{g\sin\theta}{R}$

$a_c = g\sin\theta - \frac{g}{5}\sin\theta = \frac{4}{5}\sin\theta$

$\Rightarrow t^2 = \dfrac{2s}{a_c} = \dfrac{2l}{\left(\dfrac{4g\sin\theta}{5}\right)} = \dfrac{5l}{2g\sin\theta}$

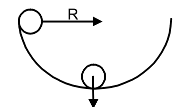
Again, $\omega = \alpha t$

K.E. = $(1/2)\ mv^2 + (1/2)\ I\omega^2 = (1/2)\ m(2as) + (1/2)\ I\ (\alpha^2 t^2)$

$= \dfrac{1}{2} m \times \dfrac{4g\sin\theta}{5} \times 2 \times l + \dfrac{1}{2} \times \dfrac{2}{3} mR^2 \times \dfrac{9}{100} \dfrac{g^2 \sin^2\theta}{R} \times \dfrac{5l}{2g\sin\theta}$

$= \dfrac{4mgl\sin\theta}{5} + \dfrac{3mgl\sin\theta}{40} = \dfrac{7}{8} mgl\sin\theta$

77. Total normal force = $mg + \dfrac{mv^2}{R-r}$



$\Rightarrow mg\ (R - r) = (1/2)\ I\omega^2 + (1/2)\ mv^2$

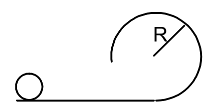
$\Rightarrow mg\ (R - r) = \dfrac{1}{2} \times \dfrac{2}{5} mv^2 + \dfrac{1}{2} mv^2$

$\Rightarrow \dfrac{7}{10} mv^2 = mg(R - r) \Rightarrow v^2 = \dfrac{10}{7}\ g(R - r)$

Therefore total normal force = $mg + \dfrac{mg + m\left(\dfrac{10}{7}\right)g(R-r)}{R-r} = mg + mg\left(\dfrac{10}{7}\right) = \dfrac{17}{7} mg$

78. At the top most point

$\dfrac{mv^2}{R-r} = mg \Rightarrow v^2 = g(R - r)$



Let the sphere is thrown with a velocity v′

Therefore applying laws of conservation of energy

$\Rightarrow (1/2)\ mv^2 + (1/2)\ I\omega'^2 = mg\ 2\ (R - r) + (1/2)\ mv^2 + (1/2)\ I\omega^2$

$\Rightarrow \dfrac{7}{10} v'^2 = g\ 2(R - r) + \dfrac{7}{10}\ v^2$

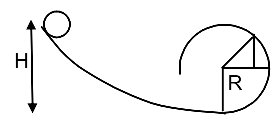
$\Rightarrow v'^2 = \dfrac{20}{7}\ g\ (R - r) + g\ (R - r)$

$\Rightarrow v' = \sqrt{\dfrac{27}{7} g(R-r)}$

79. a) Total kinetic energy y = $(1/2)\ mv^2 + (1/2)\ I\omega^2$

Therefore according to the question

$mg\ H = (1/2)\ mv^2 + (1/2)\ I\omega^2 + mg\ R\ (1 + \cos\theta)$



$\Rightarrow mg\ H - mg\ R\ (1 + \cos\theta) = (1/2)\ mv^2 + (1/2)\ I\omega^2$

$\Rightarrow (1/2)\ mv^2 + (1/2)\ I\omega^2 = mg\ (H - R - R\sin\theta)$

b) to find the acceleration components

$\Rightarrow (1/2)\ mv^2 + (1/2)\ I\omega^2 = mg\ (H - R - R\sin\theta)$

$\Rightarrow 7/10\ mv^2 = mg\ (H - R - R\sin\theta)$

$\dfrac{v^2}{R} = \dfrac{10}{7} g\left[\left(\dfrac{H}{R}\right) - 1 - \sin\theta\right] \rightarrow$ radical acceleration

$\Rightarrow v^2 = \dfrac{10}{7} g\ (H - R) - R\sin\theta$

$\Rightarrow 2v\ \dfrac{dv}{dt} = -\ \dfrac{10}{7} g\ R\cos\theta\ \dfrac{d\theta}{dt}$

$\Rightarrow \omega\ R\ \dfrac{dv}{dt} = -\ \dfrac{5}{7}\ g\ R\cos\theta\ \dfrac{d\theta}{dt}$

$\Rightarrow \dfrac{dv}{dt} = -\ \dfrac{5}{7}\ g\cos\theta \rightarrow$ tangential acceleration

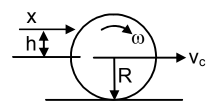
c) Normal force at $\theta = 0$

$$\Rightarrow \frac{mv^2}{R} = \frac{70}{1000} \times \frac{10}{7} \times 10\left(\frac{0.6-0.1}{0.1}\right) = 5N$$

Frictional force :-

$$f = mg - ma = m(g-a) = m\left(10 - \frac{5}{7} \times 10\right) = 0.07\left(\frac{70-50}{7}\right) = \frac{1}{100} \times 20 = 0.2N$$

80. Let the cue strikes at a height 'h' above the centre, for pure rolling, $V_c = R\omega$
Applying law of conservation of angular momentum at a point A,

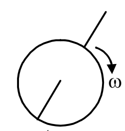


$mv_ch - \ell\omega = 0$

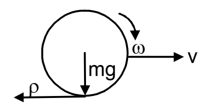
$$mv_ch = \frac{2}{3} mR^2 \times \left(\frac{v_c}{R}\right)$$

$$h = \frac{2R}{3}$$

81. A uniform wheel of radius R is set into rotation about its axis (case-I) at an angular speed ω
This rotating wheel is now placed on a rough horizontal. Because of its friction at contact, the wheel accelerates forward and its rotation decelerates. As the rotation decelerates the frictional force will act backward.
If we consider the net moment at A then it is zero.
Therefore the net angular momentum before pure rolling & after pure rolling remains constant
Before rolling the wheel was only rotating around its axis.
Therefore Angular momentum = $\ell\,\omega = (1/2)\ MR^2\,\omega$ …(1)
After pure rolling the velocity of the wheel let v
Therefore angular momentum = $\ell_{cm}\,\omega + m(V \times R)$
$= (1/2)\ mR^2\ (V/R) + mVR = 3/2\ mVR$ …(2)
Because, Eq(1) and (2) are equal
Therefore, $3/2\ mVR = ½\ mR^2\ \omega$
$\Rightarrow V = \omega R/3$

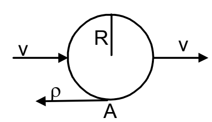


(1st case)

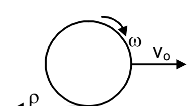


(2nd case)

82. The shell will move with a velocity nearly equal to v due to this motion a frictional force well act in the background direction, for which after some time the shell attains a pure rolling. If we consider moment about A, then it will be zero. Therefore, Net angular momentum about A before pure rolling = net angular momentum after pure rolling.
Now, angular momentum before pure rolling about A = M (V × R) and angular momentum after pure rolling :-
$(2/3)\ MR^2 \times (V_0 / R) + M\ V_0\ R$
(V_0 = velocity after pure rolling)
$\Rightarrow MVR = 2/3\ MV_0R + MV_0R$
$\Rightarrow (5/3)\ V_0 = V$
$\Rightarrow V_0 = 3V/\ 5$

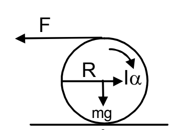


(1st case)



(2nd case)

83. Taking moment about the centre of hollow sphere we will get



$$F \times R = \frac{2}{3} MR^2\,\alpha$$

$$\Rightarrow \alpha = \frac{3F}{2MR}$$

Again, $2\pi = (1/2)\,\alpha t^2$ (From $\theta = \omega_0 t + (1/2)\,\alpha t^2$)

$$\Rightarrow t^2 = \frac{8\pi MR}{3F}$$

$$\Rightarrow a_c = \frac{F}{m}$$

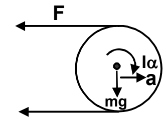
$$\Rightarrow X = (1/2)\,a_c t^2 = (1/2) = \frac{4\pi R}{3}$$

84. If we take moment about the centre, then

F × R = ℓα × f × R

⇒ F = 2/5 mRα + μmg …(1)

Again, F = ma_c – μ mg …(2)



$\Rightarrow a_c = \dfrac{F + \mu mg}{m}$

Putting the value a_c in eq(1) we get

$\Rightarrow \dfrac{2}{5} \times m \times \left(\dfrac{F + \mu mg}{m}\right) + \mu mg$

⇒ 2/5 (F + μ mg) + μ mg

$\Rightarrow F = \dfrac{2}{5}F + \dfrac{2}{5} \times 0.5 \times 10 + \dfrac{2}{7} \times 0.5 \times 10$

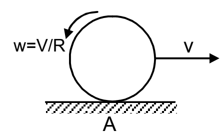
$\Rightarrow \dfrac{3F}{5} = \dfrac{4}{7} + \dfrac{10}{7} = 2$

$\Rightarrow F = \dfrac{5 \times 2}{3} = \dfrac{10}{3} = 3.33$ N

85. a) if we take moment at A then external torque will be zero

Therefore, the initial angular momentum = the angular momentum after rotation stops (i.e. only leniar velocity exits)

MV × R – ℓ ω = MV_O × R



⇒ MVR – 2/5 × MR^2 V / R = MV_O R

⇒ V_O = 3V/5

b) Again, after some time pure rolling starts

therefore ⇒ M × v_o × R = (2/5) MR^2 × (V′/R) + MV′R

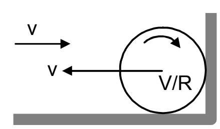
⇒ m × (3V/5) × R = (2/5) MV′R + MV′R

⇒ V′ = 3V/7

86. When the solid sphere collides with the wall, it rebounds with velocity 'v' towards left but it continues to rotate in the clockwise direction.

So, the angular momentum = mvR – (2/5) mR^2 × v/R

After rebounding, when pure rolling starts let the velocity be v′



and the corresponding angular velocity is v′ / R

Therefore angular momentum = mv′R + (2/5) mR^2 (v′/R)

So, mvR – (2/5) mR^2, v/R = mvR + (2/5) mR^2(v′/R)

mvR × (3/5) = mvR × (7/5)

v′ = 3v/7

So, the sphere will move with velocity 3v/7.

* * * *

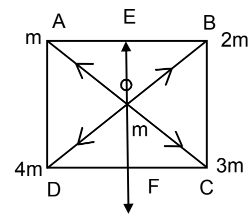
SOLUTIONS TO CONCEPTS
CHAPTER 11

1. Gravitational force of attraction,

$F = \frac{GMm}{r^2}$

$= \frac{6.67 \times 10^{-11} \times 10 \times 10}{(0.1)^2}$ = 6.67 × 10^{-7} N

2. To calculate the gravitational force on 'm' at unline due to other mouse.



$\overrightarrow{F_{OD}} = \frac{G \times m \times 4m}{(a/r^2)^2} = \frac{8Gm^2}{a^2}$

$\overrightarrow{F_{OI}} = \frac{G \times m \times 2m}{(a/r^2)^2} = \frac{6Gm^2}{a^2}$

$\overrightarrow{F_{OB}} = \frac{G \times m \times 2m}{(a/r^2)^2} = \frac{4Gm^2}{a^2}$

$\overrightarrow{F_{OA}} = \frac{G \times m \times m}{(a/r^2)^2} = \frac{2Gm^2}{a^2}$

Resultant $\overrightarrow{F_{OF}} = \sqrt{64\left(\frac{Gm^2}{a^2}\right)^2 + 36\left(\frac{Gm^2}{a^2}\right)^2} = 10\frac{Gm^2}{a^2}$

Resultant $\overrightarrow{F_{OE}} = \sqrt{64\left(\frac{Gm^2}{a^2}\right)^2 + 4\left(\frac{Gm^2}{a^2}\right)^2} = 2\sqrt{5}\frac{Gm^2}{a^2}$

The net resultant force will be,

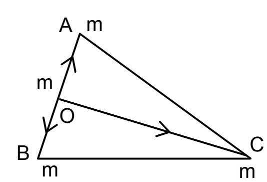
$F = \sqrt{100\left(\frac{Gm^2}{a^2}\right)^2 + 20\left(\frac{Gm^2}{a^2}\right)^2 - 2\left(\frac{Gm^2}{a^2}\right) \times 20\sqrt{5}}$

$= \sqrt{\left(\frac{Gm^2}{a^2}\right)^2 (120 - 40\sqrt{5})} = \sqrt{\left(\frac{Gm^2}{a^2}\right)^2 (120 - 89.6)}$

$= \frac{Gm^2}{a^2}\sqrt{40.4} = 4\sqrt{2}\frac{Gm^2}{a^2}$

3. a) if 'm' is placed at mid point of a side

then $\overrightarrow{F_{OA}} = \frac{4Gm^2}{a^2}$ in OA direction

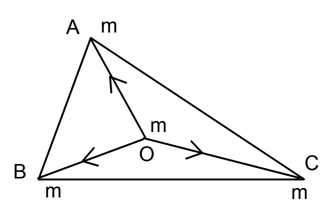


$\overrightarrow{F_{OB}} = \frac{4Gm^2}{a^2}$ in OB direction

Since equal & opposite cancel each other

$\overrightarrow{F_{oc}} = \frac{Gm^2}{\left[(r^3/2)a\right]^2} = \frac{4Gm^2}{3a^2}$ in OC direction

Net gravitational force on m = $\frac{4Gm^2}{a^2}$



b) If placed at O (centroid)

the $\overrightarrow{F_{OA}} = \frac{Gm^2}{(a/r_3)} = \frac{3Gm^2}{a^2}$

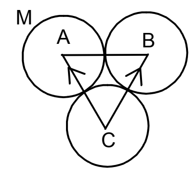
$\overrightarrow{F_{OB}} = \dfrac{3Gm^2}{a^2}$

Resultant $\vec{F} = \sqrt{2\left(\dfrac{3Gm^2}{a^2}\right)^2 - 2\left(\dfrac{3Gm^2}{a^2}\right)^2 \times \dfrac{1}{2}} = \dfrac{3Gm^2}{a^2}$

Since $\overrightarrow{F_{OC}} = \dfrac{3Gm^2}{a^2}$, equal & opposite to F, cancel

Net gravitational force = 0

4. $\overrightarrow{F_{CB}} = \dfrac{Gm^2}{4a^2}\cos 60\hat{i} - \dfrac{Gm^2}{4a^2}\sin 60\hat{j}$

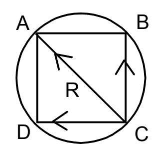


$\overrightarrow{F_{CA}} = \dfrac{Gm^2}{-4a^2}\cos 60\hat{i} - \dfrac{Gm^2}{4a^2}\sin 60\hat{j}$

$\vec{F} = \overrightarrow{F_{CB}} + \overrightarrow{F_{CA}}$

$= \dfrac{-2Gm^2}{4a^2}\sin 60\hat{j} = \dfrac{-2Gm^2}{4a^2}\dfrac{r_3}{2} = \dfrac{r_3Gm^2}{4a^2}$

5. Force on M at C due to gravitational attraction.



$\overrightarrow{F_{CB}} = \dfrac{Gm^2}{2R^2}\hat{j}$

$\overrightarrow{F_{CD}} = \dfrac{-GM^2}{4R^2}\hat{i}$

$\overrightarrow{F_{CA}} = \dfrac{-GM^2}{4R^2}\cos 45\hat{j} + \dfrac{GM^2}{4R^2}\sin 45\hat{j}$

So, resultant force on C,

$\therefore \overrightarrow{F_C} = \overrightarrow{F_{CA}} + \overrightarrow{F_{CB}} + \overrightarrow{F_{CD}}$

$= -\dfrac{GM^2}{4R^2}\left(2 + \dfrac{1}{\sqrt{2}}\right)\hat{i} + \dfrac{GM^2}{4R^2}\left(2 + \dfrac{1}{\sqrt{2}}\right)\hat{j}$

$\therefore F_C = \dfrac{GM^2}{4R^2}\left(2\sqrt{2} + 1\right)$

For moving along the circle, $\vec{F} = \dfrac{mv^2}{R}$

or $\dfrac{GM^2}{4R^2}\left(2\sqrt{2}+1\right) = \dfrac{MV^2}{R}$ or $V = \sqrt{\dfrac{GM}{R}\left(\dfrac{2\sqrt{2}+1}{4}\right)}$

6. $\dfrac{GM}{(R+h)^2} = \dfrac{6.67 \times 10^{-11} \times 7.4 \times 10^{22}}{(1740+1000)^2 \times 10^6} = \dfrac{49.358 \times 10^{11}}{2740 \times 2740 \times 10^6}$

$= \dfrac{49.358 \times 10^{11}}{0.75 \times 10^{13}} = 65.8 \times 10^{-2} = 0.65\ m/s^2$

7. The linear momentum of 2 bodies is 0 initially. Since gravitational force is internal, final momentum is also zero.

So (10 kg)v_1 = (20 kg) v_2

Or $v_1 = v_2$ …(1)

Since P.E. is conserved

Initial P.E. = $\dfrac{-6.67 \times 10^{-11} \times 10 \times 20}{1} = -13.34 \times 10^{-9}$ J

When separation is 0.5 m,

$-13.34 \times 10^{-9} + 0 = \frac{-13.34 \times 10^{-9}}{(1/2)} + (1/2) \times 10\, v_1^2 + (1/2) \times 20\, v_2^2$ …(2)

$\Rightarrow -13.34 \times 10^{-9} = -26.68 \times 10^{-9} + 5\, v_1^2 + 10\, v_2^2$

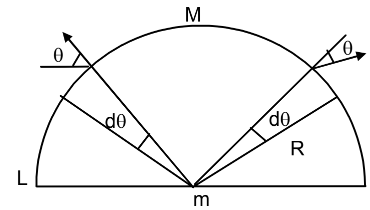
$\Rightarrow -13.34 \times 10^{-9} = -26.68 \times 10^{-9} + 30\, v_2^2$

$\Rightarrow v_2^2 = \frac{13.34 \times 10^{-9}}{30} = 4.44 \times 10^{-10}$

$\Rightarrow v_2 = 2.1 \times 10^{-5}$ m/s.

So, $v_1 = 4.2 \times 10^{-5}$ m/s.

8. In the semicircle, we can consider, a small element of d, then R dθ = (M/L) R dθ = dM.



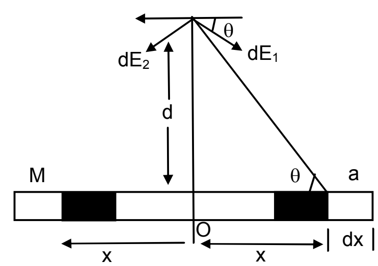
$F = \frac{GMRd\theta m}{LR^2}$

$dF_3 = 2\, dF$ since $= \frac{2GMm}{LR} \sin\theta\, d\theta$.

$\therefore F = \int_0^{\pi/2} \frac{2GMm}{LR} \sin\theta d\theta = \frac{2GMm}{LR} \left[-\cos\theta\right]_0^{\pi/2}$

$\therefore = -2\frac{GMm}{LR}(-1) = \frac{2GMm}{LR} = \frac{2GMm}{L \times L/A} = \frac{2\pi GMm}{L^2}$

9. A small section of rod is considered at 'x' distance mass of the element = (M/L). dx = dm



$dE_1 = \frac{G(dm) \times 1}{(d^2 + x^2)} = dE_2$

Resultant $dE = 2\, dE_1 \sin\theta$

$= 2 \times \frac{G(dm)}{(d^2 + x^2)} \times \frac{d}{\sqrt{(d^2 + x^2)}} = \frac{2 \times GM \times d\, dx}{L(d^2 + x^2)\left(\sqrt{(d^2 + x^2)}\right)}$

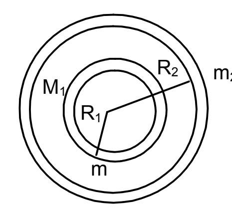
Total gravitational field

$E = \int_0^{L/2} \frac{2Gmd\, dx}{L(d^2 + x^2)^{3/2}}$

Integrating the above equation it can be found that,

$E = \frac{2GM}{d\sqrt{L^2 + 4d^2}}$

10. The gravitational force on 'm' due to the shell of M_2 is 0.



M is at a distance $\frac{R_1 + R_2}{2}$

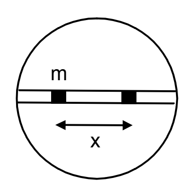
Then the gravitational force due to M is given by

$= \frac{GM_1m}{(R_1 + R_{2/2})} = \frac{4GM_1m}{(R_1 + R_2)^2}$

11. Man of earth $M = (4/3)\, \pi R^3 \rho$

Man of the imaginary sphere, having

Radius = x, $M' = (4/3)\pi x^3 \rho$



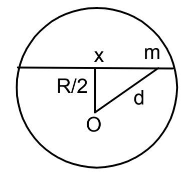
or $\frac{M'}{M} = \frac{x^3}{R^3}$

$\therefore$ Gravitational force on $F = \frac{GM'm}{m^2}$

or $F = \frac{GMx^3m}{R^3x^2} = \frac{GMmx}{R^3}$

12. Let d be the distance from centre of earth to man 'm' then

$$D = \sqrt{x^2 + \left(\frac{R^2}{4}\right)} = (1/2)\ \sqrt{4x^2 + R^2}$$



M be the mass of the earth, M′ the mass of the sphere of radius d/2.

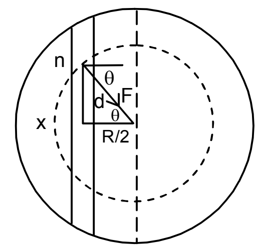
Then M = (4/3) $\pi R^3 \rho$

M′ = (4/3)$\pi d^3\ \tau$

or $\frac{M'}{M} = \frac{d^3}{R^3}$

∴ Gravitational force is m,

$$F = \frac{Gm'm}{d^2} = \frac{Gd^3Mm}{R^3d^2} = \frac{GMmd}{R^3}$$



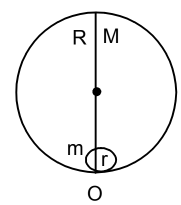
So, Normal force exerted by the wall = F cosθ.

$= \frac{GMmd}{R^3} \times \frac{R}{2d} = \frac{GMm}{2R^2}$ (therefore I think normal force does not depend on x)

13. a) m′ is placed at a distance x from 'O'.

If r < x , 2r, Let's consider a thin shell of man

$$dm = \frac{m}{(4/3)\pi r^2} \times \frac{4}{3}\pi x^3 = \frac{mx^3}{r^3}$$



Thus $\int dm = \frac{mx^3}{r^3}$

Then gravitational force $F = \frac{Gmdm}{x^2} = \frac{Gmx^3/r^3}{x^2} = \frac{Gmx}{r^3}$

b) 2r < x < 2R, then F is due to only the sphere.

$$\therefore F = \frac{Gmm'}{(x-r)^2}$$

c) if x > 2R, then Gravitational force is due to both sphere & shell, then due to shell,

$$F = \frac{GMm'}{(x-R)^2}$$

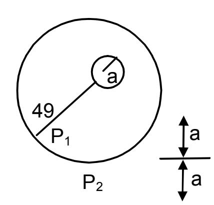
due to the sphere $= \frac{Gmm'}{(x-r)^2}$

So, Resultant force $= \frac{Gmm'}{(x-r)^2} + \frac{GMm'}{(x-R)^2}$

14. At P_1, Gravitational field due to sphere $M = \frac{GM}{(3a+a)^2} = \frac{GM}{16a^2}$

At P_2, Gravitational field is due to sphere & shell,

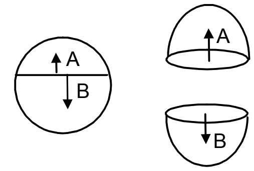
$$= \frac{GM}{(a+4a+a)^2} + \frac{GM}{(4a+a)^2} = \frac{GM}{a^2}\left(\frac{1}{36} + \frac{1}{25}\right) = \left(\frac{61}{900}\right)\frac{GM}{a^2}$$



15. We know in the thin spherical shell of uniform density has gravitational field at its internal point is zero.

At A and B point, field is equal and opposite and cancel each other so Net field is zero.

Hence, $E_A = E_B$



16. Let 0.1 kg man is x m from 2kg mass and (2 – x) m from 4 kg mass.

$$\therefore \frac{2 \times 0.1}{x^2} = -\frac{4 \times 0.1}{(2-x)^2}$$

or $\frac{0.2}{x^2} = -\frac{0.4}{(2-x)^2}$

or $\frac{1}{x^2} = \frac{2}{(2-x)^2}$ or $(2 – x)^2 = 2 x^2$

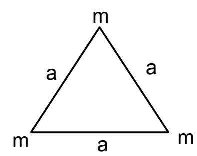
or $2 – x = \sqrt{2}$ x or $x(r_2 + 1) = 2$

or $x = \frac{2}{2.414}$ = 0.83 m from 2kg mass.

17. Initially, the ride of Δ is a

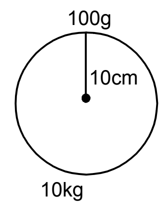
To increase it to 2a,

work done = $\frac{Gm^2}{2a} + \frac{Gm^2}{a} = \frac{3Gm^2}{2a}$



18. Work done against gravitational force to take away the particle from sphere,

$= \frac{G\times 10\times 0.1}{0.1\times 0.1} = \frac{6.67\times 10^{-11}\times 1}{1\times 10^{-1}} = 6.67 \times 10^{-10}$ J



19. $\vec{E}$ = (5 N/kg) $\hat{i}$ + (12 N/kg) $\hat{j}$

a) $\vec{F} = \vec{E}$ m

= 2kg [(5 N/kg) $\hat{i}$ + (12 N/kg) $\hat{j}$] = (10 N) $\hat{i}$ + (12 N) $\hat{j}$

$|\vec{F}| = \sqrt{100+576}$ = 26 N

b) $\vec{V} = \vec{E}$ r

At (12 m, 0), $\vec{V}$ = – (60 J/kg) $\hat{i}$ $|\vec{V}|$ = 60 J

At (0, 5 m), $\vec{V}$ = – (60 J/kg) $\hat{j}$ $|\vec{V}|$ = – 60 J

c) $\Delta\vec{V} = \int_{(0,0)}^{(1,2,5)} \vec{E}\,mdr = \left[\left[(10N)\hat{i} + (24N)\hat{j}\right] r\right]_{(0,0)}^{(12,5)}$

= – (120 J $\hat{i}$ + 120 J $\hat{i}$) = 240 J

d) Δ v = $-\left[r(10N\hat{i} + 24N\hat{j})\right]_{(12m,0)}^{(0,5m)}$

= –120 $\hat{j}$ + 120 $\hat{i}$ = 0

20. a) V = (20 N/kg) (x + y)

$\frac{GM}{R} = \frac{MLT^{-2}}{M}L$ or $\frac{M^{-1}L^3T^{-2}M^1}{L} = \frac{ML^2T^{-2}}{M}$

Or $M^0 L^2 T^{-2} = M^0 L^2 T^{-2}$

$\therefore$ L.H.S = R.H.S

b) $\vec{E}_{(x,y)}$ = – 20(N/kg) $\hat{i}$ – 20(N/kg) $\hat{j}$

c) $\vec{F} = \vec{E}$m

= 0.5kg [– (20 N/kg) $\hat{i}$ – (20 N/kg) $\hat{j}$ = – 10N $\hat{i}$ - 10 N $\hat{j}$

$\therefore$ $|\vec{F}| = \sqrt{100+100} = 10\sqrt{2}$ N

21. $\vec{E} = 2\hat{i} + 3\hat{j}$

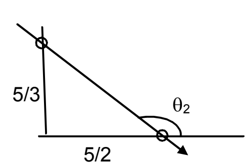
The field is represented as

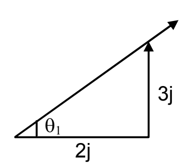
$\tan\theta_1 = 3/2$

Again the line 3y + 2x = 5 can be represented as

$\tan\theta_2 = – 2/3$

$m_1 m_2 = –1$





Since, the direction of field and the displacement are perpendicular, is done by the particle on the line.

22. Let the height be h

$\therefore (1/2)\ \frac{GM}{R^2} = \frac{GM}{(R+h)^2}$

Or $2R^2 = (R + h)^2$

Or $\sqrt{2}\ R = R + h$

Or $h = (r_2 - 1)R$

23. Let g′ be the acceleration due to gravity on mount everest.

$g' = g\left(1-\frac{2h}{R}\right)$

$=9.8\left(1-\frac{17696}{6400000}\right) = 9.8\ (1 - 0.00276) = 9.773\ m/s^2$

24. Let g′ be the acceleration due to gravity in mine.

Then $g' = g\left(1-\frac{d}{R}\right)$

$= 9.8\ \left(1-\frac{640}{6400\times10^3}\right) = 9.8 \times 0.9999 = 9.799\ m/s^2$

25. Let g′ be the acceleration due to gravity at equation & that of pole = g

$g' = g - \omega^2 R$

$= 9.81 - (7.3 \times 10^{-5})^2 \times 6400 \times 10^3$

$= 9.81 - 0.034$

$= 9.776\ m/s^2$

$mg' = 1\ kg \times 9.776\ m/s^2$

$= 9.776$ N or 0.997 kg

The body will weigh 0.997 kg at equator.

26. At equator, $g' = g - \omega^2 R$ …(1)

Let at 'h' height above the south pole, the acceleration due to gravity is same.

Then, here $g' = g\ \left(1-\frac{2h}{R}\right)$ …(2)

$\therefore\ g - \omega^2 R = g\ \left(1-\frac{2h}{R}\right)$

or $1-\frac{\omega^2 R}{g} = 1-\frac{2h}{R}$

or $h = \frac{\omega^2 R^2}{2g} = \frac{(7.3\times10^{-5})^2\times(6400\times10^3)^2}{2\times9.81}$ = 11125 N = 10Km (approximately)

27. The apparent 'g' at equator becomes zero.

i.e. $g' = g - \omega^2 R = 0$

or $g = \omega^2 R$

or $\omega = \sqrt{\frac{g}{R}} = \sqrt{\frac{9.8}{6400\times10^3}} = \sqrt{1.5\times10^{-6}} = 1.2 \times 10^{-3}$ rad/s.

$T = \frac{2\pi}{\omega} = \frac{2\times3.14}{1.2\times10^{-3}} = 1.5 \times 10^{-6}$ sec. = 1.41 hour

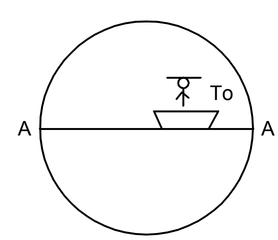
28. a) Speed of the ship due to rotation of earth $v = \omega R$

b) $T_0 = mgr = mg - m\omega^2 R$

$\therefore\ T_0 - mg = m\omega^2 R$

c) If the ship shifts at speed 'v'

$T = mg - m\omega_1^2 R$



$= T_0 - \left(\frac{(v-\omega R)^2}{R^2}\right)R$

$= T_0 - \left(\frac{v^2+\omega^2R^2-2\omega Rv}{R}\right)$ m

$\therefore$ T = T_0 + $2\omega v$ m

29. According to Kepler's laws of planetary motion,

$T^2 \; \alpha \; R^3$

$\frac{T_m{}^2}{T_e{}^2} = \frac{R_{ms}{}^3}{R_{es}{}^3}$

$\left(\frac{R_{ms}}{R_{es}}\right)^3 = \left(\frac{1.88}{1}\right)^2$

$\therefore \frac{R_{ms}}{R_{es}}$ = $(1.88)^{2/3}$ = 1.52

30. $T = 2\pi\sqrt{\frac{r^3}{GM}}$

$27.3 = 2 \times 3.14\sqrt{\frac{(3.84\times10^5)^3}{6.67\times10^{-11}\times M}}$

or $2.73 \times 2.73 = \frac{2\times3.14\times(3.84\times10^5)^3}{6.67\times10^{-11}\times M}$

or $M = \frac{2\times(3.14)^2\times(3.84)^3\times10^{15}}{3.335\times10^{-11}(27.3)^2}$ = 6.02×10^{24} kg

$\therefore$ mass of earth is found to be 6.02×10^{24} kg.

31. $T = 2\pi\sqrt{\frac{r^3}{GM}}$

$\Rightarrow 27540 = 2 \times 3.14\sqrt{\frac{(9.4\times10^3\times10^3)^3}{6.67\times10^{-11}\times M}}$

or $(27540)^2 = (6.28)^2 \; \frac{(9.4\times10^6)^2}{6.67\times10^{-11}\times M}$

or $M = \frac{(6.28)^2\times(9.4)^3\times10^{18}}{6.67\times10^{-11}\times(27540)^2}$ = 6.5×10^{23} kg.

32. a) $V = \sqrt{\frac{GM}{r+h}} = \sqrt{\frac{gr^2}{r+h}}$

$= \sqrt{\frac{9.8\times(6400\times10^3)^2}{10^6\times(6.4+2)}}$ = 6.9×10^3 m/s = 6.9 km/s

b) K.E. = (1/2) mv^2

= (1/2) 1000 × (47.6 × 10^6) = 2.38 × 10^{10} J

c) P.E. = $\frac{GMm}{-(R+h)}$

$= -\frac{6.67\times10^{-11}\times6\times10^{24}\times10^3}{(6400+2000)\times10^3} = -\frac{40\times10^{13}}{8400}$ = – 4.76 × 10^{10} J

d) $T = \frac{2\pi(r+h)}{V} = \frac{2\times3.14\times8400\times10^3}{6.9\times10^3}$ = 76.6 × 10^2 sec = 2.1 hour

33. Angular speed f earth & the satellite will be same

$\frac{2\pi}{T_e} = \frac{2\pi}{T_s}$

or $\frac{1}{24 \times 3600} = \frac{1}{2\pi\sqrt{\frac{(R+h)^3}{gR^2}}}$ or 12 I 3600 = 3.14 $\sqrt{\frac{(R+h)^3}{gR^2}}$

or $\frac{(R+h)^2}{gR^2} = \frac{(12 \times 3600)^2}{(3.14)^2}$ or $\frac{(6400+h)^3 \times 10^9}{9.8 \times (6400)^2 \times 10^6} = \frac{(12 \times 3600)^2}{(3.14)^2}$

or $\frac{(6400+h)^3 \times 10^9}{6272 \times 10^9} = 432 \times 10^4$

or $(6400 + h)^3 = 6272 \times 432 \times 10^4$

or $6400 + h = (6272 \times 432 \times 10^4)^{1/3}$

or $h = (6272 \times 432 \times 10^4)^{1/3} - 6400$

= 42300 cm.

b) Time taken from north pole to equator = (1/2) t

$= (1/2) \times 6.28 \sqrt{\frac{(43200+6400)^3}{10 \times (6400)^2 \times 10^6}} = 3.14 \sqrt{\frac{(497)^3 \times 10^6}{(64)^2 \times 10^{11}}}$

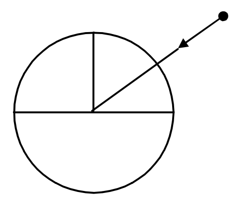
$= 3.14 \sqrt{\frac{497 \times 497 \times 497}{64 \times 64 \times 10^5}}$ = 6 hour.

34. For geo stationary satellite,

$r = 4.2 \times 10^4$ km

$h = 3.6 \times 10^4$ km

Given mg = 10 N



$mgh = mg\left(\frac{R^2}{(R+h)^2}\right)$

$= 10\left[\frac{(6400 \times 10^3)^2}{(6400 \times 10^3 + 3600 \times 10^3)^2}\right] = \frac{4096}{17980} = 0.23$ N

35. $T = 2\pi\sqrt{\frac{R_2^{\,3}}{gR_1^{\,2}}}$

Or $T^2 = 4\pi^2 \frac{R_2^{\,3}}{gR_1^{\,2}}$

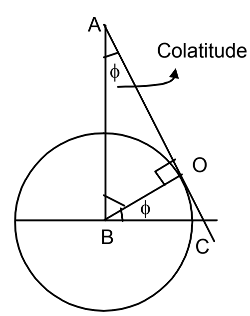
Or $g = \frac{4\pi^2}{T^2}\frac{R_2^{\,3}}{R_1^{\,2}}$

$\therefore$ Acceleration due to gravity of the planet is = $\frac{4\pi^2}{T^2}\frac{R_2^{\,3}}{R_1^{\,2}}$

36. The colattitude is given by ϕ.

$\angle OAB = 90° - \angle ABO$

Again $\angle OBC = \phi = \angle OAB$



$\therefore \sin\phi = \frac{6400}{42000} = \frac{8}{53}$

$\therefore \phi = \sin^{-1}\left(\frac{8}{53}\right) = \sin^{-1} 0.15.$

37. The particle attain maximum height = 6400 km.
On earth's surface, its P.E. & K.E.
$E_e = (1/2)\ mv^2 + \left(\frac{-GMm}{R}\right)$ …(1)
In space, its P.E. & K.E.
$E_s = \left(-\frac{GMm}{R+h}\right) + 0$
$E_s = \left(-\frac{GMm}{2R}\right)$ …(2) ($\because$ h = R)
Equating (1) & (2)
$-\frac{GMm}{R} + \frac{1}{2}mv^2 = -\frac{GMm}{2R}$
Or $(1/2)\ mv^2 = GMm\left(-\frac{1}{2R} + \frac{1}{R}\right)$
Or $v^2 = \frac{GM}{R}$
$= \frac{6.67 \times 10^{-11} \times 6 \times 10^{24}}{6400 \times 10^3}$
$= \frac{40.02 \times 10^{13}}{6.4 \times 10^6}$
$= 6.2 \times 10^7 = 0.62 \times 10^8$
Or $v = \sqrt{0.62 \times 10^8} = 0.79 \times 10^4$ m/s = 7.9 km/s.

38. Initial velocity of the particle = 15km/s
Let its speed be 'v' at interstellar space.
$\therefore (1/2)\ m[(15 \times 10^3)^2 - v^2] = \int_R^\infty \frac{GMm}{x^2}dx$
$\Rightarrow (1/2)\ m[(15 \times 10^3)^2 - v^2] = GMm\left[-\frac{1}{x}\right]_R^\infty$
$\Rightarrow (1/2)\ m[(225 \times 10^6) - v^2] = \frac{GMm}{R}$
$\Rightarrow 225 \times 10^6 - v^2 = \frac{2 \times 6.67 \times 10^{-11} \times 6 \times 10^{24}}{6400 \times 10^3}$
$\Rightarrow v^2 = 225 \times 10^6 - \frac{40.02}{32} \times 10^8$
$\Rightarrow v^2 = 225 \times 10^6 - 1.2 \times 10^8 = 10^8\ (1.05)$
Or $v = 1.01 \times 10^4$ m/s or
= 10 km/s

39. The man of the sphere = 6×10^{24} kg.
Escape velocity = 3×10^8 m/s
$V_c = \sqrt{\frac{2GM}{R}}$
Or $R = \frac{2GM}{V_c^{\ 2}}$
$= \frac{2 \times 6.67 \times 10^{-11} \times 6 \times 10^{24}}{(3 \times 10^8)^2} = \frac{80.02}{9} \times 10^{-3} = 8.89 \times 10^{-3}$ m $\approx$ 9 mm.

♣ ♣ ♣ ♣ ♣

SOLUTIONS TO CONCEPTS
CHAPTER 12

1. Given, r = 10cm.
   At t = 0, x = 5 cm.
   T = 6 sec.
   So, w = $\frac{2\pi}{T}$ = $\frac{2\pi}{6}$ = $\frac{\pi}{3}$ sec^{-1}
   At, t = 0, x = 5 cm.
   So, 5 = 10 sin (w × 0 + ϕ) = 10 sin ϕ [y = r sin wt]
   Sin ϕ = 1/2 $\Rightarrow$ ϕ = $\frac{\pi}{6}$
   $\therefore$ Equation of displacement x = (10cm) sin $\left(\frac{\pi}{3}\right)$
   (ii) At t = 4 second
   x = 10 sin $\left[\frac{\pi}{3}\times 4+\frac{\pi}{6}\right]$ = 10 sin $\left[\frac{8\pi+\pi}{6}\right]$
   = 10 sin $\left(\frac{3\pi}{2}\right)$ = 10 sin $\left(\pi+\frac{\pi}{2}\right)$ = - 10 sin $\left(\frac{\pi}{2}\right)$ = -10
   Acceleration a = – w^2x = –$\left(\frac{\pi^2}{9}\right)$ × (–10) = 10.9 ≈ 0.11 cm/sec.
2. Given that, at a particular instant,
   X = 2cm = 0.02m
   V = 1 m/sec
   A = 10 $msec^{-2}$
   We know that a = ω^2x
   $\Rightarrow \omega = \sqrt{\frac{a}{x}} = \sqrt{\frac{10}{0.02}} = \sqrt{500} = 10\sqrt{5}$
   $T = \frac{2\pi}{\omega} = \frac{2\pi}{10\sqrt{5}} = \frac{2\times 3.14}{10\times 2.236}$ = 0.28 seconds.
   Again, amplitude r is given by v = $\omega\left(\sqrt{r^2-x^2}\right)$
   $\Rightarrow v^2 = \omega^2(r^2 - x^2)$
   $1 = 500\ (r^2 - 0.0004)$
   $\Rightarrow$ r = 0.0489 ≈ 0.049 m
   $\therefore$ r = 4.9 cm.
3. r = 10cm
   Because, K.E. = P.E.
   So (1/2) m ω^2 (r^2– y^2) = (1/2) m ω^2y^2
   $r^2 - y^2 = y^2 \Rightarrow 2y^2 = r^2 \Rightarrow y = \frac{r}{\sqrt{2}} = \frac{10}{\sqrt{2}} = 5\sqrt{2}$ cm form the mean position.
4. v_{max} = 10 cm/sec.
   $\Rightarrow r\omega = 10$
   $\Rightarrow \omega^2 = \frac{100}{r^2}$ …(1)
   $A_{max} = \omega^2 r$ = 50 cm/sec
   $\Rightarrow \omega^2 = \frac{50}{y} = \frac{50}{r}$ …(2)

$\therefore \frac{100}{r^2} = \frac{50}{r} \Rightarrow r = 2$ cm.

$\therefore \omega = \sqrt{\frac{100}{r^2}} = 5\ \text{sec}^2$

Again, to find out the positions where the speed is 8m/sec,

$v^2 = \omega^2 (r^2 - y^2)$

$\Rightarrow 64 = 25 (4 - y^2)$

$\Rightarrow 4 - y^2 = \frac{64}{25} \Rightarrow y^2 = 1.44 \Rightarrow y = \sqrt{1.44} \Rightarrow y = \pm 1.2$ cm from mean position.

5. $x = (2.0\text{cm})\sin [(100\text{s}^{-1})\, t + (\pi/6)]$

m = 10g.

a) Amplitude = 2cm.

$\omega = 100\ \text{sec}^{-1}$

$\therefore T = \frac{2\pi}{100} = \frac{\pi}{50}$ sec = 0.063 sec.

We know that $T = 2\pi \sqrt{\frac{m}{k}} \Rightarrow T^2 = 4\pi^2 \times \frac{m}{k} \Rightarrow k = \frac{4\pi^2}{T^2} m$

$= 10^5$ dyne/cm = 100 N/m. [because $\omega = \frac{2\pi}{T} = 100\ \text{sec}^{-1}$]

b) At t = 0

$x = 2\text{cm} \sin\left(\frac{\pi}{6}\right) = 2 \times (1/2) = 1$ cm. from the mean position.

We know that $x = A \sin (\omega t + \phi)$

$v = A \cos (\omega t + \phi)$

$= 2 \times 100 \cos (0 + \pi/6) = 200 \times \frac{\sqrt{3}}{2} = 100\sqrt{3}\ \text{sec}^{-1} = 1.73\text{m/s}$

c) $a = -\omega^2 x = 100^2 \times 1 = 100\ \text{m/s}^2$

6. $x = 5 \sin (20t + \pi/3)$

a) Max. displacement from the mean position = Amplitude of the particle.

At the extreme position, the velocity becomes '0'.

$\therefore$ x = 5 = Amplitude.

$\therefore 5 = 5 \sin (20t + \pi/3)$

$\sin (20t + \pi/3) = 1 = \sin (\pi/2)$

$\Rightarrow 20t + \pi/3 = \pi/2$

$\Rightarrow t = \pi/120$ sec., So at $\pi/120$ sec it first comes to rest.

b) $a = \omega^2 x = \omega^2 [5 \sin (20t + \pi/3)]$

For a = 0, $5 \sin (20t + \pi/3) = 0 \Rightarrow \sin (20t + \pi/3) = \sin (\pi)$

$\Rightarrow 20\, t = \pi - \pi/3 = 2\pi/3$

$\Rightarrow t = \pi/30$ sec.

c) $v = A\, \omega \cos (\omega t + \pi/3) = 20 \times 5 \cos (20t + \pi/3)$

when, v is maximum i.e. $\cos (20t + \pi/3) = -1 = \cos \pi$

$\Rightarrow 20t = \pi - \pi/3 = 2\pi/3$

$\Rightarrow t = \pi/30$ sec.

7. a) $x = 2.0 \cos (50\pi t + \tan^{-1} 0.75) = 2.0 \cos (50\pi t + 0.643)$

$v = \frac{dx}{dt} = -100 \sin (50\pi t + 0.643)$

$\Rightarrow \sin (50\pi t + 0.643) = 0$

As the particle comes to rest for the 1st time

$\Rightarrow 50\pi t + 0.643 = \pi$

$\Rightarrow t = 1.6 \times 10^{-2}$ sec.

b) Acceleration $a = \frac{dv}{dt} = -100\pi \times 50\,\pi \cos(50\pi t + 0.643)$

For maximum acceleration $\cos(50\pi t + 0.643) = -1 \cos \pi$ (max) (so a is max)

$\Rightarrow t = 1.6 \times 10^{-2}$ sec.

c) When the particle comes to rest for second time,

$50\pi t + 0.643 = 2\pi$

$\Rightarrow t = 3.6 \times 10^{-2}$ s.

8. $y_1 = \frac{r}{2}$, $y_2 = r$ (for the two given position)

Now, $y_1 = r \sin \omega t_1$

$\Rightarrow \frac{r}{2} = r \sin \omega t_1 \Rightarrow \sin \omega t_1 = \frac{1}{2} \Rightarrow \omega t_1 = \frac{\pi}{2} \Rightarrow \frac{2\pi}{t} \times t_1 = \frac{\pi}{6} \Rightarrow t_1 = \frac{t}{12}$

Again, $y_2 = r \sin \omega t_2$

$\Rightarrow r = r \sin \omega t_2 \Rightarrow \sin \omega t_2 = 1 \Rightarrow \omega t_2 = \pi/2 \Rightarrow \left(\frac{2\pi}{t}\right) t_2 = \frac{\pi}{2} \Rightarrow t_2 = \frac{t}{4}$

So, $t_2 - t_1 = \frac{t}{4} - \frac{t}{12} = \frac{t}{6}$

9. k = 0.1 N/m

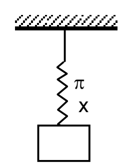
$T = 2\pi\sqrt{\frac{m}{k}} = 2$ sec [Time period of pendulum of a clock = 2 sec]

So, $4\pi^{2+}\left(\frac{m}{k}\right) = 4$

$\therefore m = \frac{k}{\pi^2} = \frac{0.1}{10} = 0.01\text{kg} \approx 10$ gm.

10. Time period of simple pendulum $= 2\pi\sqrt{\frac{1}{g}}$

Time period of spring is $2\pi\sqrt{\frac{m}{k}}$



$T_p = T_s$ [Frequency is same]

$\Rightarrow \sqrt{\frac{1}{g}} = \sqrt{\frac{m}{k}} \qquad \Rightarrow \frac{1}{g} = \frac{m}{k}$

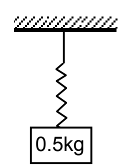
$\Rightarrow 1 = \frac{mg}{k} = \frac{F}{k} = x$. (Because, restoring force = weight = F =mg)

$\Rightarrow 1 = x$ (proved)

11. x = r = 0.1 m

T = 0.314 sec

m = 0.5 kg.



Total force exerted on the block = weight of the block + spring force.

$T = 2\pi\sqrt{\frac{m}{k}} \Rightarrow 0.314 = 2\pi\sqrt{\frac{0.5}{k}} \Rightarrow k = 200$ N/m

$\therefore$ Force exerted by the spring on the block is

$F = kx = 201.1 \times 0.1 = 20$N

$\therefore$ Maximum force = F + weight = 20 + 5 = 25N

12. m = 2kg.

T = 4 sec.

$T = 2\pi\sqrt{\frac{m}{k}} \Rightarrow 4 = 2\pi\sqrt{\frac{2}{K}} \Rightarrow 2 = \pi\sqrt{\frac{2}{K}}$

$\Rightarrow 4 = \pi^2 \left(\frac{2}{k}\right) \Rightarrow k = \frac{2\pi^2}{4} \Rightarrow k = \frac{\pi^2}{2} = 5$ N/m

But, we know that F = mg = kx

$\Rightarrow x = \frac{mg}{k} = \frac{2 \times 10}{5} = 4$

$\therefore$ Potential Energy = (1/2) k x^2 = (1/2) × 5 × 16 = 5 × 8 = 40J

13. x = 25cm = 0.25m

E = 5J

f = 5

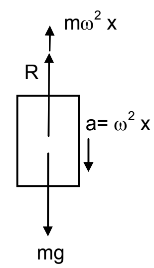
So, T = 1/5sec.

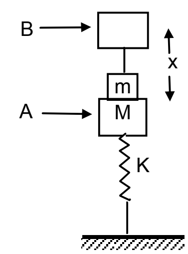
Now P.E. = (1/2) kx^2

$\Rightarrow$(1/2) $kx^2 = 5 \Rightarrow$ (1/2) k $(0.25)^2 = 5 \Rightarrow$ k = 160 N/m.

Again, $T = 2\pi\sqrt{\frac{m}{k}} \Rightarrow \frac{1}{5} = 2\pi\sqrt{\frac{m}{160}} \Rightarrow$ m = 0.16 kg.

14. a) From the free body diagram,





$\therefore R + m\omega^2 x - mg = 0$ …(1)

Resultant force $m\omega^2 x = mg - R$

$\Rightarrow m\omega^2 x = m\left(\frac{k}{M+m}\right) \Rightarrow x = \frac{mkx}{M+m}$

[$\omega = \sqrt{k/(M+m)}$ for spring mass system]

b) $R = mg - m\omega^2 x = mg - m\frac{k}{M+m}x = mg - \frac{mkx}{M+m}$

For R to be smallest, $m\omega^2 x$ should be max. i.e. x is maximum.

The particle should be at the high point.

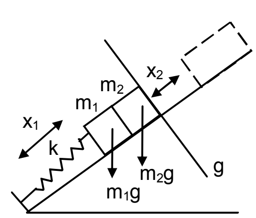
c) We have $R = mg - m\omega^2 x$

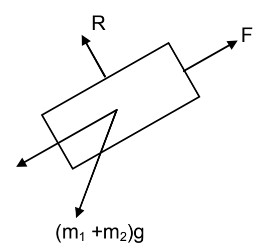
The tow blocks may oscillates together in such a way that R is greater than 0. At limiting condition, R = 0, $mg = m\omega^2 x$

$X = \frac{mg}{m\omega^2} = \frac{mg(M+m)}{mk}$

So, the maximum amplitude is = $\frac{g(M+m)}{k}$

15. a) At the equilibrium condition,





$kx = (m_1 + m_2)$ g sin θ

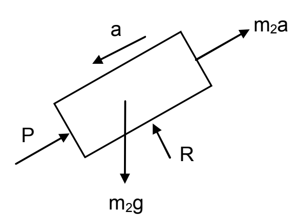
$\Rightarrow x = \frac{(m_1 + m_2)g\sin\theta}{k}$

b) $x_1 = \frac{2}{k}(m_1 + m_2)$ g sin θ (Given)

when the system is released, it will start to make SHM

where $\omega = \sqrt{\frac{k}{m_1 + m_2}}$

When the blocks lose contact, P = 0



So m_2 g sin θ = $m_2 x_2 \omega^2 = m_2 x_2 \left(\frac{k}{m_1 + m_2}\right)$

$\Rightarrow x_2 = \frac{(m_1 + m_2)g\sin\theta}{k}$

So the blocks will lose contact with each other when the springs attain its natural length.

c) Let the common speed attained by both the blocks be v.

$1/2\ (m_1 + m_2)\ v^2 - 0 = 1/2\ k(x_1 + x_2)^2 - (m_1 + m_2)\ g \sin\theta\ (x + x_1)$

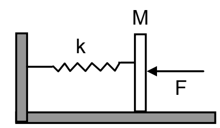
[$x + x_1$ = total compression]

$\Rightarrow (1/2)\ (m_1 + m_2)\ v^2 = [(1/2)\ k\ (3/k)\ (m_1 + m_2)\ g \sin\theta - (m_1 + m_2)\ g \sin\theta]\ (x + x_1)$

$\Rightarrow (1/2)\ (m_1 + m_2)\ v^2 = (1/2)\ (m_1 + m_2)\ g \sin\theta \times (3/k)\ (m_1 + m_2)\ g \sin\theta$

$\Rightarrow v = \sqrt{\dfrac{3}{k(m_1+m_2)}}\ g \sin\theta.$

16. Given, k = 100 N/m, M = 1kg and F = 10 N



a) In the equilibrium position,

compression $\delta = F/k = 10/100 = 0.1$ m = 10 cm

b) The blow imparts a speed of 2m/s to the block towards left.

$\therefore$ P.E. + K.E. $= 1/2\ k\delta^2 + 1/2\ Mv^2$

$= (1/2) \times 100 \times (0.1)^2 + (1/2) \times 1 \times 4 = 0.5 + 2 = 2.5$ J

c) Time period $= 2\pi\sqrt{\dfrac{M}{k}} = 2\pi\sqrt{\dfrac{1}{100}} = \dfrac{\pi}{5}$ sec

d) Let the amplitude be 'x' which means the distance between the mean position and the extreme position.

So, in the extreme position, compression of the spring is $(x + \delta)$.

Since, in SHM, the total energy remains constant.

$(1/2)\ k\ (x + \delta)^2 = (1/2)\ k\delta^2 + (1/2)\ mv^2 + Fx = 2.5 + 10x$

[because $(1/2)\ k\delta^2 + (1/2)\ mv^2 = 2.5$]

So, $50(x + 0.1)^2 = 2.5 + 10x$

$\therefore\ 50\ x^2 + 0.5 + 10x = 2.5 + 10x$

$\therefore 50x^2 = 2 \Rightarrow x^2 = \dfrac{2}{50} = \dfrac{4}{100} \Rightarrow x = \dfrac{2}{10}$ m = 20cm.

e) Potential Energy at the left extreme is given by,

P.E. $= (1/2)\ k\ (x + \delta)^2 = (1/2) \times 100\ (0.1 + 0.2)^2 = 50 \times 0.09 = 4.5$J

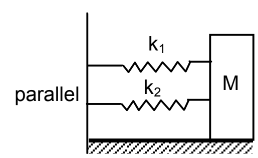
f) Potential Energy at the right extreme is given by,

P.E. $= (1/2)\ k\ (x + \delta)^2 - F(2x)$ [2x = distance between two extremes]

$= 4.5 - 10(0.4) = 0.5$J

The different values in (b) (e) and (f) do not violate law of conservation of energy as the work is done by the external force 10N.

17. a) Equivalent spring constant $k = k_1 + k_2$ (parallel)



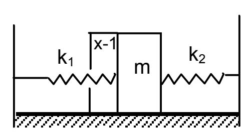
(a)

$T = 2\pi\sqrt{\dfrac{M}{k}} = 2\pi\sqrt{\dfrac{m}{k_1+k_2}}$

b) Let us, displace the block m towards left through displacement 'x'

Resultant force $F = F_1 + F_2 = (k_1 + k_2)x$

Acceleration (F/m) $= \dfrac{(k_1+k_2)x}{m}$

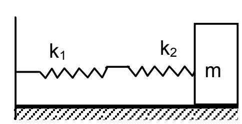


Time period $T = 2\pi\sqrt{\dfrac{\text{displacement}}{\text{Acceleration}}} = 2\pi\sqrt{\dfrac{x}{\dfrac{m(k_1+k_2)}{m}}} = 2\pi\sqrt{\dfrac{m}{k_1+k_2}}$

The equivalent spring constant $k = k_1 + k_2$

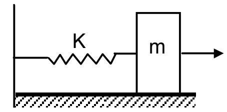
c) In series conn equivalent spring constant be k.

So, $\dfrac{1}{k} = \dfrac{1}{k_1} + \dfrac{1}{k_2} = \dfrac{k_2+k_1}{k_1k_2} \Rightarrow k = \dfrac{k_1k_2}{k_1+k_2}$



$T = 2\pi\sqrt{\dfrac{M}{k}} = 2\pi\sqrt{\dfrac{m(k_1+k_2)}{k_1k_2}}$

18. a) We have F = kx ⇒ $x = \frac{F}{k}$



Acceleration = $\frac{F}{m}$

Time period $T = 2\pi\sqrt{\frac{\text{displacement}}{\text{Acceleration}}} = 2\pi\sqrt{\frac{F/k}{F/m}} = 2\pi\sqrt{\frac{m}{k}}$

Amplitude = max displacement = F/k

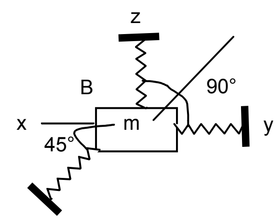
b) The energy stored in the spring when the block passes through the equilibrium position

$(1/2)\,kx^2 = (1/2)\,k\,(F/k)^2 = (1/2)\,k\,(F^2/k^2) = (1/2)\,(F^2/k)$

c) At the mean position, P.E. is 0. K.E. is $(1/2)\,kx^2 = (1/2)\,(F^2/x)$

19. Suppose the particle is pushed slightly against the spring 'C' through displacement 'x'.

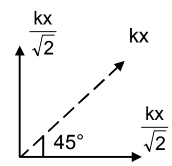
Total resultant force on the particle is kx due to spring C and $\frac{kx}{\sqrt{2}}$ due to spring A and B.



∴ Total Resultant force = $kx + \sqrt{\left(\frac{kx}{\sqrt{2}}\right)^2 + \left(\frac{kx}{\sqrt{2}}\right)^2} = kx + kx = 2kx.$

Acceleration = $\frac{2kx}{m}$

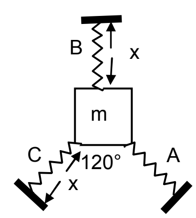
Time period $T = 2\pi\sqrt{\frac{\text{displacement}}{\text{Acceleration}}} = 2\pi\sqrt{\frac{x}{\frac{2kx}{m}}} = 2\pi\sqrt{\frac{m}{2k}}$



[Cause:- When the body pushed against 'C' the spring C, tries to pull the block towards XL. At that moment the spring A and B tries to pull the block with force $\frac{kx}{\sqrt{2}}$ and $\frac{kx}{\sqrt{2}}$ respectively towards xy and xz respectively. So the total force on the block is due to the spring force 'C' as well as the component of two spring force A and B.]

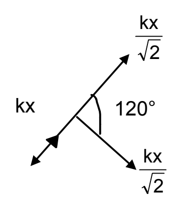
20. In this case, if the particle 'm' is pushed against 'C' a by distance 'x'.

Total resultant force acting on man 'm' is given by,



$F = kx + \frac{kx}{2} = \frac{3kx}{2}$

[Because net force A & B = $\sqrt{\left(\frac{kx}{2}\right)^2 + \left(\frac{kx}{2}\right)^2 + 2\left(\frac{kx}{2}\right)\left(\frac{kx}{2}\right)\cos 120°} = \frac{kx}{2}$



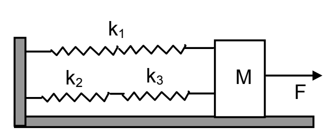
∴ $a = \frac{F}{m} = \frac{3kx}{2m}$

⇒ $\frac{a}{x} = \frac{3k}{2m} = \omega^2$ ⇒ $\omega = \sqrt{\frac{3k}{2m}}$

∴Time period $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{2m}{3k}}$

21. K_2 and K_3 are in series.

Let equivalent spring constant be K_4



∴ $\frac{1}{K_4} = \frac{1}{K_2} + \frac{1}{K_3} = \frac{K_2 + K_3}{K_2K_3} \Rightarrow K_4 = \frac{K_2K_3}{K_2 + K_3}$

Now K_4 and K_1 are in parallel.

So equivalent spring constant $k = k_1 + k_4 = \frac{K_2K_3}{K_2 + K_3} + k_1 = \frac{k_2k_3 + k_1k_2 + k_1k_3}{k_2 + k_3}$

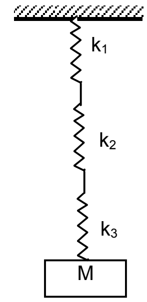
∴ $T = 2\pi\sqrt{\frac{M}{k}} = 2\pi\sqrt{\frac{M(k_2 + k_3)}{k_2k_3 + k_1k_2 + k_1k_3}}$

b) frequency = $\frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k_2k_3 + k_1k_2 + k_1k_3}{M(k_2 + k_3)}}$

c) Amplitude x = $\frac{F}{k} = \frac{F(k_2 + k_3)}{k_1k_2 + k_2k_3 + k_1k_3}$

22. k_1, k_2, k_3 are in series,

$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3}$ $\Rightarrow k = \frac{k_1k_2k_3}{k_1k_2 + k_2k_3 + k_1k_3}$



Time period T = $2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m(k_1k_2 + k_2k_3 + k_1k_3)}{k_1k_2k_3}} = 2\pi\sqrt{m\left(\frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3}\right)}$

Now, Force = weight = mg.

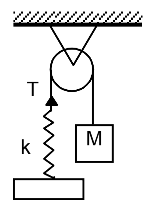
$\therefore$ At k_1 spring, $x_1 = \frac{mg}{k_1}$

Similarly $x_2 = \frac{mg}{k_2}$ and $x_3 = \frac{mg}{k_3}$

$\therefore PE_1 = (1/2)\, k_1\, x_1^2 = \frac{1}{2}k_1\left(\frac{Mg}{k_1}\right)^2 = \frac{1}{2}k_1 \frac{m^2g^2}{k_1^2} = \frac{m^2g^2}{2k_1}$

Similarly $PE_2 = \frac{m^2g^2}{2k_2}$ and $PE_3 = \frac{m^2g^2}{2k_3}$

23. When only 'm' is hanging, let the extension in the spring be 'ℓ'
So $T_1 = k\ell = mg$.
When a force F is applied, let the further extension be 'x'
$\therefore T_2 = k(x + \ell)$
$\therefore$ Driving force = $T_2 - T_1 = k(x + \ell) - k\ell = kx$
$\therefore$ Acceleration = $\frac{K\ell}{m}$



$T = 2\pi\sqrt{\frac{\text{displacement}}{\text{Acceleration}}} = 2\pi\sqrt{\frac{x}{\frac{kx}{m}}} = 2\pi\sqrt{\frac{m}{k}}$

24. Let us solve the problem by 'energy method'.
Initial extension of the sprig in the mean position,

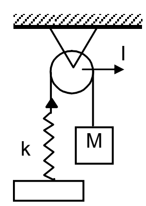
$\delta = \frac{mg}{k}$

During oscillation, at any position 'x' below the equilibrium position, let the velocity of 'm' be v and angular velocity of the pulley be 'ω'. If r is the radius of the pulley, then v = rω.
At any instant, Total Energy = constant (for SHM)

$\therefore (1/2)\, mv^2 + (1/2)\, I\, \omega^2 + (1/2)\, k[(x + \delta)^2 - \delta^2] - mgx = \text{Cosntant}$

$\Rightarrow (1/2)\, mv^2 + (1/2)\, I\, \omega^2 + (1/2)\, kx^2 - kx\delta - mgx = \text{Cosntant}$

$\Rightarrow (1/2)\, mv^2 + (1/2)\, I\, (v^2/r^2) + (1/2)\, kx^2 = \text{Constant}$ $(\delta = mg/k)$



Taking derivative of both sides eith respect to 't',

$mv\frac{dv}{dt} + \frac{I}{r^2}v\frac{dv}{dt} + k \times \frac{dv}{dt} = 0$

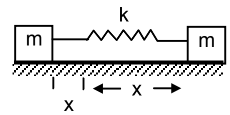
$\Rightarrow a\left(m + \frac{I}{r^2}\right) = kx$ $(\therefore x = \frac{dx}{dt}$ and $a = \frac{dx}{dt})$

$\Rightarrow \frac{a}{x} = \frac{k}{m + \frac{I}{r^2}} = \omega^2 \Rightarrow T = 2\pi\sqrt{\frac{m + \frac{I}{r^2}}{k}}$

25. The centre of mass of the system should not change during the motion. So, if the block 'm' on the left moves towards right a distance 'x', the block on the right moves towards left a distance 'x'. So, total compression of the spring is 2x.

By energy method, $\frac{1}{2}k\,(2x)^2 + \frac{1}{2}\,mv^2 + \frac{1}{2}mv^2 = C \Rightarrow mv^2 + 2kx^2 = C.$

Taking derivative of both sides with respect to 't'.



$m \times 2v\,\frac{dv}{dt} + 2k \times 2x\,\frac{dx}{dt} = 0$

$\therefore ma + 2kx = 0$ [because v = dx/dt and a = dv/dt]

$\Rightarrow \frac{a}{x} = -\frac{2k}{m} = \omega^2 \Rightarrow \omega = \sqrt{\frac{2k}{m}}$

$\Rightarrow$ Time period $T = 2\pi\sqrt{\frac{m}{2k}}$

26. Here we have to consider oscillation of centre of mass

Driving force F = mg sin θ

Acceleration = $a = \frac{F}{m} = g\sin\theta.$

For small angle θ, $\sin\theta = \theta$.

$\therefore a = g\,\theta = g\left(\frac{x}{L}\right)$ [where g and L are constant]

$\therefore a \propto x,$

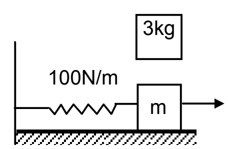
So the motion is simple Harmonic

Time period $T = 2\pi\sqrt{\frac{\text{Displacement}}{\text{Acceleration}}} = 2\pi\sqrt{\frac{x}{\left(\frac{gx}{L}\right)}} = 2\pi\sqrt{\frac{L}{g}}$

27. Amplitude = 0.1m

Total mass = 3 + 1 = 4kg (when both the blocks are moving together)

$\therefore T = 2\pi\sqrt{\frac{M}{k}} = 2\pi\sqrt{\frac{4}{100}} = \frac{2\pi}{5}$ sec.



$\therefore$ Frequency = $\frac{5}{2\pi}$ Hz.

Again at the mean position, let 1kg block has velocity v.

KE. = $(1/2)\,mv^2 = (1/2)\,mx^2$ where $x \rightarrow$ Amplitude = 0.1m.

$\therefore (1/2) \times (1 \times v^2) = (1/2) \times 100\,(0.1)^2$

$\Rightarrow v = 1$m/sec …(1)

After the 3kg block is gently placed on the 1kg, then let, 1kg +3kg = 4kg block and the spring be one system. For this mass spring system, there is so external force. (when oscillation takes place). The momentum should be conserved. Let, 4kg block has velocity v′.

$\therefore$ Initial momentum = Final momentum

$\therefore 1 \times v = 4 \times v' \Rightarrow v' = 1/4$ m/s (As v = 1m/s from equation (1))

Now the two blocks have velocity 1/4 m/s at its mean poison.

$KE_{mass} = (1/2)\,m'v'^2 = (1/2)\,4 \times (1/4)^2 = (1/2) \times (1/4).$

When the blocks are going to the extreme position, there will be only potential energy.

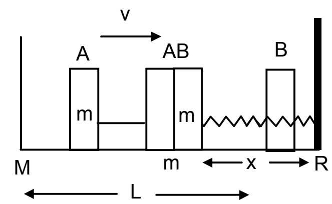
$\therefore PE = (1/2)\,k\delta^2 = (1/2) \times (1/4)$ where $\delta \rightarrow$ new amplitude.

$\therefore 1/4 = 100\,\delta^2 \Rightarrow \delta = \sqrt{\frac{1}{400}} = 0.05\text{m} = 5\text{cm}.$

So Amplitude = 5cm.

28. When the block A moves with velocity 'V' and collides with the block B, it transfers all energy to the block B. (Because it is a elastic collision). The block A will move a distance 'x' against the spring, again the block B will return to the original point and completes half of the oscillation.

So, the time period of B is $\dfrac{2\pi\sqrt{\frac{m}{k}}}{2} = \pi\sqrt{\dfrac{m}{k}}$



The block B collides with the block A and comes to rest at that point. The block A again moves a further distance 'L' to return to its original position.

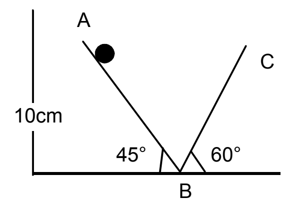
$\therefore$ Time taken by the block to move from M → N and N → M

is $\dfrac{L}{V} + \dfrac{L}{V} = 2\left(\dfrac{L}{V}\right)$

$\therefore$ So time period of the periodic motion is $2\left(\dfrac{L}{V}\right) + \pi\sqrt{\dfrac{m}{k}}$

29. Let the time taken to travel AB and BC be t_1 and t_2 respectively

Fro part AB, $a_1 = g \sin 45°$. $s_1 = \dfrac{0.1}{\sin 45°} = 2m$



Let, v = velocity at B

$\therefore v^2 - u^2 = 2a_1 s_1$

$\Rightarrow v^2 = 2 \times g \sin 45° \times \dfrac{0.1}{\sin 45°} = 2$

$\Rightarrow v = \sqrt{2}$ m/s

$\therefore t_1 = \dfrac{v-u}{a_1} = \dfrac{\sqrt{2}-0}{\frac{g}{\sqrt{2}}} = \dfrac{2}{g} = \dfrac{2}{10} = 0.2$ sec

Again for part BC, $a_2 = -g \sin 60°$, $u = \sqrt{2}$, $v = 0$

$\therefore t_2 = \dfrac{0-\sqrt{2}}{-g\left(\frac{\sqrt{3}}{2}\right)} = \dfrac{2\sqrt{2}}{\sqrt{3}g} = \dfrac{2\times(1.414)}{(1.732)\times 10} = 0.165$sec.

So, time period = $2(t_1 + t_2) = 2(0.2 + 0.155) = 0.71$sec

30. Let the amplitude of oscillation of 'm' and 'M' be x_1 and x_2 respectively.

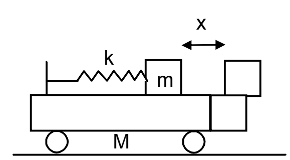
a) From law of conservation of momentum,

$mx_1 = Mx_2$ …(1) [because only internal forces are present]

Again, $(1/2)\, kx_0^2 = (1/2)\, k\, (x_1 + x_2)^2$

$\therefore x_0 = x_1 + x_2$ …(2)

[Block and mass oscillates in opposite direction. But x → stretched part]



From equation (1) and (2)

$\therefore x_0 = x_1 + \dfrac{m}{M}x_1 = \left(\dfrac{M+m}{M}\right)x_1$

$\therefore x_1 \dfrac{Mx_0}{M+m}$

So, $x_2 = x_0 - x_1 = x_0\left[1 - \dfrac{M}{M+m}\right] = \dfrac{mx_0}{M+m}$ respectively.

b) At any position, let the velocities be v_1 and v_2 respectively.

Here, v_1 = velocity of 'm' with respect to M.

By energy method

Total Energy = Constant

$(1/2)\, Mv^2 + (1/2)\, m(v_1 - v_2)^2 + (1/2)\, k(x_1 + x_2)^2 = \text{Constant}$ …(i)

[$v_1 - v_2$ = Absolute velocity of mass 'm' as seen from the road.]

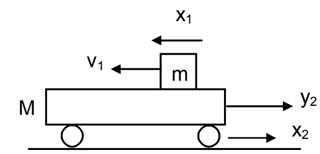
Again, from law of conservation of momentum,

$mx_2 = mx_1 \Rightarrow x_1 = \frac{M}{m} x_2$ ...(1)

$mv_2 = m(v_1 - v_2) \Rightarrow (v_1 - v_2) = \frac{M}{m} v_2$ …(2)

Putting the above values in equation (1), we get

$\frac{1}{2} Mv_2^2 + \frac{1}{2} m \frac{M^2}{m^2} v_2^2 + \frac{1}{2} kx_2^2 \left(1+\frac{M}{m}\right)^2$ = constant



$\therefore M\left(1+\frac{M}{m}\right) v_2 + k\left(1+\frac{M}{m}\right)^2 x_2^2$ = Constant.

$\Rightarrow mv_2^2 + k\left(1+\frac{M}{m}\right) x_2^2$ = constant

Taking derivative of both sides,

$M \times 2v_2 \frac{dv_2}{dt} + k\frac{(M+m)}{m} - ex_2^2 \frac{dx_2}{dt} = 0$

$\Rightarrow ma_2 + k\left(\frac{M+m}{m}\right) x_2 = 0$ [because, $v_2 = \frac{dx_2}{dt}$]

$\Rightarrow \frac{a_2}{x_2} = -\frac{k(M+m)}{Mm} = \omega^2$

$\therefore \omega = \sqrt{\frac{k(M+m)}{Mm}}$

So, Time period, $T = 2\pi \sqrt{\frac{Mm}{k(M+m)}}$

31. Let 'x' be the displacement of the plank towards left. Now the centre of gravity is also displaced through 'x'
In displaced position
$R_1 + R_2 = mg$.
Taking moment about G, we get
$R_1(\ell/2 - x) = R_2(\ell/2 + x) = (mg - R_1)(\ell/2 + x)$ …(1)\
So, $R_1 (\ell/2 - x) = (mg - R_1)(\ell/2 + x)$

$\Rightarrow R_1 \frac{\ell}{2} - R_1 x = mg \frac{\ell}{2} - R_1 x + mgx - R_1 \frac{\ell}{2}$

$\Rightarrow R_1 \frac{\ell}{2} + R_1 \frac{\ell}{2} = mg (x + \frac{\ell}{2})$

$\Rightarrow R_1 \left(\frac{\ell}{2}+\frac{\ell}{2}\right) = mg \left(\frac{2x+\ell}{2}\right)$

$\Rightarrow R_1 \ell = \frac{mg(2x+\ell)}{2}$

$\Rightarrow R_1 = \frac{mg(2x+\ell)}{2\ell}$ …(2)

Now $F_1 = \mu R_1 = \frac{\mu mg(\ell+2x)}{2\ell}$

Similarly $F_2 = \mu R_2 = \frac{\mu mg(\ell-2x)}{2\ell}$

Since, $F_1 > F_2$. $\Rightarrow F_1 - F_2 = ma = \frac{2\mu mg}{\ell} x$

$\Rightarrow \frac{a}{x} = \frac{2\mu g}{\ell} = \omega^2 \Rightarrow \omega = \sqrt{\frac{2\mu g}{\ell}}$

$\therefore$ Time period $= 2\pi \sqrt{\frac{\ell}{2rg}}$

32. T = 2sec.

$T = 2\pi\sqrt{\frac{\ell}{g}}$

$\Rightarrow 2 = 2\pi\sqrt{\frac{\ell}{10}} \Rightarrow \frac{\ell}{10} = \frac{1}{\pi^2} \Rightarrow \ell = 1cm \qquad (\because \pi^2 \approx 10)$

33. From the equation,

$\theta = \pi \sin [\pi \sec^{-1} t]$

$\therefore \omega = \pi \sec^{-1}$ (comparing with the equation of SHM)

$\Rightarrow \frac{2\pi}{T} = \pi \Rightarrow T = 2$ sec.

We know that $T = 2\pi\sqrt{\frac{\ell}{g}} \quad \Rightarrow 2 = 2\sqrt{\frac{\ell}{g}} \quad \Rightarrow 1 = \sqrt{\frac{\ell}{g}} \quad \Rightarrow \ell = 1m.$

$\therefore$ Length of the pendulum is 1m.

34. The pendulum of the clock has time period 2.04sec.

Now, No. or oscillation in 1 day = $\frac{24 \times 3600}{2}$ = 43200

But, in each oscillation it is slower by (2.04 – 2.00) = 0.04sec.

So, in one day it is slower by,

= 43200 × (0.04) = 12 sec = 28.8 min

So, the clock runs 28.8 minutes slower in one day.

35. For the pendulum, $\frac{T_1}{T_2} = \sqrt{\frac{g_2}{g_1}}$

Given that, T_1 = 2sec, g_1 = 9.8m/s^2

$T_2 = \frac{24 \times 3600}{\left(\frac{24 \times 3600 - 24}{2}\right)} = 2 \times \frac{3600}{3599}$

Now, $\frac{g^2}{g_1} = \left(\frac{T_1}{T_2}\right)^2$

$\therefore g_2 = (9.8)\left(\frac{3599}{3600}\right)^2 = 9.795m/s^2$

36. L = 5m.

a) $T = 2\pi\sqrt{\frac{\ell}{g}} = 2\pi\sqrt{0.5} = 2\pi(0.7)$

$\therefore$ In $2\pi(0.7)$sec, the body completes 1 oscillation,

In 1 second, the body will complete $\frac{1}{2\pi(0.7)}$ oscillation

$\therefore f = \frac{1}{2\pi(0.7)} = \frac{10}{14\pi} = \frac{0.70}{\pi}$ times

b) When it is taken to the moon

$T = 2\pi\sqrt{\frac{\ell}{g'}}$ where $g' \rightarrow$ Acceleration in the moon.

$= 2\pi\sqrt{\frac{5}{1.67}}$

$\therefore f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{1.67}{5}} = \frac{1}{2\pi}(0.577) = \frac{1}{2\pi\sqrt{3}}$ times.

37. The tension in the pendulum is maximum at the mean position and minimum on the extreme position.

Here (1/2) $mv^2 - 0 = mg\,\ell(1 - \cos\theta)$

$v^2 = 2g\ell(1 - \cos\theta)$

Now, $T_{max} = mg + 2\,mg\,(1 - \cos\theta)$ $\quad$ $[\,T = mg + (mv^2/\ell)]$

Again, $T_{min} = mg\cos\theta$.

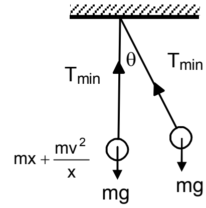
According to question, $T_{max} = 2T_{min}$

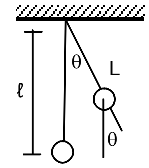
$\Rightarrow mg + 2mg - 2mg\cos\theta = 2mg\cos\theta$

$\Rightarrow 3mg = 4mg\cos\theta$

$\Rightarrow \cos\theta = 3/4$

$\Rightarrow \theta = \cos^{-1}(3/4)$





38. Given that, R = radius.

Let N = normal reaction.

Driving force $F = mg\sin\theta$.

Acceleration $= a = g\sin\theta$

As, $\sin\theta$ is very small, $\sin\theta \to \theta$

$\therefore$ Acceleration $a = g\theta$

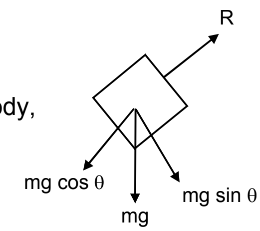
Let 'x' be the displacement from the mean position of the body,

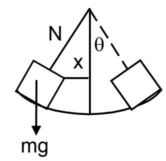
$\therefore\ \theta = x/R$

$\Rightarrow a = g\theta = g(x/R) \Rightarrow (a/x) = (g/R)$

So the body makes S.H.M.

$$\therefore T = 2\pi\sqrt{\frac{\text{Displacement}}{\text{Acceleration}}} = 2\pi\sqrt{\frac{x}{gx/R}} = 2\pi\sqrt{\frac{R}{g}}$$





39. Let the angular velocity of the system about the point os suspension at any time be 'ω'

So, $v_c = (R - r)\omega$

Again $v_c = r\omega_1$ [where, ω_1 = rotational velocity of the sphere]

$$\omega_1 = \frac{v_c}{r} = \left(\frac{R + -r}{r}\right)\omega \qquad \ldots(1)$$

By Energy method, Total energy in SHM is constant.

So, $mg(R - r)(1 - \cos\theta) + (1/2)\,mv_c^2 + (1/2)\,I\omega_1^2 = \text{constant}$

$$\therefore\ mg(R - r)(1 - \cos\theta) + (1/2)\,m(R - r)^2\omega^2 + (1/2)\,mr^2\left(\frac{R - r}{r}\right)^2\omega^2 = \text{constant}$$

$$\Rightarrow g(R - r)\ 1 - \cos\theta) + (R - r)^2\omega^2\left[\frac{1}{2} + \frac{1}{5}\right] = \text{constant}$$

$$\text{Taking derivative, } g(R - r)\sin\theta\frac{d\theta}{dt} = \frac{7}{10}(R - r)^2 2\omega\frac{d\omega}{dt}$$

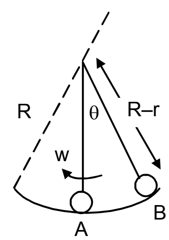
$$\Rightarrow g\sin\theta = 2 \times \frac{7}{10}(R - r)\alpha$$

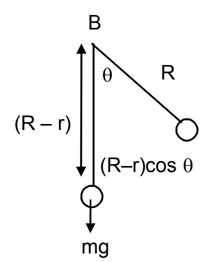
$$\Rightarrow g\sin\theta = \frac{7}{5}(R - r)\alpha$$

$$\Rightarrow \alpha = \frac{5g\sin\theta}{7(R - r)} = \frac{5g\theta}{7(R - r)}$$

$$\therefore \frac{\alpha}{\theta} = \omega^2 = \frac{5g\theta}{7(R - r)} = \text{constant}$$

$$\text{So the motion is S.H.M. Again } \omega = \omega\sqrt{\frac{5g}{7(R - r)}} \Rightarrow T = 2\pi\sqrt{\frac{7(R - r)}{5g}}$$





40. Length of the pendulum = 40cm = 0.4m.

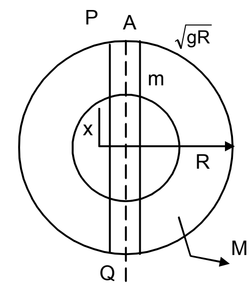
Let acceleration due to gravity be g at the depth of 1600km.

$$\therefore gd = g(1\text{-}d/R) = 9.8\left(1 - \frac{1600}{6400}\right) = 9.8\left(1 - \frac{1}{4}\right) = 9.8 \times \frac{3}{4} = 7.35\text{m/s}^2$$

$\therefore$ Time period $T' = 2\pi\sqrt{\dfrac{\ell}{g\delta}}$

$= 2\pi\sqrt{\dfrac{0.4}{7.35}} = 2\pi\sqrt{0.054} = 2\pi \times 0.23 = 2 \times 3.14 \times 0.23 = 1.465 \approx 1.47$sec.

41. Let M be the total mass of the earth.
At any position x,



$\therefore \dfrac{M'}{M} = \dfrac{\rho \times \left(\frac{4}{3}\right)\pi \times x^3}{\rho \times \left(\frac{4}{3}\right)\pi \times R^3} = \dfrac{x^3}{R^3} \Rightarrow M' = \dfrac{Mx^3}{R^3}$

So force on the particle is given by,

$\therefore F_X = \dfrac{GM'm}{x^2} = \dfrac{GMm}{R^3}x$ ...(1)

So, acceleration of the mass 'M' at that position is given by,

$a_x = \dfrac{GM}{R^2}x \Rightarrow \dfrac{a_x}{x} = w^2 = \dfrac{GM}{R^3} = \dfrac{g}{R}$ $\left(\because g = \dfrac{GM}{R^2}\right)$

So, $T = 2\pi\sqrt{\dfrac{R}{g}}$ = Time period of oscillation.

a) Now, using velocity – displacement equation.

$V = \omega\sqrt{(A^2 - R^2)}$ [Where, A = amplitude]

Given when, $y = R$, $v = \sqrt{gR}$, $\omega = \sqrt{\dfrac{g}{R}}$

$\Rightarrow \sqrt{gR} = \sqrt{\dfrac{g}{R}}\sqrt{(A^2 - R^2)}$ [because $\omega = \sqrt{\dfrac{g}{R}}$]

$\Rightarrow R^2 = A^2 - R^2 \Rightarrow A = \sqrt{2}R$

[Now, the phase of the particle at the point P is greater than $\pi/2$ but less than π and at Q is greater than π but less than $3\pi/2$. Let the times taken by the particle to reach the positions P and Q be t_1 & t_2 respectively, then using displacement time equation]

$y = r \sin \omega t$

We have, $R = \sqrt{2}\,R \sin \omega t_1$ $\Rightarrow \omega t_1 = 3\pi/4$

& $-R = \sqrt{2}\,R \sin \omega t_2$ $\Rightarrow \omega t_2 = 5\pi/4$

So, $\omega(t_2 - t_1) = \pi/2 \Rightarrow t_2 - t_1 = \dfrac{\pi}{2\omega} = \dfrac{\pi}{2\sqrt{(R/g)}}$

Time taken by the particle to travel from P to Q is $t_2 - t_1 = \dfrac{\pi}{2\sqrt{(R/g)}}$ sec.

b) When the body is dropped from a height R, then applying conservation of energy, change in P.E. = gain in K.E.

$\Rightarrow \dfrac{GMm}{R} - \dfrac{GMm}{2R} = \dfrac{1}{2}mv^2$ $\Rightarrow v = \sqrt{gR}$

Since, the velocity is same at P, as in part (a) the body will take same time to travel PQ.

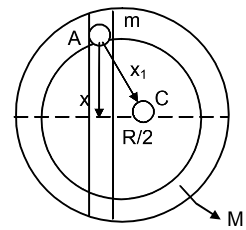
c) When the body is projected vertically upward from P with a velocity $\sqrt{gR}$, its velocity will be Zero at the highest point.

The velocity of the body, when reaches P, again will be $v = \sqrt{gR}$, hence, the body will take same time $\dfrac{\pi}{2\sqrt{(R/g)}}$ to travel PQ.

42. $M = 4/3\ \pi R^3 \rho$.

$M^1 = 4/3\ \pi x_1^3 \rho$

$M^1 = \left(\frac{M}{R^3}\right) x_1^3$

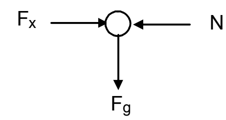


a) F = Gravitational force exerted by the earth on the particle of mass 'x' is,

$$F = \frac{GM^1 m}{x_1^2} = \frac{GMm}{R^3}\frac{x_1^3}{x_1^2} = \frac{GMm}{R^3} x_1 = \frac{GMm}{R^3}\sqrt{x^2 + \left(\frac{R^2}{4}\right)}$$

b) $F_y = F \cos\theta = \frac{GMmx_1}{R^3}\frac{x}{x_1} = \frac{GMmx}{R^3}$

$F_x = F \sin\theta = \frac{GMmx_1}{R^3}\frac{R}{2x_1} = \frac{GMm}{2R^2}$



c) $F_x = \frac{GMm}{2R^2}$ [since Normal force exerted by the wall $N = F_x$]

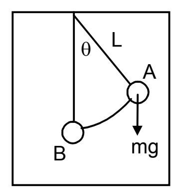
d) Resultant force = $\frac{GMmx}{R^3}$

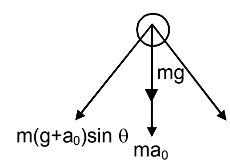
e) Acceleration = $\frac{\text{Driving force}}{\text{mass}} = \frac{GMmx}{R^3 m} = \frac{GMx}{R^3}$

So, a α x (The body makes SHM)

$\therefore \frac{a}{x} = w^2 = \frac{GM}{R^3} \Rightarrow w = \sqrt{\frac{GM}{R^3}} \Rightarrow T = 2\pi\sqrt{\frac{R^3}{GM}}$

43. Here driving force $F = m(g + a_0)\sin\theta$ …(1)





Acceleration $a = \frac{F}{m} = (g + a_0)\sin\theta = \frac{(g + a_0)x}{\ell}$

(Because when θ is small $\sin\theta \to \theta = x/\ell$)

$\therefore a = \frac{(g + a_0)x}{\ell}$.

$\therefore$ acceleration is proportional to displacement.

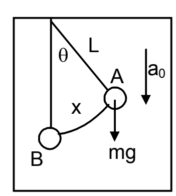
So, the motion is SHM.

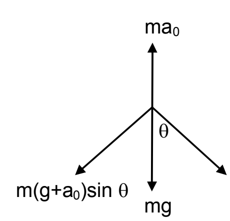
Now $\omega^2 = \frac{(g + a_0)}{\ell}$

$\therefore T = 2\pi\sqrt{\frac{\ell}{g + a_0}}$

b) When the elevator is going downwards with acceleration a_0

Driving force = $F = m(g - a_0)\sin\theta$.





Acceleration = $(g - a_0)\sin\theta = \frac{(g - a_0)x}{\ell} = \omega^2 x$

$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{\ell}{g - a_0}}$

c) When moving with uniform velocity $a_0 = 0$.

For, the simple pendulum, driving force = $\frac{mgx}{\ell}$

$\Rightarrow a = \frac{gx}{\ell} \Rightarrow \frac{x}{a} = \frac{\ell}{g}$

$T = 2\pi\sqrt{\frac{\text{displacement}}{\text{acceleration}}} = 2\pi\sqrt{\frac{\ell}{g}}$

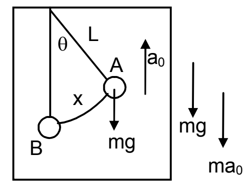
44. Let the elevator be moving upward accelerating 'a_0'

Here driving force $F = m(g + a_0) \sin\theta$

Acceleration $= (g + a_0) \sin\theta$

$= (g + a_0)\theta \qquad (\sin\theta \to \theta)$

$= \frac{(g+a_0)x}{\ell} = \omega^2 x$



$T = 2\pi\sqrt{\frac{\ell}{g+a_0}}$

Given that, $T = \pi/3$ sec, $\ell = 1$ft and $g = 32$ ft/sec^2

$\frac{\pi}{3} = 2\pi\sqrt{\frac{1}{32+a_0}}$

$\frac{1}{9} = 4\left(\frac{1}{32+a}\right)$

$\Rightarrow 32 + a = 36 \qquad \Rightarrow a = 36 - 32 = 4$ ft/sec^2

45. When the car moving with uniform velocity

$T = 2\pi\sqrt{\frac{\ell}{g}} \Rightarrow 4 = 2\pi\sqrt{\frac{\ell}{g}} \qquad \ldots(1)$

When the car makes accelerated motion, let the acceleration be a_0

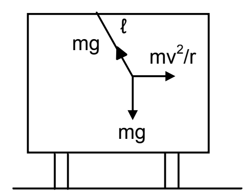
$T = 2\pi\sqrt{\frac{\ell}{\sqrt{g^2+a_0^2}}}$

$\Rightarrow 3.99 = 2\pi\sqrt{\frac{\ell}{\sqrt{g^2+a_0^2}}}$

Now $\frac{T}{T'} = \frac{4}{3.99} = \frac{(g^2+a_0^2)^{1/4}}{\sqrt{g}}$

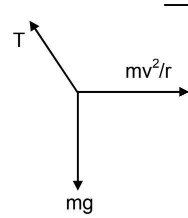
Solving for 'a_0' we can get $a_0 = g/10$ ms^{-2}

46. From the freebody diagram,



$T = \sqrt{(mg)^2 + \left(\frac{mv^2}{r^2}\right)}$

$= m\sqrt{g^2 + \frac{v^4}{r^2}} = ma$, where a = acceleration $= \left(g^2 + \frac{v^4}{r^2}\right)^{1/2}$

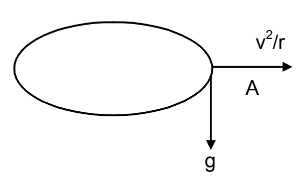


The time period of small accellations is given by,

$T = 2\pi\sqrt{\frac{\ell}{g}} = 2\pi\sqrt{\frac{\ell}{\left(g^2 + \frac{v^4}{r^2}\right)^{1/2}}}$

47. a) $\ell = 3$cm $= 0.03$m.

$T = 2\pi\sqrt{\frac{\ell}{g}} = 2\pi\sqrt{\frac{0.03}{9.8}} = 0.34$ second.



b) When the lady sets on the Merry-go-round the ear rings also experience centrepetal acceleration

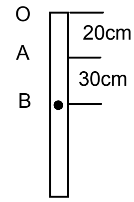
$a = \frac{v^2}{r} = \frac{4^2}{2} = 8$ m/s^2

Resultant Acceleration $A = \sqrt{g^2 + a^2} = \sqrt{100+64} = 12.8$ m/s^2

Time period $T = 2\pi\sqrt{\frac{\ell}{A}} = 2\pi\sqrt{\frac{0.03}{12.8}} = 0.30$ second.

48. a) M.I. about the pt A = I = $I_{C.G.}$ + Mh^2

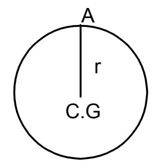
$$= \frac{m\ell^2}{12} + MH^2 = \frac{m\ell^2}{12} + m(0.3)^2 = M\left(\frac{1}{12} + 0.09\right) = M\left(\frac{1+1.08}{12}\right) = M\left(\frac{2.08}{12}\right)$$



$\therefore$ T = $2\pi\sqrt{\frac{I}{mg\ell'}} = 2\pi\sqrt{\frac{2.08m}{m \times 9.8 \times 0.3}}$ (ℓ' = dis. between C.G. and pt. of suspension)

$\approx$ 1.52 sec.

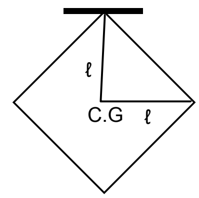
b) Moment of in isertia about A



I = $I_{C.G.}$+ mr^2 = mr^2 + mr^2 = 2 mr^2

$\therefore$ Time period = $2\pi\sqrt{\frac{I}{mg\ell}} = 2\pi\sqrt{\frac{2mr^2}{mgr}} = 2\pi\sqrt{\frac{2r}{g}}$

c) I_{ZZ} (corner) = $m\left(\frac{a^2+a^2}{3}\right) = \frac{2ma^2}{3}$



In the ΔABC, $\ell^2 + \ell^2 = a^2$

$\therefore \ell = \frac{a}{\sqrt{2}}$

$\therefore$ T = $2\pi\sqrt{\frac{I}{mg\ell}} = 2\pi\sqrt{\frac{2ma^2}{3mg\ell}} = 2\pi\sqrt{\frac{2a^2}{3ga\sqrt{2}}} = 2\pi\sqrt{\frac{\sqrt{8}a}{3g}}$

d) h = r/2, ℓ = r/2 = Dist. Between C.G and suspension point.

M.I. about A, I = $I_{C.G.}$+ Mh^2 = $\frac{mc^2}{2} + n\left(\frac{r}{2}\right)^2 = mr^2\left(\frac{1}{2}+\frac{1}{4}\right) = \frac{3}{4}mr^2$

$\therefore$ T = $2\pi\sqrt{\frac{I}{mg\ell}} = 2\pi\sqrt{\frac{3mr^2}{4mg\ell}} = 2\pi\sqrt{\frac{3r^2}{4g\left(\frac{r}{2}\right)}} = 2\pi\sqrt{\frac{3r}{2g}}$

49. Let A $\rightarrow$ suspension of point.

B $\rightarrow$ Centre of Gravity.

$\ell' = \ell/2$, h = $\ell/2$

Moment of inertia about A is

I = $I_{C.G.}$ + mh^2 = $\frac{m\ell^2}{12} + \frac{m\ell^2}{4} = \frac{m\ell^2}{3}$

$\Rightarrow$ T = $2\pi\sqrt{\frac{I}{mg\left(\frac{\ell}{2}\right)}} = 2\pi\sqrt{\frac{2m\ell^2}{3mgl}} = 2\pi\sqrt{\frac{2\ell}{3g}}$

Let, the time period 'T' is equal to the time period of simple pendulum of length 'x'.

$\therefore$ T = $2\pi\sqrt{\frac{x}{g}}$. So, $\frac{2\ell}{3g} = \frac{x}{g} \Rightarrow x = \frac{2\ell}{3}$

$\therefore$ Length of the simple pendulum = $\frac{2\ell}{3}$

50. Suppose that the point is 'x' distance from C.G.

Let m = mass of the disc., Radius = r

Here ℓ = x

M.I. about A = $I_{C.G.}$ + mx^2 = $mr^2/2+mx^2$ = $m(r^2/2 + x^2)$

$$T = 2\pi\sqrt{\frac{I}{mg\ell}} = 2\pi\sqrt{\frac{m\left(\frac{r^2}{2}+x^2\right)}{mgx}} = 2\pi\sqrt{\frac{m(r^2+2x^2)}{2mgx}} = 2\pi\sqrt{\frac{r^2+2x^2}{2gx}} \quad \ldots(1)$$

For T is minimum $\frac{dt^2}{dx} = 0$

$\therefore \frac{d}{dx} T^2 = \frac{d}{dx}\left(\frac{4\pi^2 r^2}{2gx} + \frac{4\pi^2 2x^2}{2gx}\right)$

$\Rightarrow \frac{2\pi^2 r^2}{g}\left(-\frac{1}{x^2}\right) + \frac{4\pi^2}{g} = 0$

$\Rightarrow -\frac{\pi^2 r^2}{gx^2} + \frac{2\pi^2}{g} = 0$

$\Rightarrow \frac{\pi^2 r^2}{gx^2} = \frac{2\pi^2}{g} \Rightarrow 2x^2 = r^2 \Rightarrow x = \frac{r}{\sqrt{2}}$

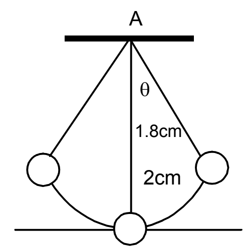
So putting the value of equation (1)

$T = 2\pi\sqrt{\frac{r^2 + 2\left(\frac{r^2}{2}\right)}{2gx}} = 2\pi\sqrt{\frac{2r^2}{2gx}} = 2\pi\sqrt{\frac{r^2}{g\left(\frac{r}{\sqrt{2}}\right)}} = 2\pi\sqrt{\frac{\sqrt{2}\,r^2}{gr}} = 2\pi\sqrt{\frac{\sqrt{2}r}{g}}$

51. According to Energy equation,
$mg\ell(1 - \cos\theta) + (1/2) I\omega^2 = \text{const.}$
$mg(0.2)(1 - \cos\theta) + (1/2) I\omega^2 = C.$ (I)
Again, $I = 2/3\, m(0.2)^2 + m(0.2)^2$

$= m\left[\frac{0.008}{3} + 0.04\right]$

$= m\left(\frac{0.1208}{3}\right)$m. Where I → Moment of Inertia about the pt of suspension A



From equation
Differenting and putting the value of I and 1 is

$\frac{d}{dt}\left[mg(0.2)(1 - \cos\theta) + \frac{1}{2}\frac{0.1208}{3} m\omega^2\right] = \frac{d}{dt}(C)$

$\Rightarrow mg\,(0.2)\sin\theta\,\frac{d\theta}{dt} + \frac{1}{2}\left(\frac{0.1208}{3}\right) m2\omega\frac{d\omega}{dt} = 0$

$\Rightarrow 2\sin\theta = \frac{0.1208}{3}\alpha$ [because, $g = 10m/s^2$]

$\Rightarrow \frac{\alpha}{\theta} = \frac{6}{0.1208} = \omega^2 = 58.36$

$\Rightarrow \omega = 7.3$. So $T = \frac{2\pi}{\omega} = 0.89$sec.

For simple pendulum $T = 2\pi\sqrt{\frac{0.19}{10}} = 0.86$sec.

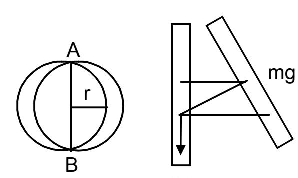
% more $= \frac{0.89 - 0.86}{0.89} = 0.3$.

∴ It is about 0.3% larger than the calculated value.

52. (For a compound pendulum)

a) $T = 2\pi\sqrt{\frac{I}{mg\ell}} = 2\pi\sqrt{\frac{I}{mgr}}$

The MI of the circular wire about the point of suspension is given by
∴ $I = mr^2 + mr^2 = 2\,mr^2$ is Moment of inertia about A.



$\therefore 2 = 2\pi \sqrt{\dfrac{2mr^2}{mgr}} = 2\pi \sqrt{\dfrac{2r}{g}}$

$\Rightarrow \dfrac{2r}{g} = \dfrac{1}{\pi^2} \Rightarrow r = \dfrac{g}{2\pi^2} = 0.5\pi = 50cm.$ (Ans)

b) $(1/2)\ \omega^2 - 0 = mgr\ (1 - \cos\theta)$

$\Rightarrow (1/2)\ 2mr^2 - \omega^2 = mgr\ (1 - \cos 2°)$

$\Rightarrow \omega^2 = g/r\ (1 - \cos 2°)$

$\Rightarrow \omega = 0.11$ rad/sec [putting the values of g and r]

$\Rightarrow v = \omega \times 2r = 11$ cm/sec.

c) Acceleration at the end position will be centripetal.

$= a_n = \omega^2\ (2r) = (0.11)^2 \times 100 = 1.2\ cm/s^2$

The direction of 'a_n' is towards the point of suspension.

d) At the extreme position the centrepetal acceleration will be zero. But, the particle will still have acceleration due to the SHM.

Because, T = 2 sec.

Angular frequency $\omega = \dfrac{2\pi}{T}$ (π= 3.14)

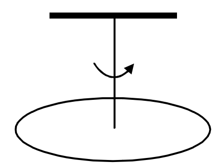
So, angular acceleration at the extreme position,

$\alpha = \omega^2\theta = \pi^2 \times \dfrac{2\pi}{180} = \dfrac{2\pi^3}{180}$ [$1° = \dfrac{\pi}{180}$ radious]

So, tangential acceleration $= \alpha\ (2r) = \dfrac{2\pi^3}{180} \times 100 = 34\ cm/s^2$.

53. M.I. of the centre of the disc. $= mr^2/2$

$T = 2\pi \sqrt{\dfrac{I}{k}} = 2\pi \sqrt{\dfrac{mr^2}{2K}}$ [where K = Torsional constant]



$T^2 = 4\pi^2 \dfrac{mr^2}{2K} = 2\pi^2 \dfrac{mr^2}{K}$

$\Rightarrow 2\pi^2\ mr^2 = KT^2 \quad \Rightarrow K = \dfrac{2mr^2\pi^2}{T^2}$

$\therefore$ Torsional constant $K = \dfrac{2mr^2\pi^2}{T^2}$

54. The M.I of the two ball system

$I = 2m\ (L/2)^2 = m\ L^2/2$

At any position θ during the oscillation, [fig-2]

Torque = $k\theta$

So, work done during the displacement 0 to θ_0,

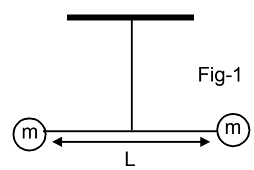
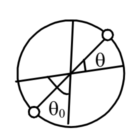


Fig-1

$W = \int_0^{\theta} k\theta d\theta = k\ \theta_0^2/2$

By work energy method,



$(1/2)\ I\omega^2 - 0 = \text{Work done} = k\ \theta_0^2/2$

$\therefore \omega^2 = \dfrac{k\theta_0^2}{2I} = \dfrac{k\theta_0^2}{mL^2}$

Now, from the freebody diagram of the rod,

$T_2 = \sqrt{(m\omega^2 L)^2 + (mg)^2}$

$= \sqrt{\left(m\dfrac{k\theta_0^2}{mL^2} \times L\right)^2 + m^2g^2} \quad = \dfrac{k^2\theta_0^4}{L^2} + m^2g^2$

55. The particle is subjected to two SHMs of same time period in the same direction/
Given, r_1 = 3cm, r_2 = 4cm and ϕ = phase difference.

Resultant amplitude = R = $\sqrt{r_1^2 + r_2^2 + 2r_1r_2\cos\phi}$

a) When ϕ = 0°,

R = $\sqrt{(3^2 + 4^2 + 2\times3\times4\cos0^\circ}$ = 7 cm

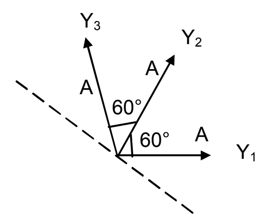
b) When ϕ = 60°

R = $\sqrt{(3^2 + 4^2 + 2\times3\times4\cos60^\circ}$ = 6.1 cm

c) When $\phi = 90°$

R = $\sqrt{(3^2 + 4^2 + 2\times3\times4\cos90^\circ}$ = 5 cm

56. Three SHMs of equal amplitudes 'A' and equal time periods in the same dirction combine.
The vectors representing the three SHMs are shown it the figure.
Using vector method,
Resultant amplitude = Vector sum of the three vectors
= A + A cos 60° + A cso 60° = A + A/2 + A/2 = 2A
So the amplitude of the resultant motion is 2A.



57. x_1 = 2 sin 100 πt
x_2 = w sin (120πt + π/3)
So, resultant displacement is given by,
x = x_1 + x_2 = 2 [sin (100πt) + sin (120πt + π/3)]
a) At t = 0.0125s,
x = 2 [sin (100π × 0.0125) + sin (120π ×0.0125 + π/3)]
= 2 [sin 5π/4 + sin (3π/2 + π/3)]
= 2 [(–0.707) + (–0.5)] = – 2.41cm.
b) At t = 0.025s.
x = 2 [sin (100π × 0.025) + sin (120π ×0.025 + π/3)]
= 2 [sin 5π/2 + sin (3π + π/3)]
=2[1+(–0.8666)] = 0.27 cm.

58. The particle is subjected to two simple harmonic motions represented by,
x = x_0 sin wt
s = s_0 sin wt
and, angle between two motions = θ = 45°
$\therefore$Resultant motion will be given by,

R = $\sqrt{(x^2 + s^2 + 2xs\cos45^\circ)}$

= $\sqrt{\{x_0^2\sin^2 wt + s_0^2\sin^2 wt + 2x_0s_0\sin^2 wtx(1/\sqrt{2})\}}$

= $[x_0^2 + s_0^2 = \sqrt{2}\,x_0s_0]^{1/2}$ sin wt

$\therefore$ Resultant amplitude = $[x_0^2 + s_0^2 = \sqrt{2}\,x_0s_0]^{1/2}$

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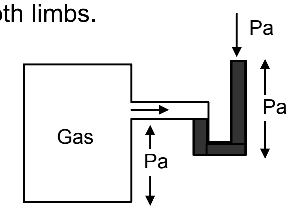
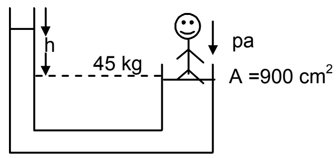
SOLUTIONS TO CONCEPTS
CHAPTER 13

1. $p = h \rho g$
It is necessary to specify that the tap is closed. Otherwise pressure will gradually decrease, as h decrease, because, of the tap is open, the pressure at the tap is atmospheric.
2. a) Pressure at the bottom of the tube should be same when considered for both limbs.
From the figure are shown,
$p_g + \rho_{Hg} \times h_2 \times g = p_a + \rho_{Hg} \times h_1 \times g$
$\Rightarrow \quad p_g = p_a + \rho_{Hg} \times g(h_1 - h_2)$
b) Pressure of mercury at the bottom of u tube
$p \quad = p_a + \rho_{Hg}\, h_1 \times g$



3. From the figure shown
$p_a + h\rho g = p_a + mg/A$
$\Rightarrow h\rho g = mg/A$
$\Rightarrow h = \dfrac{m}{A\rho}$



4. a) Force exerted at the bottom.
= Force due to cylindrical water colum + atm. Force
$= A \times h \times \rho_w \times g + p_a \times A$
$= A(h\, \rho_w\, g + p_a)$
b) To find out the resultant force exerted by the sides of the glass, from the freebody, diagram of water inside the glass
$p_a \times A + mg = A \times h \times \rho_w \times g + F_s + p_a \times A$
$\Rightarrow mg = A \times h \times \rho_w \times g + F_s$
This force is provided by the sides of the glass.
5. If the glass will be covered by a jar and the air is pumped out, the atmospheric pressure has no effect.
So,
a) Force exerted on the bottom.
$= (h\, \rho_w\, g) \times A$
b) $mg = h \times \rho_w \times g \times A \times F_s$.
c) It glass of different shape is used provided the volume, height and area remain same, no change in answer will occur.
6. Standard atmospheric pressure is always pressure exerted by 76 cm Hg column
$= (76 \times 13.6 \times g)$ Dyne/cm^2.
If water is used in the barometer.
Let $h \rightarrow$ height of water column.
$\therefore h \times \rho_w \times g$
7. a) $F = P \times A = (h\, \rho_w \times g)\, A$
b) The force does not depend on the orientation of the rock as long as the surface area remains same.
8. a) $F = A\, h\, \rho\, g$.
b) The force exerted by water on the strip of width δx as shown,
$dF = p \times A$
$= (x\rho g) \times A$
c) Inside the liquid force act in every direction due to adhesion.
$di = F \times r$
d) The total force by the water on that side is given by
$$F = \int_0^1 20000\, x\delta x \Rightarrow F = 20{,}000\ [x^2/2]_0^1$$
e) The torque by the water on that side will be,

$i = \int_0^1 20000\, x\delta x\,(1-x) \Rightarrow 20{,}000\ [x^2/2 - x^3/3]_0^1$

9. Here, $m_0 = m_{Au} + m_{cu} = 36$ g …(1)
   Let V be the volume of the ornament in cm^3
   So, $V \times \rho_w \times g = 2 \times g$
   $\Rightarrow (V_{au} + V_{cu}) \times \rho_w \times g = 2 \times g$
   $\Rightarrow \left(\frac{m}{\rho_{au}} + \frac{m}{\rho_{au}}\right)\rho_w \times g = 2 \times g$
   $\Rightarrow \left(\frac{m_{Au}}{19.3} + \frac{m_{Au}}{8.9}\right) \times 1 = 2$
   $\Rightarrow 8.9\ m_{Au} + 19.3\ m_{cu} = 2 \times 19.3 \times 8.9 = 343.54$ …(2)
   From equation (1) and (2), $8.9\ m_{Au} + 19.3\ m_{cu} = 343.54$
   $\Rightarrow \frac{8.9(m_{Au} + m_{cu}) = 8.9 \times 36}{m_{cu} = 2.225g}$
   So, the amount of copper in the ornament is 2.2 g.
10. $\left(\frac{M_{Au}}{\rho_{Au}} + V_c\right)\rho_w \times g = 2 \times g$ (where V_c = volume of cavity)
11. mg = U + R (where U = Upward thrust)
    $\Rightarrow$ mg – U = R
    $\Rightarrow R = mg - v\,\rho_w\,g$ (because, $U = v\rho_w g$)
    $= mg - \frac{m}{\rho} \times \rho_w \times g$
12. a) Let $V_i \rightarrow$ volume of boat inside water = volume of water displace in m^3.
    Since, weight of the boat is balanced by the buoyant force.
    $\Rightarrow mg = V_i \times \rho_w \times g$
    b) Let, $v^1 \rightarrow$ volume of boat filled with water before water starts coming in from the sides.
    $mg + v^1\,\rho_w \times g = V \times \rho_w \times g$.
13. Let $x \rightarrow$ minimum edge of the ice block in cm.
    So, $mg + W_{ice} = U$. (where U = Upward thrust)
    $\Rightarrow 0.5 \times g + x^3 \times \rho_{ice} \times g = x^3 \times \rho_w \times g$
14. $V_{ice} = V_k + V_w$
    $V_{ice} \times \rho_{ice} \times g = V_k \times \rho_k \times g + V_w \times \rho_w \times g$
    $\Rightarrow (V_k + V_w) \times \rho_{ice} = V_k \times \rho_k + V_w \times \rho_w$
    $\Rightarrow \frac{V_w}{V_k} = 1$.
15. $V_{ii}g = V\,\rho_w\,g$
16. $(m_w + m_{pb})g = (V_w + V_{pb})\,\rho \times g$
    $\Rightarrow (m_w + m_{pb}) = \left(\frac{m_w}{\rho_w} + \frac{m_{pb}}{\rho_{pb}}\right)\rho$
17. $Mg = w \Rightarrow (m_w + m_{pb})g = V_w \times \rho \times g$
18. Given, x = 12 cm
    Length of the edge of the block $\rho_{Hg} = 13.6$ gm/cc
    Given that, initially 1/5 of block is inside mercuty.
    Let $\rho_b \rightarrow$ density of block in gm/cc.
    $\therefore (x)^3 \times \rho_b \times g = (x)^2 \times (x/5) \times \rho_{Hg} \times g$
    $\Rightarrow 12^3 \times \rho_b = 12^2 \times 12/5 \times 13.6$
    $\Rightarrow \rho_b = \frac{13.6}{5}$ gm/cc

After water poured, let x = height of water column.

$V_b = V_{Hg} + V_w = 12^3$

Where V_{Hg} and V_w are volume of block inside mercury and water respectively

$\therefore (V_b \times \rho_b \times g) = (V_{Hg} \times \rho_{Hg} \times g) + (V_w \times \rho_w \times g)$

$\Rightarrow (V_{Hg} + V_w)\rho_b = V_{Hg} \times \rho_{Hg} + V_w \times \rho_w$.

$\Rightarrow (V_{Hg} + V_w) \times \dfrac{13.6}{5} = V_{Hg} \times 13.6 + V_w \times 1$

$\Rightarrow (12)^3 \times \dfrac{13.6}{5} = (12 - x) \times (12)^2 \times 13.6 + (x) \times (12)^2 \times 1$

$\Rightarrow x = 10.4$ cm

19. Here, Mg = Upward thrust

$\Rightarrow V\rho g = (V/2)(\rho_w) \times g$ (where ρ_w = density of water)

$\Rightarrow \left(\frac{4}{3}\pi r_2^3 - \frac{4}{3}\pi r_1^3\right)\rho = \left(\frac{1}{2}\right)\left(\frac{4}{3}\pi r_2^3\right) \times \rho_w$

$\Rightarrow (r_2^3 - r_1^3) \times \rho = \frac{1}{2} r_2^3 \times 1 = 865 \text{ kg/m}^3$.

20. $W_1 + W_2 = U$.

$\Rightarrow mg + V \times \rho_s \times g = V \times \rho_w \times g$ (where ρ_s = density of sphere in gm/cc)

$\Rightarrow 1 - \rho_s = 0.19$

$\Rightarrow \rho_s = 1 - (0.19) = 0.8$ gm/cc

So, specific gravity of the material is 0.8.

21. $W_i = mg - V_i\, \rho_{air} \times g = \left(m - \frac{m}{\rho_i}\rho_{air}\right)g$

$W_w = mg - V_w\, \rho_{air}\, g = \left(m - \frac{m}{\rho_w}\rho_{air}\right)g$

22. Driving force $U = V\rho_w g$

$\Rightarrow a = \pi r^2 (X) \times \rho_w\, g \;\Rightarrow T = 2\pi\sqrt{\frac{\text{displacement}}{\text{Acceleration}}}$

23. a) F + U = mg (where F = kx)

$\Rightarrow kx + V\rho_w g = mg$

b) $F = kX + V\rho_w \times g$

$\Rightarrow ma = kX + \pi r^2 \times (X) \times \rho_w \times g = (k + \pi r^2 \times \rho_w \times g)X$

$\Rightarrow \omega^2 \times (X) = \dfrac{(k + \pi r^2 \times \rho_w \times g)}{m} \times (X)$

$\Rightarrow T = 2\pi\sqrt{\dfrac{m}{K + \pi r^2 \times \rho_w \times g}}$

24. a) $mg = kX + V\rho_w g$

b) $a = kx/m$

$w^2 x = kx/m$

$T = 2\pi\sqrt{m/k}$

25. Let $x \rightarrow$ edge of ice block

When it just leaves contact with the bottom of the glass.

$h \rightarrow$ height of water melted from ice

W = U

$\Rightarrow x^3 \times \rho_{ice} \times g = x^2 \times h \times \rho_w \times g$

Again, volume of water formed, from melting of ice is given by,

$4^3 - x^3 = \pi \times r^2 \times h - x^2 h$ (because amount of water = $(\pi r^2 - x^2)h$)

$\Rightarrow 4^3 - x^3 = \pi \times 3^2 \times h - x^2 h$

Putting $h = 0.9\,x \Rightarrow x = 2.26$ cm.

26. If $p_a \rightarrow$ atm. Pressure
    $A \rightarrow$ area of cross section
    $h \rightarrow$ increase in hright
    $p_aA + A \times L \times \rho \times a_0 = pa^A + h\rho g \times A$
    $\Rightarrow hg = a_0L \quad \Rightarrow \quad a_0L/g$
27. Volume of water, discharged from Alkananda + vol are of water discharged from Bhagirathi = Volume of water flow in Ganga.
28. a) $a_A \times V_A = Q_A$
    b) $a_A \times V_A = a_B \times V_B$
    c) $1/2\ \rho v_A^2 + p_A = 1/2\ \rho v_B^2 + p_B$
    $\Rightarrow (p_A - p_B) = 1/2\ \rho\ (v_B^2 - v_A^2)$
29. From Bernoulli's equation, $1/2\ \rho v_A^2 + \rho g h_A + p_A$
    $= 1/2\ \rho v_B^2 + \rho g h_B + p_B$.
    $\Rightarrow P_A - P_B = (1/2)\ \rho\ (v_B^2 - v_A^2) + \rho g\ (h_B - h_A)$
30. $1/2\ \rho v_B^2 + \rho g h_B + p_B = 1/2\ \rho v_A^2 + \rho g h_A + p_A$
31. $1/2\ \rho v_A^2 + \rho g h_A + p_A = 1/2\ \rho v_B^2 + \rho g h_B + p_B$
    $\Rightarrow P_B - P_A = 1/2\ \rho(v_A^2 - v_B^2) + \rho g\ (h_A - h_B)$
32. $\vec{v}_A a_A = \vec{v}_B \times a_B$
    $\Rightarrow 1/2\ \rho v_A^2 + \rho g h_A + p_A = 1/2\ \rho v_B^2 + \rho g h_B + p_B$
    $\Rightarrow 1/2\ \rho v_A^2 + p_A = 1/2\ \rho v_B^2 + p_B$
    $\Rightarrow P_A - P_B = 1/2\ \rho(v_B^2 - v_B^2)$
    Rate of flow $= v_a \times a_A$
33. $V_A\ a_A = v_B\ a_B \Rightarrow \dfrac{v_A}{B} = \dfrac{a_B}{a_A}$

    $5v_A = 2v_B \Rightarrow v_B = (5/2)v_A$
    $1/2\ \rho v_A^2 + \rho g h_A + p_A = 1/2\ \rho v_B^2 + \rho g h_B + p_B$
    $\Rightarrow P_A - P_B = 1/2\ \rho\ (v_B^2 - v_B^2)$ (because $P_A - P_B = h\rho_m g$)
34. $P_A + (1/2)\rho v_A^2 = P_B + (1/2)\ \rho v_B^2 \Rightarrow p_A - p_B = (1/2)\rho v_B^2\ \{v_A = 0\}$
    $\Rightarrow \rho g h = (1/2)\ \rho v_B^2\ \{p_A = p_{atm} + \rho g h\}$
    $\Rightarrow v_B = \sqrt{2gh}$

    a) $v = \sqrt{2gh}$

    b) $v = \sqrt{2g(h/2)} = \sqrt{gh}$

    c) $v = \sqrt{2gh}$
    $v = av \times dt$
    $AV = av$

    $\Rightarrow A \times \dfrac{dh}{dt} = a \times \sqrt{2gh} \Rightarrow dh = \dfrac{a \times \sqrt{2gh} \times dt}{A}$

    d) $dh = \dfrac{a \times \sqrt{2gh} \times dt}{A} \Rightarrow T = \dfrac{A}{a}\sqrt{\dfrac{2}{g}}[\sqrt{H_1} - \sqrt{H_2}]$

35. $v = \sqrt{2g(H-h)}$

    $t = \sqrt{2h/g}$

    $x = v \times t = \sqrt{2g(H-h) \times 2h/g} = 4\sqrt{(Hh - h^2)}$

    So, $\Rightarrow \left(\dfrac{d}{dh}\right)(Hh - h^2) = 0 \Rightarrow 0 = H - 2h \Rightarrow h = H/2$.

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SOLUTIONS TO CONCEPTS
CHAPTER 14

1. F = mg

   Stress = $\frac{F}{A}$

   Strain = $\frac{\Delta L}{L}$

   Y = $\frac{FL}{A\Delta L} \Rightarrow \frac{\Delta L}{L} = \frac{F}{YA}$

2. ρ = stress = mg/A

   e = strain = ρ/Y

   Compression ΔL = eL

3. y = $\frac{F}{A}\frac{L}{\Delta L} \Rightarrow \Delta L = \frac{FL}{AY}$

4. $L_{steel} = L_{cu}$ and $A_{steel} = A_{cu}$

   a) $\frac{\text{Stress of cu}}{\text{Stress of st}} = \frac{F_{cu}}{A_{cu}}\frac{A_g}{F_g}$ = $\frac{F_{cu}}{F_{st}} = 1$

   b) Strain = $\frac{\Delta Lst}{\Delta lcu} = \frac{F_{st}L_{st}}{A_{st}Y_{st}} \cdot \frac{A_{cu}Y_{cu}}{F_{cu}l_{cu}}$ ($\because$ $L_{cu} = l_{st}$; $A_{cu} = A_{st}$)

5. $\left(\frac{\Delta L}{L}\right)_{st} = \frac{F}{AY_{st}}$

   $\left(\frac{\Delta L}{L}\right)_{cu} = \frac{F}{AY_{cu}}$

   $\frac{\text{strain steel wire}}{\text{Strain om copper wire}} = \frac{F}{AY_{st}} \times \frac{AY_{cu}}{F} (\because A_{cu} = A_{st}) = \frac{Y_{cu}}{Y_{st}}$

6. Stress in lower rod = $\frac{T_1}{A_1} \Rightarrow \frac{m_1 g + \omega g}{A_1}$ $\Rightarrow$ w = 14 kg

   Stress in upper rod = $\frac{T_2}{A_u} \Rightarrow \frac{m_2 g + m_1 g + wg}{A_u} \Rightarrow$ w = .18 kg

   For same stress, the max load that can be put is 14 kg. If the load is increased the lower wire will break first.

   $\frac{T_1}{A_1} = \frac{m_1 g + \omega g}{A_1}$ = $8 \times 10^8 \Rightarrow$ w = 14 kg

   $\frac{T_2}{A_u} \Rightarrow \frac{m_2 g + m_1 g + \omega g}{A_u}$ = $8 \times 10^8 \Rightarrow \omega_0$ = 2 kg

   The maximum load that can be put is 2 kg. Upper wire will break first if load is increased.

7. $Y = \frac{F}{A}\frac{L}{\Delta L}$

8. $Y = \frac{F}{A}\frac{L}{\Delta L} \Rightarrow F = \frac{YA\ \Delta L}{L}$

9. $m_2 g - T = m_2 a$ …(1)

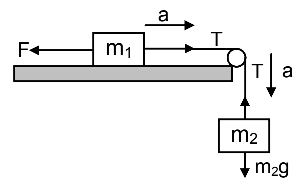
   and $T - F = m_1 a$ …(2)

   $\Rightarrow a = \frac{m_2 g - F}{m_1 + m_2}$

From equation (1) and (2), we get $\frac{m_2 g}{2(m_1 + m_2)}$

Again, T = F + m_1a

$\Rightarrow T = \frac{m_2 g}{2} + m_1 \frac{m_2 g}{2(m_1 + m_2)} \Rightarrow \frac{m_2^2 g + 2m_1 m_2 g}{2(m_1 + m_2)}$



Now Y = $\frac{FL}{A\ \Delta L} \Rightarrow \frac{\Delta L}{L} = \frac{F}{AY}$

$\Rightarrow \frac{\Delta L}{L} = \frac{(m_2^2 + 2m_1 m_2)g}{2(m_1 + m_2)AY} = \frac{m_2 g(m_2 + 2m_1)}{2AY(m_1 + m_2)}$

10. At equilibrium $\Rightarrow$ T = mg

When it moves to an angle θ, and released, the tension the T′ at lowest point is

$\Rightarrow T' = mg + \frac{mv^2}{r}$

The change in tension is due to centrifugal force $\Delta T = \frac{mv^2}{r}$ …(1)

$\Rightarrow$ Again, by work energy principle,

$\Rightarrow \frac{1}{2}mv^2 - 0 = mgr(1 - \cos\theta)$

$\Rightarrow v^2 = 2gr\ (1 - \cos\theta)$ …(2)

So, $\Delta T = \frac{m[2gr(1-\cos\theta)]}{r} = 2mg(1-\cos\theta)$

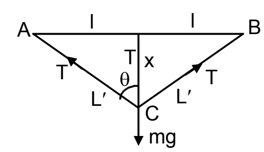
$\Rightarrow F = \Delta T$

$\Rightarrow F = \frac{YA\ \Delta L}{L} = 2mg - 2mg\cos\theta \Rightarrow 2mg\cos\theta = 2mg - \frac{YA\ \Delta L}{L}$

$= \cos\theta = 1 - \frac{YA\ \Delta L}{L(2mg)}$

11. From figure $\cos\theta = \frac{x}{\sqrt{x^2 + l^2}} = \frac{x}{l}\left[1 + \frac{x^2}{l^2}\right]^{-1/2}$

$= x / l$ … (1)



Increase in length ΔL = (AC + CB) – AB

Here, AC = $(l^2 + x^2)^{1/2}$

So, $\Delta L = 2(l^2 + x^2)^{1/2} - 100$ …(2)

$Y = \frac{F}{A}\frac{l}{\Delta l}$ …(3)

From equation (1), (2) and (3) and the freebody diagram,

2l cosθ = mg.

12. $Y = \frac{FL}{A\Delta L} \Rightarrow \frac{\Delta L}{L} = \frac{F}{Ay}$

$\sigma = \frac{\Delta D/D}{\Delta L/L} \Rightarrow \frac{\Delta D}{D} = \frac{\Delta L}{L}$

Again, $\frac{\Delta A}{A} = \frac{2\Delta r}{r}$

$\Rightarrow \Delta A = \frac{2\Delta r}{r}$

13. B = $\frac{Pv}{\Delta v}$ $\Rightarrow$ P = $B\left(\frac{\Delta v}{v}\right)$

14. $\rho_0 = \frac{m}{V_0} = \frac{m}{V_d}$

so, $\frac{\rho_d}{\rho_0} = \frac{V_0}{V_d}$ …(1)

vol.strain = $\frac{V_0 - V_d}{V_0}$

B = $\frac{\rho_0 gh}{(V_0 - V_d)/V_0}$ $\Rightarrow 1 - \frac{V_d}{V_0} = \frac{\rho_0 gh}{B}$

$\Rightarrow \frac{vD}{v_0} = \left(1 - \frac{\rho_0 gh}{B}\right)$ …(2)

Putting value of (2) in equation (1), we get

$\frac{\rho_d}{\rho_0} = \frac{1}{1 - \rho_0 gh/B} \Rightarrow \rho_d = \frac{1}{(1 - \rho_0 gh/B)} \times \rho_0$

15. $\eta = \frac{F}{A\theta}$

Lateral displacement = $l\theta$.

16. F = T l

17. a) $P = \frac{2T_{Hg}}{r}$ b) $P = \frac{4T_g}{r}$ c) $P = \frac{2T_g}{r}$

18. a) F = P_0A
b) Pressure = P_0 + (2T/r)
F = P′A = (P_0 + (2T/r)A
c) P = 2T/r
F = PA = $\frac{2T}{r}A$

19. a) $h_A = \frac{2T\cos\theta}{r_A - \rho g}$ b) $h_B = \frac{2T\cos\theta}{r_B \rho g}$ c) $h_C = \frac{2T\cos\theta}{r_C \rho g}$

20. $h_{Hg} = \frac{2T_{Hg}\cos\theta_{Hg}}{r\rho_{Hg} g}$

$h_\omega = \frac{2T_\omega \cos\theta_\omega}{r\rho_\omega g}$ where, the symbols have their usual meanings.

$\frac{h_\omega}{h_{Hg}} = \frac{T_\omega}{T_{Hg}} \times \frac{\rho_{Hg}}{\rho_\omega} \times \frac{\cos\theta_\omega}{\cos\theta_{Hg}}$

21. $h = \frac{2T\cos\theta}{r\rho g}$

22. P = $\frac{2T}{r}$

P = F/r

23. A = πr^2

24. $\frac{4}{3}\pi R^3 = \frac{4}{3}\pi r^3 \times 8$

$\Rightarrow$ r = R/2 = 2

Increase in surface energy = TA′ – TA

25. $h = \frac{2T\cos\theta}{r\rho g}$, $h' = \frac{2T\cos\theta}{r\rho g}$

$\Rightarrow \cos\theta = \frac{h' r\rho g}{2T}$

So, $\theta = \cos^{-1}(1/2) = 60°$.

26. a) $h = \frac{2T\cos\theta}{r\rho g}$

b) $T \times 2\pi r \cos\theta = \pi r^2 h \times \rho \times g$

$\therefore \cos\theta = \frac{hr\rho g}{2T}$

27. $T(2l) = [1 \times (10^{-3}) \times h]\rho g$

28. Surface area = $4\pi r^2$

29. The length of small element = $r\, d\theta$

$dF = T \times r\, d\theta$

considering symmetric elements,

$dF_y = 2T\, rd\theta \,.\, \sin\theta$ $[dF_x = 0]$

so, $F = 2Tr\int_0^{\pi/2} \sin\theta d\theta = 2Tr[\cos\theta]_0^{\pi/2} = T \times 2r$

Tension $\Rightarrow 2T_1 = T \times 2r \Rightarrow T_1 = Tr$

30. a) Viscous force = $6\pi\eta rv$

b) Hydrostatic force = $B = \left(\frac{4}{3}\right)\pi r^3 \sigma g$

c) $6\pi\eta\, rv + \left(\frac{4}{3}\right)\pi r^3 \sigma g = mg$

$v = \frac{2}{9}\frac{r^2(\rho - \sigma)g}{\eta} \Rightarrow \frac{2}{9}r^2 \frac{\left(\frac{m}{(4/3)\pi r^3} - \sigma\right)g}{n}$

31. To find the terminal velocity of rain drops, the forces acting on the drop are,

i) The weight $(4/3)\pi\, r^3\, \rho g$ downward.

ii) Force of buoyancy $(4/3)\pi\, r^3\, \sigma g$ upward.

iii) Force of viscosity $6\,\pi\,\eta\, r\, v$ upward.

Because, σ of air is very small, the force of buoyancy may be neglected.

Thus,

$6\,\pi\,\eta\, r\, v = \left(\frac{4}{3}\right)\pi r^2 \rho g$ or $v = \frac{2r^2\rho g}{9\eta}$

32. $v = \frac{R\eta}{\rho D} \Rightarrow R = \frac{v\rho D}{\eta}$

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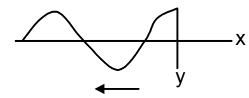
SOLUTIONS TO CONCEPTS
CHAPTER 15



1. v = 40 cm/sec
   As velocity of a wave is constant location of maximum after 5 sec
   = 40 × 5 = 200 cm along negative x-axis.
2. Given y = $Ae^{-[(x/a)+(t/T)]^2}$
   a) $[A] = [M^0L^1T^0]$, $[T] = [M^0L^0T^1]$
      $[a] = [M^0L^1T^0]$
   b) Wave speed, v = λ/T = a/T [Wave length λ = a]
   c) If y = f(t – x/v) → wave is traveling in positive direction
      and if y = f(t + x/v) → wave is traveling in negative direction
      So, y = $Ae^{-[(x/a)+(t/T)]^2}$ = $Ae^{-(1/T)\left[\frac{x}{a/T}+t\right]^2}$
      = $Ae^{-(1/T)\left[\frac{x}{v}+t\right]^2}$
      i.e. y = f{t + (x / v)}
   d) Wave speed, v = a/T
      ∴ Max. of pulse at t = T is (a/T) × T = a (negative x-axis)
      Max. of pulse at t = 2T = (a/T) × 2T = 2a (along negative x-axis)
      So, the wave travels in negative x-direction.
3. At t = 1 sec, $s_1 = vt = 10 \times 1 = 10$ cm
   t = 2 sec, $s_2 = vt = 10 \times 2 = 20$ cm
   t = 3 sec, $s_3 = vt = 10 \times 3 = 30$ cm
4. The pulse is given by, $y = [(a^3) / \{(x - vt)^2 + a^2\}]$
   a = 5 mm = 0.5 cm, v = 20 cm/s
   At t = 0s, $y = a^3 / (x^2 + a^2)$
   The graph between y and x can be plotted by taking different values of x.
   (left as exercise for the student)
   similarly, at t = 1 s, $y = a^3 / \{(x - v)^2 + a^2\}$
   and at t = 2 s, $y = a^3 / \{(x - 2v)^2 + a^2\}$
5. At x = 0, f(t) = a sin (t/T)
   Wave speed = v
   ⇒ λ = wavelength = vT (T = Time period)
   So, general equation of wave
   Y = A sin [(t/T) – (x/vT)] [because y = f((t/T) – (x/λ))
6. At t = 0, g(x) = A sin (x/a)
   a) $[M^0L^1T^0] = [L]$
      $a = [M^0L^1T^0] = [L]$
   b) Wave speed = v
      ∴ Time period, T = a/v (a = wave length = λ)
   ∴ General equation of wave
   y = A sin {(x/a) – t/(a/v)}
      = A sin {(x – vt) / a}
7. At $t = t_0$, $g(x, t_0) = A \sin (x/a)$ …(1)
   For a wave traveling in the positive x-direction, the general equation is given by
   $y = f\left(\frac{x}{a} - \frac{t}{T}\right)$
   Putting $t = -t_0$ and comparing with equation (1), we get
   ⇒ $g(x, 0) = A \sin \{(x/a) + (t_0/T)\}$
   ⇒ $g(x, t) = A \sin \{(x/a) + (t_0/T) - (t/T)\}$

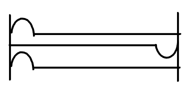
As T = a/v (a = wave length, v = speed of the wave)

$\Rightarrow$ y = $A\sin\left(\frac{x}{a}+\frac{t_0}{(a/v)}-\frac{t}{(a/v)}\right)$

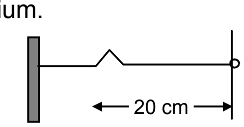
= $A\sin\left(\frac{x+v(t_0-t)}{a}\right)$

$\Rightarrow$ y = $A\sin\left[\frac{x-v(t-t_0)}{a}\right]$

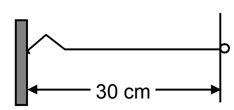
8. The equation of the wave is given by
   y = (0.1 mm) sin [(31.4 m^{-1})x +(314 s^{-1})t]   y = r sin {(2πx / λ)} + ωt)
   a) Negative x-direction
   b) k = 31.4 m^{-1}
   $\Rightarrow$ 2λ/λ = 31.4 $\Rightarrow$ λ = 2π/31.4 = 0.2 mt = 20 cm
   Again, ω = 314 s^{-1}
   $\Rightarrow$ 2πf = 314 $\Rightarrow$ f = 314 / 2π = 314 / (2 × (3/14)} = 50 sec^{-1}
   ∴ wave speed, v = λf = 20 × 50 = 1000 cm/s
   c) Max. displacement = 0.10 mm
   Max. velocity = aω = 0.1 × 10^{-1} × 314 = 3.14 cm/sec.
9. Wave speed, v = 20 m/s
   A = 0.20 cm
   λ = 2 cm
   a) Equation of wave along the x-axis
   y = A sin (kx – wt)
   ∴ k = 2π/λ = 2π/2 = π cm^{-1}
   T = λ/v = 2/2000 = 1/1000 sec = 10^{-3} sec
   $\Rightarrow$ ω = 2π/T = 2π × 10^{-3} sec^{-1}
   So, the wave equation is,
   ∴ y = (0.2 cm)sin[(π cm^{-1})x – (2π × 10^3 sec^{-1})t]
   b) At x = 2 cm, and t = 0,
   y = (0.2 cm) sin (π/2) = 0
   ∴ v = rω cos πx = 0.2 × 2000 π × cos 2π = 400 π
   = 400 × (3.14) = 1256 cm/s
   = 400 π cm/s = 4π m/s
10. Y = (1 mm) sin π $\left[\frac{x}{2cm}-\frac{t}{0.01sec}\right]$
   a) T = 2 × 0.01 = 0.02 sec = 20 ms
   λ = 2 × 2 = 4 cm
   b) v = dy/dt = d/dt [sin 2π (x/4 – t/0.02)] = –cos2π {x/4) – (t/0.02)} × 1/(0.02)
   $\Rightarrow$ v = –50 cos 2π {(x/4) – (t/0.02)}
   at x = 1 and t = 0.01 sec, v = –50 cos 2* [(1/4) – (1/2)] = 0
   c) i) at x = 3 cm, t = 0.01 sec
   v = –50 cos 2π (3/4 – ½) = 0
   ii) at x = 5 cm, t = 0.01 sec, v = 0 (putting the values)
   iii) at x = 7 cm, t = 0.01 sec, v = 0
   at x = 1 cm and t = 0.011 sec
   v = –50 cos 2π {(1/4) – (0.011/0.02)} = –50 cos (3π/5) = –9.7 cm/sec
   (similarly the other two can be calculated)
11. Time period, T = 4 × 5 ms = 20 × 10^{-3} = 2 × 10^{-2} s
   λ = 2 × 2 cm = 4 cm
   frequency, f = 1/T = 1/(2 × 10^{-2}) = 50 s^{-1} = 50 Hz
   Wave speed = λf = 4 × 50 m/s = 2000 m/s = 2 m/s

12. Given that, $v = 200$ m/s
    a) Amplitude, A = 1 mm
    b) Wave length, $\lambda = 4$ cm
    c) wave number, $n = 2\pi/\lambda = (2 \times 3.14)/4 = 1.57\ cm^{-1}$ (wave number = k)
    d) frequency, $f = 1/T = (26/\lambda)/20 = 20/4 = 5$ Hz
    (where time period $T = \lambda/v$)
13. Wave speed = v = 10 m/sec
    Time period = T = 20 ms = $20 \times 10^{-3} = 2 \times 10^{-2}$ sec
    a) wave length, $\lambda = vT = 10 \times 2 \times 10^{-2} = 0.2$ m = 20 cm
    b) wave length, $\lambda = 20$ cm
    $\therefore$ phase $diff^n = (2\pi/\lambda)\ x = (2\pi / 20) \times 10 = \pi$ rad
    $y_1 = a \sin(\omega t - kx) \Rightarrow 1.5 = a \sin(\omega t - kx)$
    So, the displacement of the particle at a distance x = 10 cm.
    [$\phi = \frac{2\pi x}{\lambda} = \frac{2\pi \times 10}{20} = \pi$] is given by
    $y_2 = a \sin(\omega t - kx + \pi) \Rightarrow -a \sin(\omega t - kx) = -1.5$ mm
    $\therefore$ displacement = –1.5 mm
14. mass = 5 g, length l = 64 cm
    $\therefore$ mass per unit length = m = 5/64 g/cm
    $\therefore$ Tension, $T = 8N = 8 \times 10^5$ dyne
    $V = \sqrt{(T/m)} = \sqrt{(8 \times 10^5 \times 64)/5} = 3200$ cm/s = 32 m/s
15. 

    a) Velocity of the wave, $v = \sqrt{(T/m)} = \sqrt{(16 \times 10^5)/0.4} = 2000$ cm/sec
    $\therefore$ Time taken to reach to the other end = 20/2000 = 0.01 sec
    Time taken to see the pulse again in the original position = $0.01 \times 2 = 0.02$ sec
    b) At t = 0.01 s, there will be a 'though' at the right end as it is reflected.
16. The crest reflects as a crest here, as the wire is traveling from denser to rarer medium.
    $\Rightarrow$ phase change = 0

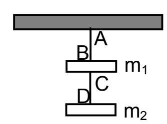


    a) To again original shape distance travelled by the wave S = 20 + 20 = 40 cm.
    Wave speed, v = 20 m/s $\Rightarrow$ time = s/v = 40/20 = 2 sec
    b) The wave regains its shape, after traveling a periodic distance = $2 \times 30 = 60$ cm
    $\therefore$ Time period = 60/20 = 3 sec.

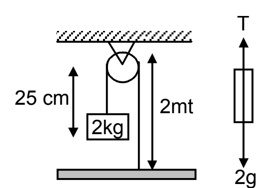


    c) Frequency, $n = (1/3\ sec^{-1})$
    $n = (1/2l)\sqrt{(T/m)}$ m = mass per unit length = 0.5 g/cm
    $\Rightarrow 1/3 = 1/(2 \times 30)\ \sqrt{(T/0.5)}$
    $\Rightarrow T = 400 \times 0.5 = 200$ dyne $= 2 \times 10^{-3}$ Newton.
17. Let v_1 = velocity in the 1^{st} string
    $\Rightarrow v_1 = \sqrt{(T/m_1)}$
    Because m_1 = mass per unit length = $(\rho_1 a_1 l_1 / l_1) = \rho_1 a_1$ where a_1 = Area of cross section
    $\Rightarrow v_1 = \sqrt{(T/\rho_1 a_1)}$ …(1)
    Let v_2 = velocity in the second string
    $\Rightarrow v_2 = \sqrt{(T/m^2)}$
    $\Rightarrow v_2 = \sqrt{(T/\rho_2 a_2)}$ …(2)
    Given that, $v_1 = 2v_2$
    $\Rightarrow \sqrt{(T/\rho_1 a_1)} = 2\sqrt{(T/\rho_2 a_2)} \Rightarrow (T/a_1\rho_1) = 4(T/a_2\rho_2)$
    $\Rightarrow \rho_1/\rho_2 = 1/4 \Rightarrow \rho_1 : \rho_2 = 1 : 4$ (because $a_1 = a_2$)

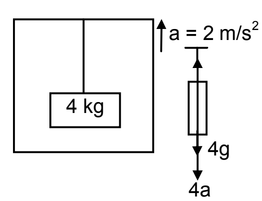
18. m = mass per unit length = 1.2×10^{-4} kg/mt
    Y = (0.02m) sin [($1.0\ m^{-1}$)x + ($30\ s^{-1}$)t]
    Here, k = $1\ m^{-1} = 2\pi/\lambda$
    $\omega = 30\ s^{-1} = 2\pi f$
    $\therefore$ velocity of the wave in the stretched string
    $v = \lambda f = \omega/k = 30/I = 30$ m/s
    $\Rightarrow v = \sqrt{T/m} \Rightarrow 30\sqrt{(T/1.2)\times 10^{-4} N)}$
    $\Rightarrow T = 10.8 \times 10^{-2}$ N $\Rightarrow T = 1.08 \times 10^{-1}$ Newton.
19. Amplitude, A = 1 cm, Tension T = 90 N
    Frequency, f = 200/2 = 100 Hz
    Mass per unit length, m = 0.1 kg/mt
    a) $\Rightarrow V = \sqrt{T/m} = 30$ m/s
       λ = V/f = 30/100 = 0.3 m = 30 cm
    b) The wave equation y = (1 cm) cos 2π (t/0.01 s) – (x/30 cm)
       [because at x = 0, displacement is maximum]
    c) y = 1 cos 2π(x/30 – t/0.01)
       $\Rightarrow$ v = dy/dt = (1/0.01)2π sin 2π {(x/30) – (t/0.01)}
       a = dv/dt = – {$4\pi^2$ / $(0.01)^2$} cos 2π {(x/30) – (t/0.01)}
       When, x = 50 cm, t = 10 ms = 10×10^{-3} s
       x = (2π / 0.01) sin 2π {(5/3) – (0.01/0.01)}
       = (p/0.01) sin ($2\pi \times 2$ / 3) = (1/0.01) sin (4π/3) = –200 π sin (π/3) = –200 πx $(\sqrt{3}/2)$
       = 544 cm/s = 5.4 m/s
       Similarly
       a = {$4\pi^2$ / $(0.01)^2$} cos 2π {(5/3) – 1}
       = $4\pi^2 \times 10^4 \times$ ½ $\Rightarrow 2 \times 10^5\ cm/s^2 \Rightarrow 2\ km/s^2$
20. l = 40 cm, mass = 10 g
    $\therefore$ mass per unit length, m = 10 / 40 = 1/4 (g/cm)
    spring constant K = 160 N/m
    deflection = x = 1 cm = 0.01 m
    $\Rightarrow$ T = kx = 160 $\times$ 0.01 = 1.6 N = 16×10^4 dyne
    Again v = $\sqrt{(T/m)}$ = $\sqrt{(16\times 10^4/(1/4)}$ = 8×10^2 cm/s = 800 cm/s
    $\therefore$ Time taken by the pulse to reach the spring
    t = 40/800 = 1/20 = 0/05 sec.
21. $m_1 = m_2$ = 3.2 kg
    mass per unit length of AB = 10 g/mt = 0.01 kg.mt
    mass per unit length of CD = 8 g/mt = 0.008 kg/mt
    for the string CD, T = 3.2 $\times$ g
    $\Rightarrow v = \sqrt{(T/m)} = \sqrt{(3.2\times 10)/0.008} = \sqrt{(32\times 10^3)/8} = 2\times 10\sqrt{10}$ = 20 $\times$ 3.14 = 63 m/s
    for the string AB, T = 2 $\times$ 3.2 g = 6.4 $\times$ g = 64 N
    $\Rightarrow v = \sqrt{(T/m)} = \sqrt{(64/0.01)} = \sqrt{6400}$ = 80 m/s



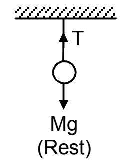
22. Total length of string 2 + 0.25 = 2.25 mt
    Mass per unit length m = $\dfrac{4.5\times 10^{-3}}{2.25}$ = 2×10^{-3} kg/m
    T = 2g = 20 N
    Wave speed, v = $\sqrt{(T/m)} = \sqrt{20}/(2\times 10^{-3}) = \sqrt{10^4} = 10^2$ m/s = 100 m/s
    Time taken to reach the pully, t = (s/v) = 2/100 = 0.02 sec.



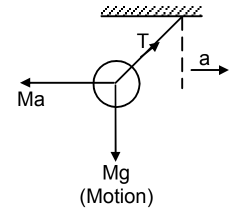
23. m = 19.2×10^{-3} kg/m
    from the freebody diagram,
    T – 4g – 4a = 0
    $\Rightarrow$ T = 4(a + g) = 48 N
    wave speed, v = $\sqrt{(T/m)}$ = 50 m/s



24. Let M = mass of the heavy ball
(m = mass per unit length)
Wave speed, $v_1 = \sqrt{(T/m)} = \sqrt{(Mg/m)}$ (because T = Mg)
$\Rightarrow 60 = \sqrt{(Mg/m)} \Rightarrow Mg/m = 60^2$ …(1)

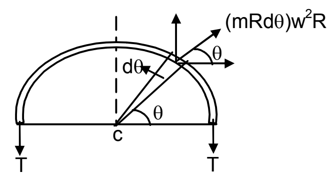


From the freebody diagram (2),
$v_2 = \sqrt{(T'/m)}$
$\Rightarrow v_2 = \dfrac{[(Ma)^2 + (Mg)^2]^{1/4}}{m^{1/2}}$ (because T' = $\sqrt{(Ma)^2 + (Mg)^2}$)
$\Rightarrow 62 = \dfrac{[(Ma)^2 + (Mg)^2]^{1/4}}{m^{1/2}}$
$\Rightarrow \dfrac{\sqrt{(Ma)^2 + (Mg)^2}}{m} = 62^2$ …(2)



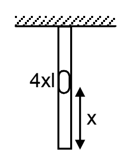
Eq(1) + Eq(2) $\Rightarrow$ (Mg/m) $\times$ [m / $\sqrt{(Ma)^2 + (Mg)^2}$] = 3600 / 3844
$\Rightarrow g / \sqrt{(a^2 + g^2)} = 0.936 \Rightarrow g^2 / (a^2 + g^2) = 0.876$
$\Rightarrow (a^2 + 100)\ 0.876 = 100$
$\Rightarrow a^2 \times 0.876 = 100 - 87.6 = 12.4$
$\Rightarrow a^2 = 12.4 / 0.876 = 14.15 \Rightarrow a = 3.76\ m/s^2$
$\therefore$ Acce^n of the car = 3.7 m/s^2

25. m = mass per unit length of the string
R = Radius of the loop
ω = angular velocity, V = linear velocity of the string
Consider one half of the string as shown in figure.
The half loop experiences cetrifugal force at every point, away from centre, which is balanced by tension 2T.
Consider an element of angular part dθ at angle θ. Consider another element symmetric to this centrifugal force experienced by the element = (mRdθ)ω^2R.

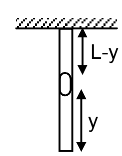


(…Length of element = Rdθ, mass = mRdθ)
Resolving into rectangular components net force on the two symmetric elements,
DF = $2mR^2\, d\theta\omega^2 \sin\theta$ [horizontal components cancels each other]
So, total F = $\int_0^{\pi/2} 2mR^2\omega^2 \sin\theta\, d\theta = 2mR^2\omega^2 [-\cos\theta] \Rightarrow 2mR^2\omega^2$
Again, $2T = 2mR^2\omega^2$ $\Rightarrow T = mR^2\omega^2$
Velocity of transverse vibration V = $\sqrt{T/m} = \omega R = V$
So, the speed of the disturbance will be V.

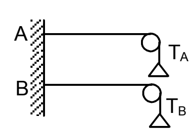
26. a) m $\rightarrow$ mass per unit of length of string
consider an element at distance 'x' from lower end.
Here wt acting down ward = (mx)g = Tension in the string of upper part
Velocity of transverse vibration = v = $\sqrt{T/m} = \sqrt{(mgx/m)} = \sqrt{(gx)}$



b) For small displacement dx, dt = dx / $\sqrt{(gx)}$
Total time T = $\int_0^L dx/\sqrt{gx} = \sqrt{(4L/g)}$



c) Suppose after time 't' from start the pulse meet the particle at distance y from lower end.
t = $\int_0^y dx/\sqrt{gx} = \sqrt{(4y/g)}$
$\therefore$ Distance travelled by the particle in this time is (L – y)



$\therefore$ S – ut + 1/2 gt^2

$\Rightarrow$ L – y (1/2)g × $\{\sqrt{(4y/g)^2}\}$ {u = 0}

$\Rightarrow$ L – y = 2y $\Rightarrow$ 3y = L

$\Rightarrow$ y = L/3. So, the particle meet at distance L/3 from lower end.

27. $m_A = 1.2 \times 10^{-2}$ kg/m, T_A = 4.8 N

$\Rightarrow V_A = \sqrt{T/m}$ = 20 m/s

$m_B = 1.2 \times 10^{-2}$ kg/m, T_B = 7.5 N

$\Rightarrow V_B = \sqrt{T/m}$ = 25 m/s

t = 0 in string A

$t_1 = 0 + 20$ ms $= 20 \times 10^{-3} = 0.02$ sec

In 0.02 sec A has travelled 20 × 0.02 = 0.4 mt

Relative speed between A and B = 25 – 20 = 5 m/s

Time taken for B for overtake A = s/v = 0.4/5 = 0.08 sec

28. r = 0.5 mm = 0.5×10^{-3} mt

f = 100 Hz, T = 100 N

v = 100 m/s

v = $\sqrt{T/m} \Rightarrow v^2 = (T/m) \Rightarrow m = (T/v^2) = 0.01$ kg/m

$P_{ave} = 2\pi^2 mvr^2f^2$

$= 2(3.14)^2(0.01) \times 100 \times (0.5 \times 10^{-3})^2 \times (100)^2 \Rightarrow 49 \times 10^{-3}$ watt = 49 mW.

29. A = 1 mm = 10^{-3} m, m = 6 g/m = 6×10^{-3} kg/m

T = 60 N, f = 200 Hz

$\therefore V = \sqrt{T/m}$ = 100 m/s

a) $P_{average} = 2\pi^2$ mv A^2f^2 = 0.47 W

b) Length of the string is 2 m. So, t = 2/100 = 0.02 sec.

Energy = $2\pi^2$ mvf^2A^2t = 9.46 mJ.

30. f = 440 Hz, m = 0.01 kg/m, T = 49 N, r = 0.5×10^{-3} m

a) v = $\sqrt{T/m}$ = 70 m/s

b) v = λf $\Rightarrow \lambda$ = v/f = 16 cm

c) $P_{average} = 2\pi^2 mvr^2f^2$ = 0.67 W.

31. Phase difference $\phi = \pi/2$

f and λ are same. So, ω is same.

y_1 = r sin wt, y_2 = rsin(wt + π/2)

From the principle of superposition

$y = y_1 + y_2$ → = r sin wt + r sin (wt + π/2)

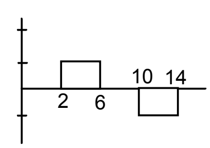
= r[sin wt + sin(wt + π/2)]

= r[2sin{(wt + wt + π/2)/2} cos {(wt – wt – π/2)/2}]

$\Rightarrow$ y = 2r sin (wt + π/4) cos (–π/4)

Resultant amplitude = $\sqrt{2}$ r = 4$\sqrt{2}$ mm (because r = 4 mm)

32. The distance travelled by the pulses are shown below.



t = 4 ms = 4×10^{-3} s s = vt = $50 \times 10 \times 4 \times 10^{-3}$ = 2 mm

t = 8 ms = 8×10^{-3} s s = vt = $50 \times 10 \times 8 \times 10^{-3}$ = 4 mm

t = 6 ms = 6×10^{-3} s s = 3 mm

t = 12 ms = 12×10^{-3} s s = $50 \times 10 \times 12 \times 10^{-3}$ = 6 mm

The shape of the string at different times are shown in the figure.

33. f = 100 Hz, λ = 2 cm = 2×10^{-2} m

$\therefore$ wave speed, v = fλ = 2 m/s

a) in 0.015 sec 1st wave has travelled

x = 0.015 × 2 = 0.03 m = path diff^n

$\therefore$ corresponding phase difference, $\phi = 2\pi x/\lambda = \{2\pi / (2 \times 10^{-2})\} \times 0.03 = 3\pi$.

b) Path different x = 4 cm = 0.04 m

$\Rightarrow \phi = (2\pi/\lambda)x = \{(2\pi/2 \times 10^{-2}) \times 0.04\} = 4\pi$.

c) The waves have same frequency, same wavelength and same amplitude.
Let, $y_1 = r \sin wt$, $y_2 = r \sin (wt + \phi)$
$\Rightarrow y = y_1 + y_2 = r[\sin wt + (wt + \phi)]$
$= 2r \sin (wt + \phi/2) \cos (\phi/2)$
$\therefore$ resultant amplitude = $2r \cos \phi/2$
So, when $\phi = 3\pi$, $r = 2 \times 10^{-3}$ m
$R_{res} = 2 \times (2 \times 10^{-3}) \cos (3\pi/2) = 0$
Again, when $\phi = 4\pi$, $R_{res} = 2 \times (2 \times 10^{-3}) \cos (4\pi/2) = 4$ mm.

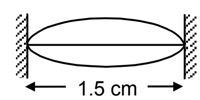
34. $l = 1$ m, $V = 60$ m/s
$\therefore$ fundamental frequency, $f_0 = V/2l = 30\ sec^{-1} = 30$ Hz.

35. $l = 2$m, $f_0 = 100$ Hz, $T = 160$ N
$f_0 = 1/2l\sqrt{(T/m)}$
$\Rightarrow m = 1$ g/m. So, the linear mass density is 1 g/m.

36. $m = (4/80)$ g/ cm $= 0.005$ kg/m
$T = 50$ N, $l = 80$ cm $= 0.8$ m
$v = \sqrt{(T/m)} = 100$ m/s
fundamental frequency $f_0 = 1/2l\sqrt{(T/m)} = 62.5$ Hz
First harmonic = 62.5 Hz
f_4 = frequency of fourth harmonic = $4f_0 = F_3 = 250$ Hz
$V = f_4 \lambda_4 \Rightarrow \lambda_4 = (v/f_4) = 40$ cm.

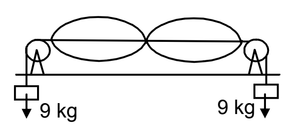
37. $l = 90$ cm $= 0.9$ m
$m = (6/90)$ g/cm $= (6/900)$ kg/mt
$f = 261.63$ Hz
$f = 1/2l\sqrt{(T/m)} \Rightarrow T = 1478.52$ N $= 1480$ N.

38. First harmonic be f_0, second harmonic be f_1
$\therefore f_1 = 2f_0$
$\Rightarrow f_0 = f_1/2$
$f_1 = 256$ Hz



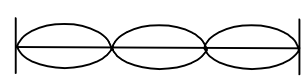
$\therefore$ 1st harmonic or fundamental frequency
$f_0 = f_1/2 = 256 / 2 = 128$ Hz
$\lambda/2 = 1.5$ m $\Rightarrow \lambda = 3$m (when fundamental wave is produced)
$\Rightarrow$ Wave speed = $V = f_0 Ql = 384$ m/s.

39. $l = 1.5$ m, mass – 12 g
$\Rightarrow m = 12/1.5$ g/m $= 8 \times 10^{-3}$ kg/m
$T = 9 \times g = 90$ N



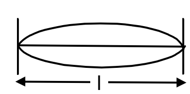
$\lambda = 1.5$ m, $f_1 = 2/2l\sqrt{T/m}$
[for, second harmonic two loops are produced]
$f_1 = 2f_0 \Rightarrow 70$ Hz.

40. A string of mass 40 g is attached to the tuning fork
$m = (40 \times 10^{-3})$ kg/m
The fork vibrates with $f = 128$ Hz



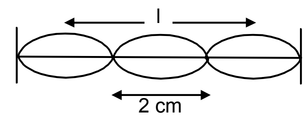
$\lambda = 0.5$ m
$v = f\lambda = 128 \times 0.5 = 64$ m/s
$v = \sqrt{T/m} \Rightarrow T = v^2 m = 163.84$ N $\Rightarrow 164$ N.

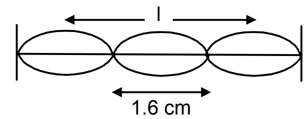
41. This wire makes a resonant frequency of 240 Hz and 320 Hz.
The fundamental frequency of the wire must be divisible by both 240 Hz and 320 Hz.



a) So, the maximum value of fundamental frequency is 80 Hz.
b) Wave speed, $v = 40$ m/s
$\Rightarrow 80 = (1/2l) \times 40 \Rightarrow 0.25$ m.

42. Let there be 'n' loops in the 1st case





$\Rightarrow$ length of the wire, $l = (n\lambda_1)/2$ $[\lambda_1 = 2 \times 2 = 4 \text{ cm}]$

So there are (n + 1) loops with the 2nd case

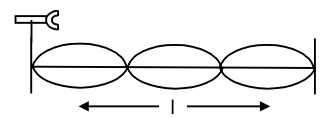
$\Rightarrow$ length of the wire, $l = \{(n+1)\lambda_2/2$ $[\lambda = 2 \times 1.6 = 3.2 \text{ cm}]$

$\Rightarrow n\lambda_1/2 = \dfrac{(n+1)\lambda_2}{2}$

$\Rightarrow n \times 4 = (n + 1)(3.2) \Rightarrow n = 4$

$\therefore$ length of the string, $l = (n\lambda_1)/2 = 8$ cm.

43. Frequency of the tuning fork, f = 660 Hz



Wave speed, $v = 220$ m/s $\Rightarrow \lambda = v/f = 1/3$ m

No.of loops = 3

a) So, $f = (3/2l)v \Rightarrow l = 50$ cm

b) The equation of resultant stationary wave is given by

$y = 2A \cos(2\pi x/Ql) \sin(2\pi vt/\lambda)$

$\Rightarrow y = (0.5 \text{ cm}) \cos(0.06\,\pi \text{ cm}^{-1}) \sin(1320\,\pi s^{-1} t)$

44. $l_1 = 30$ cm = 0.3 m

$f_1 = 196$ Hz, $f_2 = 220$ Hz

We know $f \propto (1/l)$ (as V is constant for a medium)

$\Rightarrow \dfrac{f_1}{f_2} = \dfrac{l_2}{l_1} \Rightarrow l_2 = 26.7$ cm

Again $f_3 = 247$ Hz

$\Rightarrow \dfrac{f_3}{f_1} = \dfrac{l_1}{l_3} \Rightarrow \dfrac{0.3}{l_3}$

$\Rightarrow l_3 = 0.224$ m = 22.4 cm and $l_3 = 20$ cm

45. Fundamental frequency $f_1 = 200$ Hz

Let l_4 Hz be nth harmonic

$\Rightarrow F_2/F_1 = 14000/200$

$\Rightarrow NF_1/F_1 = 70 \Rightarrow N = 70$

$\therefore$ The highest harmonic audible is 70th harmonic.

46. The resonant frequencies of a string are

$f_1 = 90$ Hz, $f_2 = 150$ Hz, $f_3 = 120$ Hz

a) The highest possible fundamental frequency of the string is f = 30 Hz
[because f_1, f_2 and f_3 are integral multiple of 30 Hz]

b) The frequencies are $f_1 = 3f$, $f_2 = 5f$, $f_3 = 7f$
So, f_1, f_2 and f_3 are 3rd harmonic, 5th harmonic and 7th harmonic respectively.

c) The frequencies in the string are f, 2f, 3f, 4f, 5f, ……….
So, 3f = 2nd overtone and 3rd harmonic
5f = 4th overtone and 5th harmonic
7f = 6th overtone and 7th harmonic

d) length of the string is $l = 80$ cm
$\Rightarrow f_1 = (3/2l)v$ (v = velocity of the wave)
$\Rightarrow 90 = \{3/(2\times 80)\} \times K$
$\Rightarrow K = (90 \times 2 \times 80) / 3 = 4800$ cm/s = 48 m/s.

47. Frequency $f = \dfrac{1}{lD}\sqrt{\dfrac{T}{\pi\rho}} \Rightarrow f_1 = \dfrac{1}{l_1 D_1}\sqrt{\dfrac{T_1}{\pi\rho_1}} \Rightarrow f_2 = \dfrac{1}{l_2 n_2}\sqrt{\dfrac{T_2}{\pi\rho_2}}$

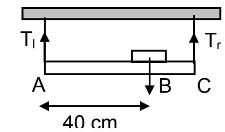
Given that, $T_1/T_2 = 2$, $r_1 / r_2 = 3 = D_1/D_2$

$\dfrac{\rho_1}{\rho_2} = \dfrac{1}{2}$

So, $\dfrac{f_1}{f_2} = \dfrac{l_2 D_2}{l_1 D_1}\sqrt{\dfrac{T_1}{T_2}}\sqrt{\dfrac{\pi\rho_2}{\pi\rho_1}}$ ($l_1 = l_2$ = length of string)

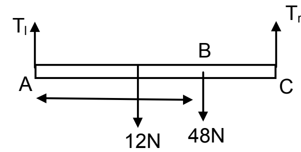
$\Rightarrow f_1 : f_2 = 2 : 3$

48. Length of the rod = L = 40 cm = 0.4 m
Mass of the rod m = 1.2 kg
Let the 4.8 kg mass be placed at a distance 'x' from the left end.
Given that, $f_l = 2f_r$



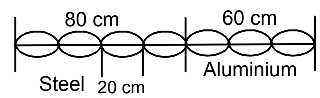
$\therefore \frac{1}{2l}\sqrt{\frac{T_l}{m}} = \frac{2}{2l}\sqrt{\frac{T_r}{m}}$

$\Rightarrow \sqrt{\frac{T_l}{T_r}} = 2 \Rightarrow \frac{T_l}{T_r} = 4$ …(1)



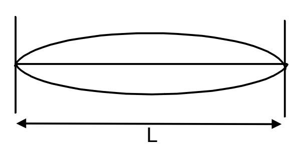
From the freebody diagram,
$T_l + T_r = 60$ N
$\Rightarrow 4T_r + T_r = 60$ N
$\therefore T_r = 12$ N and $T_l = 48$ N
Now taking moment about point A,
$T_r \times (0.4) = 48x + 12\,(0.2) \Rightarrow x = 5$ cm
So, the mass should be placed at a distance 5 cm from the left end.

49. $\rho_s = 7.8$ g/cm^3 , $\rho_A = 2.6$ g/cm^3
$m_s = \rho_s A_s = 7.8 \times 10^{-2}$ g/cm (m = mass per unit length)
$m_A = \rho_A A_A = 2.6 \times 10^{-2} \times 3$ g/cm $= 7.8 \times 10^{-3}$ kg/m



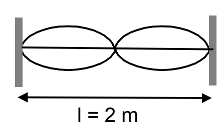
A node is always placed in the joint. Since aluminium and steel rod has same mass per unit length, velocity of wave in both of them is same.
$\Rightarrow v = \sqrt{T/m} \Rightarrow 500/7$ m/x
For minimum frequency there would be maximum wavelength for maximum wavelength minimum no of loops are to be produced.
$\therefore$ maximum distance of a loop = 20 cm
$\Rightarrow$ wavelength = $\lambda = 2 \times 20 = 40$ cm = 0.4 m
$\therefore f = v/\lambda = 180$ Hz.

50. Fundamental frequency
$V = 1/2l\ \sqrt{T/m} \Rightarrow \sqrt{T/m} = v2l$ [$\sqrt{T/m}$ = velocity of wave]



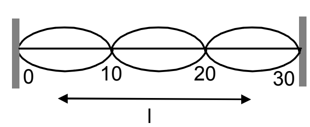
a) wavelength, λ = velocity / frequency = v2l / v = 2l
and wave number = $K = 2\pi/\lambda = 2\pi/2l = \pi/l$
b) Therefore, equation of the stationary wave is
$y = A \cos(2\pi x/\lambda) \sin(2\pi Vt / L)$
$= A \cos(2\pi x / 2l) \sin(2\pi Vt / 2L)$
$v = V/2L$ [because $v = (v/2l)$]

51. V = 200 m/s, 2A = 0.5 m
a) The string is vibrating in its 1st overtone
$\Rightarrow \lambda = 1 = 2$m
$\Rightarrow f = v/\lambda = 100$ Hz



b) The stationary wave equation is given by
$y = 2A\ \cos\frac{2\pi x}{\lambda}\sin\frac{2\pi Vt}{\lambda}$
$= (0.5\text{ cm}) \cos[(\pi m^{-1})x] \sin[(200\ \pi s^{-1})t]$

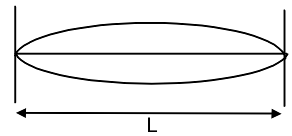
52. The stationary wave equation is given by
$y = (0.4\text{ cm}) \sin[(0.314\text{ cm} - 1)x] \cos[(6.00\ \pi s^{-1})t]$
a) $\omega = 600\ \pi \Rightarrow 2\pi f = 600\ \pi \Rightarrow f = 300$ Hz
wavelength, $\lambda = 2\pi/0.314 = (2 \times 3.14) / 0.314 = 20$ cm



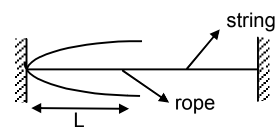
b) Therefore nodes are located at, 0, 10 cm, 20 cm, 30 cm
c) Length of the string = $3\lambda/2 = 3 \times 20/2 = 30$ cm
d) $y = 0.4 \sin(0.314\ x) \cos(600\ \pi t) \Rightarrow 0.4 \sin\{(\pi/10)x\} \cos(600\ \pi t)$
since, λ and v are the wavelength and velocity of the waves that interfere to give this vibration λ = 20 cm

$v = \omega/k = 6000$ cm/sec = 60 m/s

53. The equation of the standing wave is given by
$y = (0.4 \text{ cm}) \sin [(0.314 \text{ cm}^{-1})x] \cos [(6.00 \pi s^{-1})t]$
$\Rightarrow k = 0.314 = \pi/10$
$\Rightarrow 2\pi/\lambda = \pi/10 \Rightarrow \lambda = 20$ cm
for smallest length of the string, as wavelength remains constant, the string should vibrate in fundamental frequency
$\Rightarrow l = \lambda/2 = 20$ cm / 2 = 10 cm



54. L = 40 cm = 0.4 m, mass = 3.2 kg = 3.2×10^{-3} kg
$\therefore$ mass per unit length, m = (3.2)/(0.4) = 8×10^{-3} kg/m
change in length, $\Delta L = 40.05 - 40 = 0.05 \times 10^{-2}$ m
strain = $\Delta L/L = 0.125 \times 10^{-2}$ m
f = 220 Hz



$$f = \frac{1}{2l'}\sqrt{\frac{T}{m}} = \frac{1}{2 \times (0.4005)}\sqrt{\frac{T}{8 \times 10^{-3}}} \Rightarrow T = 248.19 \text{ N}$$

Strain = 248.19/1 mm^2 = 248.19×10^6
Y = stress / strain = 1.985×10^{11} N/m^2

55. Let, $\rho \rightarrow$ density of the block
Weight ρ Vg where V = volume of block
The same turning fork resonates with the string in the two cases

$$f_{10} = \frac{10}{2l}\sqrt{\frac{T - \rho_w Vg}{m}} = \frac{11}{2l}\sqrt{\frac{(\rho - \rho_w)Vg}{m}}$$

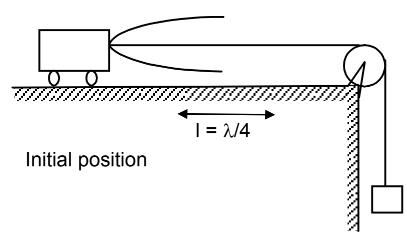
As the f of tuning fork is same,

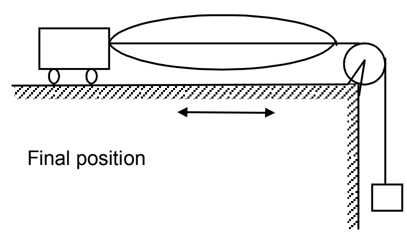
$$f_{10} = f_{11} \Rightarrow \frac{10}{2l}\sqrt{\frac{\rho Vg}{m}} = \frac{11}{2l}\sqrt{\frac{(\rho - \rho_w)Vg}{m}}$$

$$\Rightarrow \frac{10}{11} = \sqrt{\frac{\rho - \rho_w}{m}} \Rightarrow \frac{\rho - 1}{\rho} = \frac{100}{121} \qquad \text{(because, } \rho_w = 1 \text{ gm/cc)}$$

$\Rightarrow 100\rho = 121\rho - 121 \Rightarrow 5.8 \times 10^3$ kg/m^3

56. l = length of rope = 2 m
M = mass = 80 gm = 0.8 kg
mass per unit length = m = 0.08/2 = 0.04 kg/m
Tension T = 256 N
Velocity, V = $\sqrt{T/m}$ = 80 m/s
For fundamental frequency,
$l = \lambda/4 \Rightarrow \lambda = 4l = 8$ m
$\Rightarrow f = 80/8 = 10$ Hz





a) Therefore, the frequency of 1st two overtones are
1st overtone = 3f = 30 Hz
2nd overtone = 5f = 50 Hz
b) $\lambda_1 = 4l = 8$ m
$\lambda_1 = V/f_1 = 2.67$ m
$\lambda_2 = V/f_2 = 1.6$ mt
so, the wavelengths are 8 m, 2.67 m and 1.6 m respectively.

57. Initially because the end A is free, an antinode will be formed.
So, $l = Ql_1 / 4$
Again, if the movable support is pushed to right by 10 m, so that the joint is placed on the pulley, a node will be formed there.
So, $l = \lambda_2 / 2$
Since, the tension remains same in both the cases, velocity remains same.
As the wavelength is reduced by half, the frequency will become twice as that of 120 Hz i.e. 240 Hz.

♠ ♠ ♠

SOLUTIONS TO CONCEPTS
CHAPTER – 16

1. V_{air}= 230 m/s. V_s = 5200 m/s. Here S = 7 m

   So, $t = t_1 - t_2 = \left(\frac{1}{330} - \frac{1}{5200}\right)$ = 2.75×10^{-3} sec = 2.75 ms.

2. Here given S = 80 m × 2 = 160 m.

   v = 320 m/s

   So the maximum time interval will be

   t = 5/v = 160/320 = 0.5 seconds.

3. He has to clap 10 times in 3 seconds.

   So time interval between two clap = (3/10 second).

   So the time taken go the wall = (3/2 × 10) = 3/20 seconds.

   = 333 m/s.

4. a) for maximum wavelength n = 20 Hz.

   as $\left(\eta \propto \frac{1}{\lambda}\right)$

   b) for minimum wavelength, n = 20 kHz

   $\therefore \lambda = 360/(20 \times 10^3) = 18 \times 10^{-3}$ m = 18 mm

   $\Rightarrow$ x = (v/n) = 360/20 = 18 m.

5. a) for minimum wavelength n = 20 KHz

   $\Rightarrow v = n\lambda \Rightarrow \lambda = \left(\frac{1450}{20 \times 10^3}\right)$ = 7.25 cm.

   b) for maximum wavelength n should be minium

   $\Rightarrow v = n\lambda \Rightarrow \lambda = v/n \Rightarrow 1450 / 20 = 72.5$ m.

6. According to the question,

   a) λ = 20 cm × 10 = 200 cm = 2 m

   v = 340 m/s

   so, n = v/λ = 340/2 = 170 Hz.

   $N = v/\lambda \Rightarrow \frac{340}{2 \times 10^{-2}}$ = 17.000 Hz = 17 KH_2 (because λ = 2 cm = 2×10^{-2} m)

7. a) Given V_{air} = 340 m/s , n = 4.5×10^6 Hz

   $\Rightarrow \lambda_{air} = (340 / 4.5) \times 10^{-6} = 7.36 \times 10^{-5}$ m.

   b) V_{tissue} = 1500 m/s $\Rightarrow \lambda_t = (1500 / 4.5) \times 10^{-6} = 3.3 \times 10^{-4}$ m.

8. Here given $r_y = 6.0 \times 10^{-5}$ m

   a) Given $2\pi/\lambda = 1.8 \Rightarrow \lambda = (2\pi/1.8)$

   So, $\frac{r_y}{\lambda} = \frac{6.0 \times (1.8) \times 10^{-5} m/s}{2\pi}$ = 1.7×10^{-5} m

   b) Let, velocity amplitude = V_y

   V = dy/dt = 3600 cos (600 t – 1.8) × 10^{-5} m/s

   Here $V_y = 3600 \times 10^{-5}$ m/s

   Again, $\lambda = 2\pi/1.8$ and $T = 2\pi/600 \Rightarrow$ wave speed = v = λ/T = 600/1.8 = 1000 / 3 m/s.

   So the ratio of $(V_y/v) = \frac{3600 \times 3 \times 10^{-5}}{1000}$.

9. a) Here given n = 100, v = 350 m/s

   $\Rightarrow \lambda = \frac{v}{n} = \frac{350}{100}$ = 3.5 m.

   In 2.5 ms, the distance travelled by the particle is given by

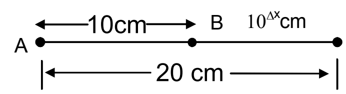
   $\Delta x = 350 \times 2.5 \times 10^{-3}$

So, phase difference $\phi = \frac{2\pi}{\lambda} \times \Delta x \Rightarrow \frac{2\pi}{(350/100)} \times 350 \times 2.5 \times 10^{-3} = (\pi/2)$.

b) In the second case, Given $\Delta\eta$ = 10 cm = 10^{-1} m

So, $\phi = \frac{2\pi}{x} \Delta x = \frac{2\pi \times 10^{-1}}{(350/100)} = 2\pi/35$.

10. a) Given Δx = 10 cm, λ = 5.0 cm



$\Rightarrow \phi = \frac{2\pi}{\lambda} \times \Delta\eta = \frac{2\pi}{5} \times 10 = 4\pi$.

So phase difference is zero.

b) Zero, as the particle is in same phase because of having same path.

11. Given that p = 1.0 × 10^5 N/m^2, T = 273 K, M = 32 g = 32 × 10^{-3} kg

V = 22.4 litre = 22.4 × 10^{-3} m^3

C/C_v = r = 3.5 R / 2.5 R = 1.4

$\Rightarrow V = \sqrt{\frac{rp}{f}} = \sqrt{\frac{1.4 \times 1.0 \times 10^{-5}}{32/22.4}}$ = 310 m/s (because ρ = m/v)

12. V_1 = 330 m/s, V_2 = ?

T_1 = 273 + 17 = 290 K, T_2 = 272 + 32 = 305 K

We know $v \propto \sqrt{T}$

$\frac{\sqrt{V_1}}{\sqrt{V_2}} = \frac{\sqrt{T_1}}{\sqrt{T_2}} \Rightarrow V_2 = \frac{V_1 \times \sqrt{T_2}}{\sqrt{T_1}}$

$= 340 \times \sqrt{\frac{305}{290}}$ = 349 m/s.

13. T_1 = 273 $V_2 = 2V_1$

V_1 = v T_2 = ?

We know that $V \propto \sqrt{T} \Rightarrow \frac{T_2}{T_1} = \frac{V_2^2}{V_1^2} \Rightarrow T_2 = 273 \times 2^2 = 4 \times 273$ K

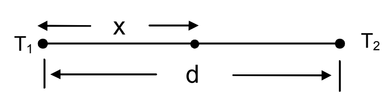
So temperature will be (4 × 273) – 273 = 819°c.

14. The variation of temperature is given by

$T = T_1 + \frac{(T_2 - T_2)}{d} x$ …(1)

We know that $V \propto \sqrt{T} \Rightarrow \frac{V_T}{V} = \sqrt{\frac{T}{273}} \Rightarrow VT = v\sqrt{\frac{T}{273}}$

$\Rightarrow dt = \frac{dx}{V_T} = \frac{du}{V} \times \sqrt{\frac{273}{T}}$



$\Rightarrow t = \frac{273}{V} \int_0^d \frac{dx}{[T_1 + (T_2 - T_1)/d)x]^{1/2}}$

$= \frac{\sqrt{273}}{V} \times \frac{2d}{T_2 - T_1} [T_1 + \frac{T_2 - T_1}{d} x]_0^d = \left(\frac{2d}{V}\right)\left(\frac{\sqrt{273}}{T_2 - T_1}\right) \times \sqrt{T_2} - \sqrt{T_1}$

$= T = \frac{2d}{V} \frac{\sqrt{273}}{\sqrt{T_2} + \sqrt{T_1}}$

Putting the given value we get

$= \frac{2 \times 33}{330} = \frac{\sqrt{273}}{\sqrt{280} + \sqrt{310}}$ = 96 ms.

15. We know that v = $\sqrt{K/\rho}$

Where K = bulk modulus of elasticity

$\Rightarrow$ K = v^2 ρ = $(1330)^2$ × 800 N/m^2

We know K = $\left(\frac{F/A}{\Delta V/V}\right)$

$\Rightarrow \Delta V = \frac{\text{Pressures}}{K} = \frac{2\times10^5}{1330\times1330\times800}$

So, ΔV = 0.15 cm^3

16. We know that,

Bulk modulus B = $\frac{\Delta p}{(\Delta V/V)} = \frac{P_0\lambda}{2\pi S_0}$

Where P_0 = pressure amplitude $\Rightarrow$ P_0 = 1.0 × 10^5

S_0 = displacement amplitude $\Rightarrow$ S_0 = 5.5 × 10^{-6} m

$\Rightarrow$ B = $\frac{14\times35\times10^{-2}m}{2\pi(5.5)\times10^{-6}m}$ = 1.4 × 10^5 N/m^2.

17. a) Here given V_{air} = 340 m/s., Power = E/t = 20 W

f = 2,000 Hz, ρ = 1.2 kg/m^3

So, intensity I = E/t.A

= $\frac{20}{4\pi r^2} = \frac{20}{4\times\pi\times6^2} = 44$ mw/m^2 (because r = 6m)

b) We know that I = $\frac{P_0^2}{2\rho V_{air}} \Rightarrow P_0 = \sqrt{1\times2\rho V_{air}}$

= $\sqrt{2\times1.2\times340\times44\times10^{-3}}$ = 6.0 N/m^2.

c) We know that I = $2\pi^2S_0^2v^2\rho V$ where S_0 = displacement amplitude

$\Rightarrow$ S_0 = $\sqrt{\frac{I}{\pi^2\rho^2\rho V_{air}}}$

Putting the value we get S_g = 1.2 × 10^{-6} m.

18. Here I_1 = 1.0 × 10^{-8} W_1/m^2 ; I_2 = ?

r_1 = 5.0 m, r_2 = 25 m.

We know that I $\propto \frac{1}{r^2}$

$\Rightarrow I_1r_1^2 = I_2r_2^2 \Rightarrow I_2 = \frac{I_1r_1^2}{r_2^2}$

= $\frac{1.0\times10^{-8}\times25}{625}$ = 4.0 × 10^{-10} W/m^2.

19. We know that β = 10 $\log_{10}\left(\frac{I}{I_0}\right)$

$\beta_A = 10\log\frac{I_A}{I_o}$, $\beta_B = 10\log\frac{I_B}{I_o}$

$\Rightarrow I_A/I_0 = 10^{(\beta_A/10)} \Rightarrow I_B/I_o = 10^{(\beta_B/10)}$

$\Rightarrow \frac{I_A}{I_B} = \frac{r_B^2}{r_A^2} = \left(\frac{50}{5}\right)^2 \Rightarrow 10^{(\beta_A\beta_B)} = 10^2$

$\Rightarrow \frac{\beta_A-\beta_B}{10} = 2 \Rightarrow \beta_A-\beta_B = 20$

$\Rightarrow \beta_B$ = 40 – 20 = 20 dβ.

20. We know that, $\beta = 10 \log_{10} J/I_0$
According to the questions
$\beta_A = 10 \log_{10} (2I/I_0)$
$\Rightarrow \beta_B - \beta_A = 10 \log (2I/I) = 10 \times 0.3010 = 3$ dB.

21. If sound level = 120 dB, then I = intensity = 1 W/m^2
Given that, audio output = 2W
Let the closest distance be x.
So, intensity = $(2 / 4\pi x^2) = 1 \Rightarrow x^2 = (2/2\pi) \Rightarrow x = 0.4$ m = 40 cm.

22. $\beta_1 = 50$ dB, $\beta_2 = 60$ dB
$\therefore I_1 = 10^{-7}$ W/m^2, $I_2 = 10^{-6}$ W/m^2
(because $\beta = 10 \log_{10} (I/I_0)$, where $I_0 = 10^{-12}$ W/m^2)
Again, $I_2/I_1 = (p_2/p_1)^2 = (10^{-6}/10^{-7}) = 10$ (where p = pressure amplitude).
$\therefore (p_2 / p_1) = \sqrt{10}$.

23. Let the intensity of each student be I.
According to the question

$$\beta_A = 10\log_{10}\frac{50\,I}{I_0}\;;\; \beta_B = 10\log_{10}\left(\frac{100\,I}{I_0}\right)$$

$$\Rightarrow \beta_B - \beta_A = 10\log_{10}\frac{50\,I}{I_0} - 10\log_{10}\left(\frac{100\,I}{I_0}\right)$$

$$= 10\log\left(\frac{100\,I}{50\,I}\right) = 10\log_{10} 2 = 3$$

So, $\beta_A = 50 + 3 = 53$ dB.

24. Distance between tow maximum to a minimum is given by, $\lambda/4 = 2.50$ cm
$\Rightarrow \lambda = 10$ cm = 10^{-1} m
We know, V = nx

$$\Rightarrow n = \frac{V}{\lambda} = \frac{340}{10^{-1}} = 3400 \text{ Hz} = 3.4 \text{ kHz}.$$

25. a) According to the data
$\lambda/4 = 16.5$ mm $\Rightarrow \lambda = 66$ mm = $66 \times 10^{-6=3}$ m

$$\Rightarrow n = \frac{V}{\lambda} = \frac{330}{66 \times 10^{-3}} = 5 \text{ kHz}.$$

b) $I_{minimum} = K(A_1 - A_2)^2 = I \Rightarrow A_1 - A_2 = 11$
$I_{maximum} = K(A_1 + A_2)^2 = 9 \Rightarrow A_1 + A_2 = 31$

$$\text{So, } \frac{A_1 + A_2}{A_1 + A_2} = \frac{3}{4} \Rightarrow A_1/A_2 = 2/1$$

So, the ratio amplitudes is 2.

26. The path difference of the two sound waves is given by
$\Delta L = 6.4 - 6.0 = 0.4$ m

The wavelength of either wave = $\lambda = \frac{V}{\rho} = \frac{320}{\rho}$ (m/s)

For destructive interference $\Delta L = \frac{(2n+1)\lambda}{2}$ where n is an integers.

$$\text{or } 0.4 \text{ m} = \frac{2n+1}{2} \times \frac{320}{\rho}$$

$$\Rightarrow \rho = n = \frac{320}{0.4} = 800\frac{2n+1}{2}\text{Hz} = (2n + 1)\ 400 \text{ Hz}$$

Thus the frequency within the specified range which cause destructive interference are 1200 Hz, 2000 Hz, 2800 Hz, 3600 Hz and 4400 Hz.

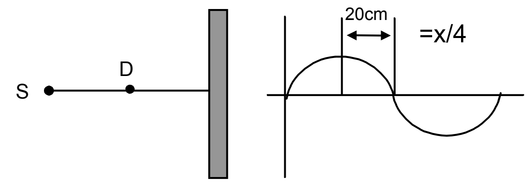
27. According to the given data

$V = 336$ m/s,

$\lambda/4$ = distance between maximum and minimum intensity

$= (20 \text{ cm}) \Rightarrow \lambda = 80$ cm

$\Rightarrow n = \text{frequency} = \dfrac{V}{\lambda} = \dfrac{336}{80 \times 10^{-2}} = 420$ Hz.



28. Here given $\lambda = d/2$

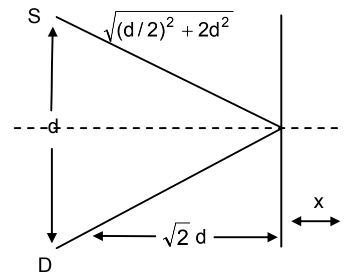
Initial path difference is given by $= 2\sqrt{\left(\dfrac{d}{2}\right)^2 + 2d^2} - d$

If it is now shifted a distance x then path difference will be

$= 2\sqrt{\left(\dfrac{d}{2}\right)^2 + (\sqrt{2}d + x)^2} - d = \dfrac{d}{4}\left(2d + \dfrac{d}{4}\right)$

$\Rightarrow \left(\dfrac{d}{2}\right)^2 + (\sqrt{2}d + x)^2 = \dfrac{169d^2}{64} \Rightarrow \dfrac{153}{64}d^2$

$\Rightarrow \sqrt{2}d + x = 1.54$ d $\Rightarrow$ x = 1.54 d – 1.414 d = 0.13 d.



29. As shown in the figure the path differences $2.4 = \Delta x = \sqrt{(3.2)^2 + (2.4)^2} - 3.2$

Again, the wavelength of the either sound waves $= \dfrac{320}{\rho}$

We know, destructive interference will be occur

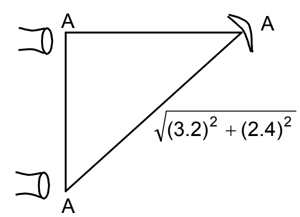
If $\Delta x = \dfrac{(2n+1)\lambda}{2}$

$\Rightarrow \sqrt{(3.2)^2 + (2.4)^2 - (3.2)} = \dfrac{(2n+1)}{2}\dfrac{320}{\rho}$

Solving we get

$\Rightarrow V = \dfrac{(2n+1)400}{2} = 200(2n+1)$

where n = 1, 2, 3, …… 49. (audible region)



30. According to the data

$\lambda = 20$ cm, $S_1S_2 = 20$ cm, BD = 20 cm

Let the detector is shifted to left for a distance x for hearing the minimum sound.

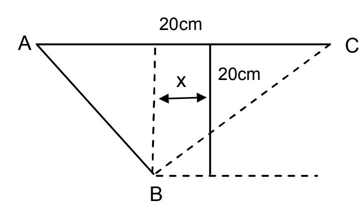
So path difference AI = BC – AB

$= \sqrt{(20)^2 + (10+x)^2} - \sqrt{(20)^2 + (10-x)^2}$

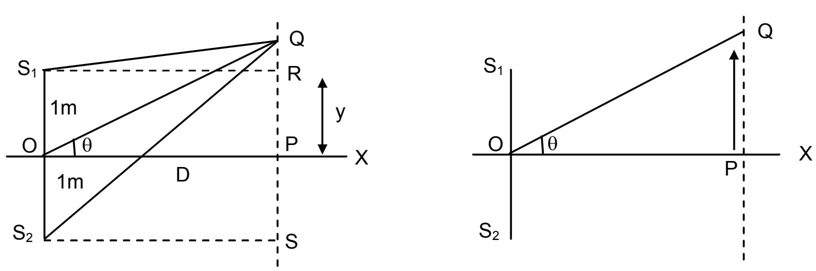
So the minimum distances hearing for minimum

$= \dfrac{(2n+1)\lambda}{2} = \dfrac{\lambda}{2} = \dfrac{20}{2} = 10$ cm

$\Rightarrow \sqrt{(20)^2 + (10+x)^2} - \sqrt{(20)^2 + (10-x)^2} = 10$ solving we get x = 12.0 cm.



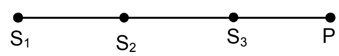
31.

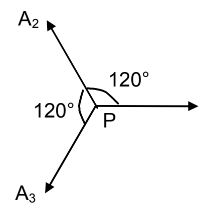


Given, F = 600 Hz, and v = 330 m/s $\Rightarrow \lambda = v/f = 330/600 = 0.55$ mm

Let OP = D, PQ = y $\Rightarrow \theta = y/R$ …(1)
Now path difference is given by, $x = S_2Q - S_1Q = yd/D$
Where d = 2m
[The proof of x = yd/D is discussed in interference of light waves]

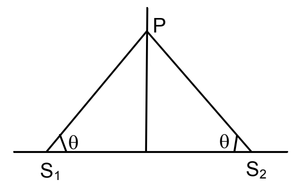
a) For minimum intensity, $x = (2n + 1)(\lambda/2)$
   $\therefore$ $yd/D = \lambda/2$ [for minimum y, $x = \lambda/2$]
   $\therefore$ $y/D = \theta = \lambda/2 = 0.55 / 4 = 0.1375$ rad $= 0.1375 \times (57.1)° = 7.9°$
b) For minimum intensity, $x = 2n(\lambda/2)$
   $yd/D = \lambda \Rightarrow y/D = \theta = \lambda/D = 0.55/2 = 0.275$ rad
   $\therefore$ $\theta = 16°$
c) For more maxima,
   $yd/D = 2\lambda, 3\lambda, 4\lambda, \ldots$
   $\Rightarrow y/D = \theta = 32°, 64°, 128°$
   But since, the maximum value of θ can be 90°, he will hear two more maximum i.e. at 32° and 64°.

32. 



Because the 3 sources have equal intensity, amplitude are equal
So, $A_1 = A_2 = A_3$
As shown in the figure, amplitude of the resultant = 0 (vector method)
So, the resultant, intensity at B is zero.

33. The two sources of sound S_1 and S_2 vibrate at same phase and frequency.
Resultant intensity at P = I_0



a) Let the amplitude of the waves at S_1 and S_2 be 'r'.
   When $\theta = 45°$, path difference $= S_1P - S_2P = 0$ (because $S_1P = S_2P$)
   So, when source is switched off, intensity of sound at P is $I_0/4$.
b) When $\theta = 60°$, path difference is also 0.
   Similarly it can be proved that, the intensity at P is $I_0 / 4$ when one is switched off.

34. If V = 340 m/s, I = 20 cm = 20×10^{-2} m
Fundamental frequency = $\dfrac{V}{21} = \dfrac{340}{2 \times 20 \times 10^{-2}}$ = 850 Hz
We know first over tone = $\dfrac{2V}{21} = \dfrac{2 \times 340}{2 \times 20 \times 10^{-2}}$ (for open pipe) = 1750 Hz
Second over tone = 3 (V/21) = 3 × 850 = 2500 Hz.

35. According to the questions V = 340 m/s, n = 500 Hz
We know that V/4I (for closed pipe)
$\Rightarrow$ I = $\dfrac{340}{4 \times 500}$ m = 17 cm.

36. Here given distance between two nodes is = 4. 0 cm,
$\Rightarrow \lambda = 2 \times 4.0 = 8$ cm
We know that $v = n\lambda$
$\Rightarrow \eta = \dfrac{328}{8 \times 10^{-2}}$ = 4.1 Hz.

37. V = 340 m/s
Distances between two nodes or antinodes
$\Rightarrow \lambda/4 = 25$ cm
$\Rightarrow \lambda = 100$ cm = 1 m
$\Rightarrow n = v/\lambda = 340$ Hz.

38. Here given that 1 = 50 cm, v = 340 m/s
As it is an open organ pipe, the fundamental frequency $f_1 = (v/21)$
$= \dfrac{340}{2 \times 50 \times 10^{-2}}$ = 340 Hz.

So, the harmonies are

$f_3 = 3 \times 340 = 1020$ Hz

$f_5 = 5 \times 340 = 1700$, $f_6 = 6 \times 340 = 2040$ Hz

so, the possible frequencies are between 1000 Hz and 2000 Hz are 1020, 1360, 1700.

39. Here given $l_2 = 0.67$ m, $l_1 = 0.2$ m, f = 400 Hz

We know that

$\lambda = 2(l_2 - l_1) \Rightarrow \lambda = 2(62 - 20) = 84$ cm = 0.84 m.

So, $v = n\lambda = 0.84 \times 400 = 336$ m/s

We know from above that,

$l_1 + d = \lambda/4 \Rightarrow d = \lambda/4 - l_1 = 21 - 20 = 1$ cm.

40. According to the questions

f_1 first overtone of a closed organ pipe $P_1 = 3v/4l = \frac{3 \times V}{4 \times 30}$

f_2 fundamental frequency of a open organ pipe $P_2 = \frac{V}{2l_2}$

Here given $\frac{3V}{4 \times 30} = \frac{V}{2l_2} \Rightarrow l_2 = 20$ cm

$\therefore$ length of the pipe P_2 will be 20 cm.

41. Length of the wire = 1.0 m

For fundamental frequency $\lambda/2 = l$

$\Rightarrow \lambda = 2l = 2 \times 1 = 2$ m

Here given n = 3.8 km/s = 3800 m/s

We know $\Rightarrow v = n\lambda \Rightarrow n = 3800 / 2 = 1.9$ kH.

So standing frequency between 20 Hz and 20 kHz which will be heard are

$= n \times 1.9$ kHz where n = 0, 1, 2, 3, … 10.

42. Let the length will be l.

Here given that V = 340 m/s and n = 20 Hz

Here $\lambda/2 = l \Rightarrow \lambda = 2l$

We know $V = n\lambda \Rightarrow l = \frac{V}{n} = \frac{340}{2 \times 20} = \frac{34}{4} = 8.5$ cm (for maximum wavelength, the frequency is minimum).

43. a) Here given l = 5 cm = 5×10^{-2} m, v = 340 m/s

$\Rightarrow$ n = $\frac{V}{2l} = \frac{340}{2 \times 5 \times 10^{-2}}$ = 3.4 KHz

b) If the fundamental frequency = 3.4 KHz

$\Rightarrow$ then the highest harmonic in the audible range (20 Hz – 20 KHz)

$= \frac{20000}{3400}$ = 5.8 = 5 (integral multiple of 3.4 KHz).

44. The resonance column apparatus is equivalent to a closed organ pipe.

Here l = 80 cm = 10×10^{-2} m ; v = 320 m/s

$\Rightarrow n_0 = v/4l = \frac{320}{4 \times 50 \times 10^{-2}}$ = 100 Hz

So the frequency of the other harmonics are odd multiple of n_0 = (2n + 1) 100 Hz

According to the question, the harmonic should be between 20 Hz and 2 KHz.

45. Let the length of the resonating column will be = 1

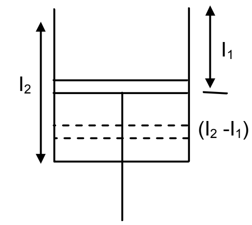
Here V = 320 m/s

Then the two successive resonance frequencies are $\frac{(n+1)v}{4\,l}$ and $\frac{nv}{4\,l}$

Here given $\frac{(n+1)v}{4\,l}$ = 2592 ; $\lambda = \frac{nv}{4\,l}$ = 1944

$\Rightarrow \frac{(n+1)v}{4\,l} - \frac{nv}{4\,l}$ = 2592 – 1944 = 548 cm = 25 cm.

46. Let, the piston resonates at length l_1 and l_2
Here, l = 32 cm; v = ?, n = 512 Hz
Now ⇒ 512 = v/λ
⇒ v = 512 × 0.64 = 328 m/s.



47. Let the length of the longer tube be L_2 and smaller will be L_1.
According to the data $440 = \frac{3 \times 330}{4 \times L_2}$ …(1) (first over tone)
and $440 = \frac{330}{4 \times L_1}$ …(2) (fundamental)
solving equation we get L_2 = 56.3 cm and L_1 = 18.8 cm.

48. Let n_0 = frequency of the turning fork, T = tension of the string
L = 40 cm = 0.4 m, m = 4g = 4×10^{-3} kg
So, m = Mass/Unit length = 10^{-2} kg/m
$n_0 = \frac{1}{2l}\sqrt{\frac{T}{m}}$.
So, 2^{nd} harmonic $2n_0 = (2/2l)\sqrt{T/m}$
As it is unison with fundamental frequency of vibration in the air column
$\Rightarrow 2n_0 = \frac{340}{4 \times 1} = 85$ Hz
$\Rightarrow 85 = \frac{2}{2 \times 0.4}\sqrt{\frac{T}{14}} \Rightarrow T = 85^2 \times (0.4)^2 \times 10^{-2} = 11.6$ Newton.

49. Given, m = 10 g = 10×10^{-3} kg, l = 30 cm = 0.3 m
Let the tension in the string will be = T
μ = mass / unit length = 33×10^{-3} kg
The fundamental frequency $\Rightarrow n_0 = \frac{1}{2l}\sqrt{\frac{T}{\mu}}$ …(1)
The fundamental frequency of closed pipe
$\Rightarrow n_0 = (v/4l)\frac{340}{4 \times 50 \times 10^2} = 170$ Hz …(2)
According equations (1) × (2) we get
$170 = \frac{1}{2 \times 30 \times 10^{-2}} \times \sqrt{\frac{T}{33 \times 10^{-3}}}$
⇒ T = 347 Newton.

50. We know that $f \propto \sqrt{T}$
According to the question $f + \Delta f \propto \sqrt{\Delta T} + T$
$\Rightarrow \frac{f + \Delta f}{f} = \sqrt{\frac{\Delta t + T}{T}} \Rightarrow 1 + \frac{\Delta f}{f} = \left(1 + \frac{\Delta T}{T}\right)^{1/2} = 1 + \frac{1}{2}\frac{\Delta T}{T} + \ldots$ (neglecting other terms)
$\Rightarrow \frac{\Delta f}{f} = (1/2)\frac{\Delta T}{T}$.

51. We know that the frequency = f, T = temperatures
$f \propto \sqrt{T}$
So $\frac{f_1}{f_2} = \frac{\sqrt{T_1}}{\sqrt{T_2}} \Rightarrow \frac{293}{f_2} = \frac{\sqrt{293}}{\sqrt{295}}$
$\Rightarrow f_2 = \frac{293 \times \sqrt{295}}{\sqrt{293}} = 294$

52. V_{rod} = ?, V_{air} = 340 m/s, $L_r = 25 \times 10^{-2}$, $d_2 = 5 \times 10^{-2}$ metres

$$\frac{V_r}{V_a} = \frac{2L_r}{D_a} \Rightarrow V_r = \frac{340 \times 25 \times 10^{-2} \times 2}{5 \times 10^{-2}} = 3400 \text{ m/s.}$$

53. a) Here given, L_r = 1.0/2 = 0.5 m, d_a = 6.5 cm = 6.5×10^{-2} m

As Kundt's tube apparatus is a closed organ pipe, its fundamental frequency

$$\Rightarrow n = \frac{V_r}{4L_r} \Rightarrow V_r = 2600 \times 4 \times 0.5 = 5200 \text{ m/s.}$$

b) $\frac{V_r}{V_a} = \frac{2L_r}{d_a} \Rightarrow v_a = \frac{5200 \times 6.5 \times 10^{-2}}{2 \times 0.5} = 338$ m/s.

54. As the tunning fork produces 2 beats with the adjustable frequency the frequency of the tunning fork will be $\Rightarrow$ n = (476 + 480) / 2 = 478.

55. A tuning fork produces 4 beats with a known tuning fork whose frequency = 256 Hz

So the frequency of unknown tuning fork = either 256 – 4 = 252 or 256 + 4 = 260 Hz

Now as the first one is load its mass/unit length increases. So, its frequency decreases.

As it produces 6 beats now original frequency must be 252 Hz.

260 Hz is not possible as on decreasing the frequency the beats decrease which is not allowed here.

56.

Group – I	Group – II
Given V = 350	v = 350
λ_1 = 32 cm	λ_2 = 32.2 cm
= 32×10^{-2} m	= 32.2×10^{-2} m
So η_1 = frequency = 1093 Hz	$\eta_2 = 350 / 32.2 \times 10^{-2}$ = 1086 Hz

So beat frequency = 1093 – 1086 = 7 Hz.

57. Given length of the closed organ pipe, l = 40 cm = 40×10^{-2} m

V_{air} = 320

So, its frequency $\rho = \frac{V}{4l} = \frac{320}{4 \times 40 \times 10^{-2}}$ = 200 Hertz.

As the tuning fork produces 5 beats with the closed pipe, its frequency must be 195 Hz or 205 Hz.

Given that, as it is loaded its frequency decreases.

So, the frequency of tuning fork = 205 Hz.

58. Here given $n_B = 600 = \frac{1}{2l}\sqrt{\frac{TB}{14}}$

As the tension increases frequency increases

It is given that 6 beats are produces when tension in A is increases.

So, $n_A \Rightarrow 606 = \frac{1}{2l}\sqrt{\frac{TA}{M}}$

$$\Rightarrow \frac{n_A}{n_B} = \frac{600}{606} = \frac{(1/2l)\sqrt{(TB/M)}}{(1/2l)\sqrt{(TA/M)}} = \frac{\sqrt{TB}}{\sqrt{TA}}$$

$$\Rightarrow \frac{\sqrt{T_A}}{\sqrt{T_B}} = \frac{606}{600} = 1.01 \qquad \Rightarrow \frac{T_A}{T_B} = 1.02.$$

59. Given that, l = 25 cm = 25×10^{-2} m

By shortening the wire the frequency increases, [f = $(1/2l)\sqrt{(TB/M)}$]

As the vibrating wire produces 4 beats with 256 Hz, its frequency must be 252 Hz or 260 Hz.

Its frequency must be 252 Hz, because beat frequency decreases by shortening the wire.

$$\text{So, } 252 = \frac{1}{2 \times 25 \times 10^{-2}}\sqrt{\frac{T}{M}} \qquad \ldots(1)$$

Let length of the wire will be l, after it is slightly shortened,

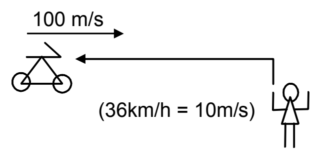
$\Rightarrow 256 = \frac{1}{2 \times l_1}\sqrt{\frac{T}{M}}$ …(2)

Dividing (1) by (2) we get

$\frac{252}{256} = \frac{l_1}{2 \times 25 \times 10^{-2}} \Rightarrow l_1 = \frac{252 \times 2 \times 25 \times 10^{-2}}{260}$ = 0.2431 m

So, it should be shorten by (25 – 24.61) = 0.39 cm.

60. Let u = velocity of sound; V_m = velocity of the medium;
v_o = velocity of the observer; v_a = velocity of the sources.



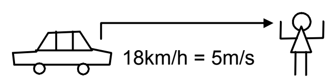
$f = \left(\frac{\vec{u} + \vec{v}_m - \vec{v}_o}{v + V_m - v_s}\right)F$

using sign conventions in Doppler's effect,

$V_m = 0$, u = 340 m/s, $v_s = 0$ and $\vec{v}_o = -10$ m (36 km/h = 10 m/s)

$= \left(\frac{340 + 0 - (-10)}{340 + 0 - 0}\right) \times 2\text{KHz}$ = 350/340 × 2 KHz = 2.06 KHz.

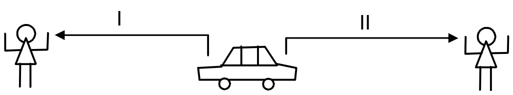
61. $f^1 = \left(\frac{\vec{u} + \vec{v}_m - \vec{v}_o}{\vec{u} + \vec{v}_m - \vec{v}_s}\right)f$ [18 km/h = 5 m/s]



using sign conventions,

app. Frequency = $\left(\frac{340 + 0 - 0}{340 + 0 - 5}\right) \times 2400$ = 2436 Hz.

62.



a) Given v_s = 72 km/hour = 20 m/s, $\rho = 1250$

apparent frequency = $\frac{340 + 0 + 0}{340 + 0 - 20} \times 1250 = 1328\ H_2$

b) For second case apparent frequency will be = $\frac{340 + 0 + 0}{340 + 0 - (-20)} \times 1250$ = 1181 Hz.

63. Here given, apparent frequency = 1620 Hz
So original frequency of the train is given by

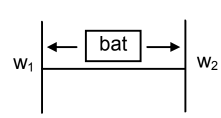
$1620 = \left(\frac{332 + 0 + 0}{332 - 15}\right)f \Rightarrow f = \left(\frac{1620 \times 317}{332}\right)$ Hz

So, apparent frequency of the train observed by the observer in

$f^1 = \left(\frac{332 + 0 + 0}{332 + 15}\right)f \times \left(\frac{1620 \times 317}{332}\right) = \frac{317}{347} \times 1620$ = 1480 Hz.

64. Let, the bat be flying between the walls W_1 and W_2.
So it will listen two frequency reflecting from walls W_2 and W_1.

So, apparent frequency, as received by wall W = $fw_2 = \frac{330 + 0 + 0}{330 - 6} \times f$ = 330/324



Therefore, apparent frequency received by the bat from wall W_2 is given by

F_{B_2} of wall W_1 = $\left(\frac{330 + 0 - (-6)}{330 + 0 + 0}\right)f_{w_2} = \left(\frac{336}{330}\right) \times \left(\frac{330}{324}\right)f$

Similarly the apparent frequency received by the bat from wall W_1 is

$f_{B_1} = (324/336)f$

So the beat frequency heard by the bat will be = $4.47 \times 10^4 = 4.3430 \times 10^4$ = 3270 Hz.

65. Let the frequency of the bullet will be f
Given, u = 330 m/s, v_s = 220 m/s

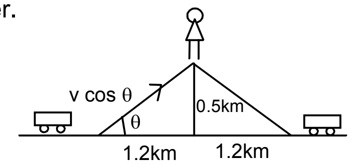
a) Apparent frequency before crossing = $f' = \left(\frac{330}{330-220}\right) f = 3f$

b) Apparent frequency after crossing = $f'' = \left(\frac{330}{530+220}\right) f = 0.6\ f$

So, $\left(\frac{f''}{f'}\right) = \frac{0.6f}{3f} = 0.2$

Therefore, fractional change = 1 – 0.2 = 0.8.

66. The person will receive, the sound in the directions BA and CA making an angle θ with the track.
Here, $\theta = \tan^{-1}(0.5/2.4) = 22°$
So the velocity of the sources will be '$v \cos \theta$' when heard by the observer.
So the apparent frequency received by the man from train B.

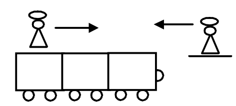


$f' = \left(\frac{340+0+0}{340 - v\cos 22°}\right) 500 = 529\text{ Hz}$

And the apparent frequency heard but the man from train C,

$f'' = \left(\frac{340+0+0}{340 - v\cos 22°}\right) \times 500 = 476\text{ Hz.}$

67. Let the velocity of the sources is = v_s



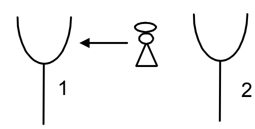
a) The beat heard by the standing man = 4
So, frequency = 440 + 4 = 444 Hz or 436 Hz

$\Rightarrow 440 = \left(\frac{340+0+0}{340 - v_s}\right) \times 400$

On solving we get V_s = 3.06 m/s = 11 km/hour.

b) The sitting man will listen less no.of beats than 4.

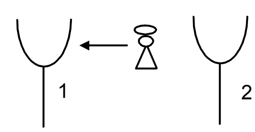
68. Here given velocity of the sources $v_s = 0$
Velocity of the observer v_0 = 3 m/s



So, the apparent frequency heard by the man = $\left(\frac{332+3}{332}\right) \times 256 = 258.3$ Hz.

from the approaching tuning form = f′
f″ = [(332–3)/332] × 256 = 253.7 Hz.
So, beat produced by them = 258.3 – 253.7 = 4.6 Hz.

69. According to the data, V_s = 5.5 m/s for each turning fork.
So, the apparent frequency heard from the tuning fork on the left,



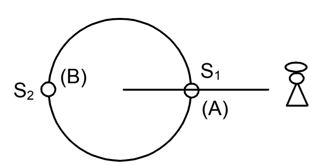
$f' = \left(\frac{330}{330-5.5}\right) \times 512 = 527.36\text{ Hz} = 527.5\text{ Hz}$

similarly, apparent frequency from the tunning fork on the right,

$f'' = \left(\frac{330}{330+5.5}\right) \times 512 = 510\text{ Hz}$

So, beats produced 527.5 – 510 = 17.5 Hz.

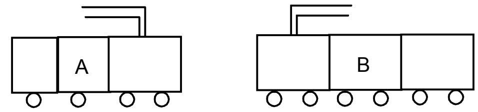
70. According to the given data
Radius of the circle = $100/\pi \times 10^{-2}$ m = $(1/\pi)$ metres; ω = 5 rev/sec.
So the linear speed $v = \omega r = 5/\pi = 1.59$
So, velocity of the source V_s = 1.59 m/s
As shown in the figure at the position A the observer will listen maximum and at the position B it will listen minimum frequency.



So, apparent frequency at A = $\frac{332}{332-1.59} \times 500 = 515$ Hz

Apparent frequency at B = $\frac{332}{332+1.59} \times 500 = 485$ Hz.

71. According to the given data V_s = 90 km/hour = 25 m/sec.
v_0 = 25 m/sec
So, apparent frequency heard by the observer in train B or observer in = $\left(\frac{350+25}{350-25}\right)$ × 500 = 577 Hz.



72. Here given $f_s = 16 \times 10^3$ Hz
Apparent frequency $f' = 20 \times 10^3$ Hz (greater than that value)
Let the velocity of the observer = v_o
Given v_s = 0

So $20 \times 10^3 = \left(\frac{330+v_o}{330+0}\right) \times 16 \times 10^3$

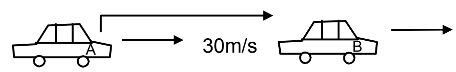
$\Rightarrow (330 + v_o) = \frac{20 \times 330}{16}$

$\Rightarrow v_o = \frac{20 \times 330 - 16 \times 330}{4} = \frac{330}{4} m/s = 297$ km/h

b) This speed is not practically attainable ordinary cars.

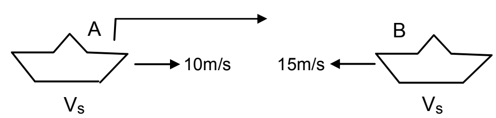
73. According to the questions velocity of car A = V_A = 108 km/h = 30 m/s
V_B = 72 km/h = 20 m/s, f = 800 Hz
So, the apparent frequency heard by the car B is given by,

$f' = \left(\frac{330-20}{330-30}\right) \times 800 \Rightarrow 826.9 = 827$ Hz.



74. a) According to the questions, v = 1500 m/s, f = 2000 Hz, v_s = 10 m/s, v_o = 15 m/s
So, the apparent frequency heard by the submarine B,

$= \left(\frac{1500+15}{1500-10}\right) \times 2000 = 2034$ Hz

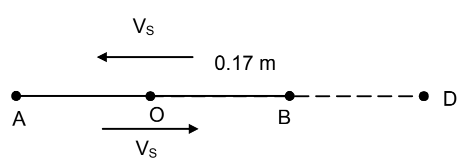


b) Apparent frequency received by submarine A,

$= \left(\frac{1500+10}{1500-15}\right) \times 2034 = 2068$ Hz.

75. Given that, r = 0.17 m, F = 800 Hz, u = 340 m/s
Frequency band = $f_1 - f_2$ = 6 Hz
Where f_1 and f_2 correspond to the maximum and minimum apparent frequencies (both will occur at the mean position because the velocity is maximum).

Now, $f_1 = \left(\frac{340}{340-v_s}\right) f$ and $f_2 = \left(\frac{340}{340+v_s}\right) f$



$\therefore f_1 - f_2 = 8$

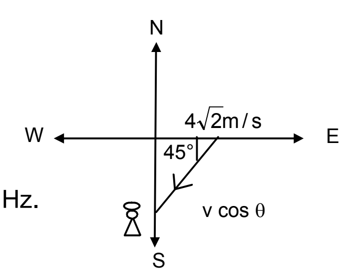
$\Rightarrow 340 f \left(\frac{1}{340-v_s} - \frac{1}{340+v_s}\right) = 8$

$\Rightarrow \frac{2v_s}{340^2 - v_s^2} = \frac{8}{340 \times 800}$

$\Rightarrow 340^2 - v_s^2 = 68000 v_s$
Solving for v_s we get, v_s = 1.695 m/s
For SHM, $v_s = r\omega \Rightarrow \omega = (1.695/0.17) = 10$
So, $T = 2\pi / \omega = \pi/5 = 0.63$ sec.

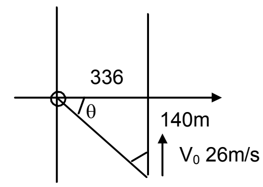
76. u = 334 m/s, $v_b = 4\sqrt{2}$ m/s, v_o = 0
so, $v_s = V_b \cos\theta = 4\sqrt{2} \times (1/\sqrt{2}) = 4$ m/s.

so, the apparent frequency $f' = \left(\frac{u+0}{u - v_b \cos\theta}\right) f = \left(\frac{334}{334-4}\right) \times 1650 = 1670$ Hz.



77. u = 330 m/s, v_0 = 26 m/s

a) Apparent frequency at, y = – 336

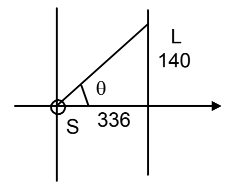


$$m = \left(\frac{v}{v - u\sin\theta}\right) \times f$$

$$= \left(\frac{330}{330 - 26\sin 23^\circ}\right) \times 660$$

[because, θ = $\tan^{-1}$ (140/336) = 23°] = 680 Hz.

b) At the point y = 0 the source and listener are on a x-axis so no apparent change in frequency is seen. So, f = 660 Hz.



c) As shown in the figure θ = $\tan^{-1}$ (140/336) = 23°

Here given, = 330 m/s ; v = V sin 23° = 10.6 m/s

So, $F'' = \frac{u}{u + v\sin 23^\circ} \times 660 = 640$ Hz.

78. V_{train} or V_s = 108 km/h = 30 m/s; u = 340 m/s

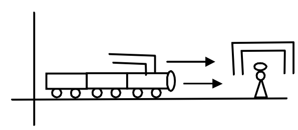
a) The frequency by the passenger sitting near the open window is 500 Hz, he is inside the train and does not hair any relative motion.

b) After the train has passed the apparent frequency heard by a person standing near the track will be,

so $f'' = \left(\frac{340 + 0}{340 + 30}\right) \times 500 = 459$ Hz

c) The person inside the source will listen the original frequency of the train.

Here, given V_m = 10 m/s



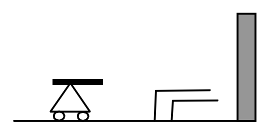
For the person standing near the track

Apparent frequency = $\frac{u + V_m + 0}{u + V_m - (-V_s)} \times 500$ = 458 Hz.

79. To find out the apparent frequency received by the wall,

a) V_s = 12 km/h = 10/3 = m/s

V_o = 0, u = 330 m/s



So, the apparent frequency is given by = $f' = \left(\frac{330}{330 - 10/3}\right) \times 1600 = 1616$ Hz

b) The reflected sound from the wall whistles now act as a sources whose frequency is 1616 Hz.

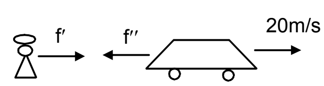
So, u = 330 m/s, V_s = 0, V_o = 10/3 m/s

So, the frequency by the man from the wall,

$\Rightarrow f'' = \left(\frac{330 + 10/3}{330}\right) \times 1616 = 1632$ m/s.

80. Here given, u = 330 m/s, f = 1600 Hz

So, apparent frequency received by the car



$f' = \left(\frac{u - V_o}{u - V_s}\right) f = \left(\frac{330 - 20}{330}\right) \times 1600$ Hz … [V_o = 20 m/s, V_s = 0]

The reflected sound from the car acts as the source for the person.

Here, V_s = –20 m/s, V_o = 0

So $f'' = \left(\frac{330 - 0}{330 + 20}\right) \times f' = \frac{330}{350} \times \frac{310}{330} \times 160 = 1417$ Hz.

∴ This is the frequency heard by the person from the car.

81. a) f = 400 Hz,, u = 335 m/s

$\Rightarrow$ λ (v/f) = (335/400) = 0.8 m = 80 cm

b) The frequency received and reflected by the wall,

$f' = \left(\frac{u - V_o}{u - V_s}\right) \times f = \frac{335}{320} \times 400$ …[V_s = 54 m/s and V_o = 0]

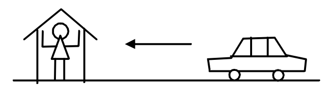
$\Rightarrow x' = (v/f) = \dfrac{320 \times 335}{335 \times 400} = 0.8 \text{ m} = 80 \text{ cm}$

c) The frequency received by the person sitting inside the car from reflected wave,

$f' = \left(\dfrac{335 - 0}{335 - 15}\right) f = \dfrac{335}{320} \times 400 = 467$ [$V_s = 0$ and $V_o = -15$ m/s]

d) Because, the difference between the original frequency and the apparent frequency from the wall is very high (437 – 440 = 37 Hz), he will not hear any beats.mm)

82. f = 400 Hz, u = 324 m/s, $f' = \dfrac{u - (-v)}{u - (0)} f = \dfrac{324 + v}{324} \times 400$ …(1)



for the reflected wave,

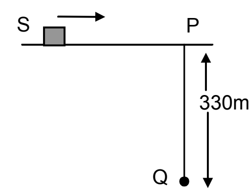
$f'' = 410 = \dfrac{u - 0}{u - v} f'$

$\Rightarrow 410 = \dfrac{324}{324 - v} \times \dfrac{324 + v}{324} \times 400$

$\Rightarrow 810 \text{ v} = 324 \times 10$

$\Rightarrow v = \dfrac{324 \times 10}{810} = 4 \text{ m/s.}$

83. f = 2 kHz, v = 330 m/s, u = 22 m/s



At t = 0, the source crosses P

a) Time taken to reach at Q is

$t = \dfrac{S}{v} = \dfrac{330}{330} = 1 \text{ sec}$

b) The frequency heard by the listner is

$f' = f\left(\dfrac{v}{v - u\cos\theta}\right)$

since, $\theta = 90°$

$f' = 2 \times (v/u) = 2$ KHz.

c) After 1 sec, the source is at 22 m from P towards right.

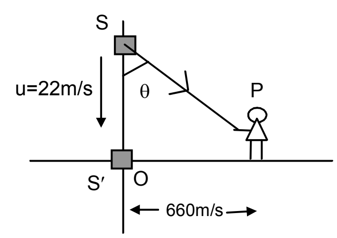
84. t = 4000 Hz, u = 22 m/s

Let ‘t’ be the time taken by the source to reach at ‘O’. Since observer hears the sound at the instant it crosses the ‘O’, ‘t’ is also time taken to the sound to reach at P.

$\therefore$ OQ = ut and QP = vt

$\cos\theta = u/v$

Velocity of the sound along QP is ($u \cos\theta$).



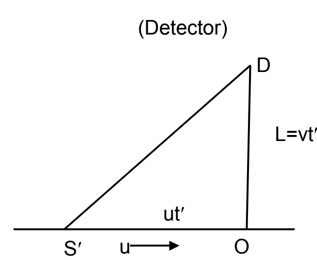
$f' = f\left(\dfrac{v - 0}{v - u\cos\theta}\right) = f\left(\dfrac{v}{v - \dfrac{u^2}{v}}\right) = f\left(\dfrac{v^2}{v^2 - u^2}\right)$

Putting the values in the above equation, $f' = 4000 \times \dfrac{330^2}{330^2 - 22^2} = 4017.8 = 4018$ Hz.

85. a) Given that, f = 1200 Hz, u = 170 m/s, L = 200 m, v = 340 m/s

From Doppler’s equation (as in problem no.84)

$f' = f\left(\dfrac{v^2}{v^2 - u^2}\right) = 1200 \times \dfrac{340^2}{340^2 - 170^2} = 1600$ Hz.



b) v = velocity of sound, u = velocity of source

let, t be the time taken by the sound to reach at D

DO = vt′ = L, and S′O = ut′

t′ = L/V

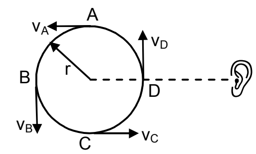
$S'D = \sqrt{S'O^2 + DO^2} = \sqrt{u^2 \frac{L^2}{v^2} + L^2} = \frac{L}{v}\sqrt{u^2 + v^2}$

Putting the values in the above equation, we get

$S'D = \frac{220}{340}\sqrt{170^2 + 340^2}$ = 223.6 m.

86. Given that, r = 1.6 m, f = 500 Hz, u = 330 m/s

a) At A, velocity of the particle is given by



$v_A = \sqrt{rg} = \sqrt{1.6 \times 10} = 4\,m/s$

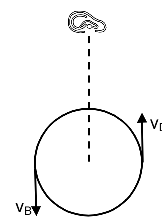
and at C, $v_c = \sqrt{5rg} = \sqrt{5 \times 1.6 \times 10} = 8.9\,m/s$

So, maximum frequency at C,

$f'_c = \frac{u}{u - v_s} f = \frac{330}{330 - 8.9} \times 500 = 513.85\,Hz.$

Similarly, maximum frequency at A is given by $f'_A = \frac{u}{u - (-v_s)} f = \frac{330}{330 + 4}(500) = 494\,Hz.$

b) Velocity at B = $\sqrt{3rg} = \sqrt{3 \times 1.6 \times 10} = 6.92\,m/s$



So, frequency at B is given by,

$f_B = \frac{u}{u + v_s} \times f = \frac{330}{330 + 6.92} \times 500$ = 490 Hz

and frequency at D is given by,

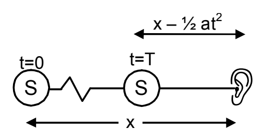
$f_D = \frac{u}{u - v_s} \times f = \frac{330}{330 - 6.92} \times 500$

87. Let the distance between the source and the observer is 'x' (initially)

So, time taken for the first pulse to reach the observer is $t_1 = x/v$

and the second pulse starts after T (where, T = 1/v)

and it should travel a distance $\left(x - \frac{1}{2}aT^2\right)$.



So, $t_2 = T + \frac{x - 1/2\,aT^2}{v}$

$t_2 - t_1 = T + \frac{x - 1/2\,aT^2}{v} = \frac{x}{v} = T - \frac{1}{2}\frac{aT^2}{v}$

Putting = T = 1/v, we get

$t_2 - t_1 = \frac{2uv - a}{2vv^2}$

so, frequency heard = $\frac{2vv^2}{2uv - a}$ (because, f = $\frac{1}{t_2 - t_1}$)

♣ ♣ ♣ ♣ ♣

SOLUTIONS TO CONCEPTS
CHAPTER 17

1. Given that, 400 m < λ < 700 nm.

$$\frac{1}{700\text{nm}} < \frac{1}{\lambda} < \frac{1}{400\text{nm}}$$

$$\Rightarrow \frac{1}{7\times10^{-7}} < \frac{1}{\lambda} < \frac{1}{4\times10^{-7}} \Rightarrow \frac{3\times10^8}{7\times10^{-7}} < \frac{c}{\lambda} < \frac{3\times10^8}{4\times10^{-7}}$$ (Where, c = speed of light = 3×10^8 m/s)

$\Rightarrow 4.3 \times 10^{14} < c/\lambda < 7.5 \times 10^{14}$

$\Rightarrow 4.3 \times 10^{14}$ Hz < f < 7.5×10^{14} Hz.

2. Given that, for sodium light, $\lambda = 589$ nm $= 589 \times 10^{-9}$ m

a) $f_a = \frac{3\times10^8}{589\times10^{-9}} = 5.09 \times 10^{14}\ \text{sec}^{-1}\left[\because f = \frac{c}{\lambda}\right]$

b) $\frac{\mu_a}{\mu_w} = \frac{\lambda_w}{\lambda_a} \Rightarrow \frac{1}{1.33} = \frac{\lambda_w}{589\times10^{-9}} \Rightarrow \lambda_w = 443$ nm

c) $f_w = f_a = 5.09 \times 10^{14}$ sec^{-1} [Frequency does not change]

d) $\frac{\mu_a}{\mu_w} = \frac{v_w}{v_a} \Rightarrow v_w = \frac{\mu_a v_a}{\mu_w} = \frac{3\times10^8}{1.33} = 2.25 \times 10^8$ m/sec.

3. We know that, $\frac{\mu_2}{\mu_1} = \frac{v_1}{v_2}$

So, $\frac{1472}{1} = \frac{3\times10^8}{v_{400}} \Rightarrow v_{400} = 2.04\times10^8 \text{m/sec}.$

[because, for air, $\mu = 1$ and $v = 3 \times 10^8$ m/s]

Again, $\frac{1452}{1} = \frac{3\times10^8}{v_{760}} \Rightarrow v_{760} = 2.07\times10^8 \text{m/sec}.$

4. $\mu_t = \frac{1\times3\times10^8}{(2.4)\times10^8} = 1.25$ $\left[\text{since, } \mu = \frac{\text{velocity of light in vaccum}}{\text{velocity of light in the given medium}}\right]$

5. Given that, d = 1 cm = 10^{-2} m, $\lambda = 5 \times 10^{-7}$ m and D = 1 m

a) Separation between two consecutive maxima is equal to fringe width.

So, $\beta = \frac{\lambda D}{d} = \frac{5\times10^{-7}\times1}{10^{-2}}$ m $= 5 \times 10^{-5}$ m = 0.05 mm.

b) When, β = 1 mm = 10^{-3} m

$10^{-3}\text{m} = \frac{5\times10^{-7}\times1}{D} \Rightarrow D = 5 \times 10^{-4}$ m = 0.50 mm.

6. Given that, β = 1 mm = 10^{-3} m, D = 2.t m and d = 1 mm = 10^{-3} m

So, $10^{-3}\text{m} = \frac{25\times\lambda}{10^{-3}} \Rightarrow \lambda = 4 \times 10^{-7}$ m = 400 nm.

7. Given that, d = 1 mm = 10^{-3} m, D = 1 m.

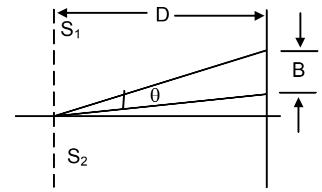
So, fringe with = $\frac{D\lambda}{d}$ = 0.5 mm.

a) So, distance of centre of first minimum from centre of central maximum = 0.5/2 mm = 0.25 mm

b) No. of fringes = 10 / 0.5 = 20.

8. Given that, d = 0.8 mm = 0.8×10^{-3} m, λ = 589 nm = 589×10^{-9} m and D = 2 m.

So, $\beta = \frac{D\lambda}{d} = \frac{589\times10^{-9}\times2}{0.8\times10^{-3}} = 1.47 \times 10^{-3}$ m = 147 mm.



9. Given that, λ = 500 nm = 500×10^{-9} m and d = 2×10^{-3} m

As shown in the figure, angular separation $\theta = \frac{\beta}{D} = \frac{\lambda D}{dD} = \frac{\lambda}{d}$

So, $\theta = \frac{\beta}{D} = \frac{\lambda}{d} = \frac{500 \times 10^{-9}}{2 \times 10^{-3}}$ = 250×10^{-6}

= 25×10^{-5} radian = 0.014 degree.

10. We know that, the first maximum (next to central maximum) occurs at $y = \frac{\lambda D}{d}$

Given that, λ_1 = 480 nm, λ_2 = 600 nm, D = 150 cm = 1.5 m and d = 0.25 mm = 0.25×10^{-3} m

So, $y_1 = \frac{D\lambda_1}{d} = \frac{1.5 \times 480 \times 10^{-9}}{0.25 \times 10^{-3}}$ = 2.88 mm

$y_2 = \frac{1.5 \times 600 \times 10^{-9}}{0.25 \times 10^{-3}}$ = 3.6 mm.

So, the separation between these two bright fringes is given by,

$\therefore$ separation = $y_2 - y_1$ = 3.60 – 2.88 = 0.72 mm.

11. Let mth bright fringe of violet light overlaps with nth bright fringe of red light.

$\therefore \frac{m \times 400nm \times D}{d} = \frac{n \times 700nm \times D}{d} \Rightarrow \frac{m}{n} = \frac{7}{4}$

$\Rightarrow$ 7th bright fringe of violet light overlaps with 4th bright fringe of red light (minimum). Also, it can be seen that 14th violet fringe will overlap 8th red fringe.

Because, m/n = 7/4 = 14/8.

12. Let, t = thickness of the plate

Given, optical path difference = $(\mu - 1)t = \lambda/2$

$\Rightarrow t = \frac{\lambda}{2(\mu - 1)}$

13. a) Change in the optical path = $\mu t - t = (\mu - 1)t$

b) To have a dark fringe at the centre the pattern should shift by one half of a fringe.

$\Rightarrow (\mu - 1)t = \frac{\lambda}{2} \Rightarrow t = \frac{\lambda}{2(\mu - 1)}$.

14. Given that, μ = 1.45, t = 0.02 mm = 0.02×10^{-3} m and λ = 620 nm = 620×10^{-9} m

We know, when the transparent paper is pasted in one of the slits, the optical path changes by $(\mu - 1)t$.

Again, for shift of one fringe, the optical path should be changed by λ.

So, no. of fringes crossing through the centre is given by,

$n = \frac{(\mu - 1)t}{\lambda} = \frac{0.45 \times 0.02 \times 10^{-3}}{620 \times 10^{-9}}$ = 14.5

15. In the given Young's double slit experiment,

μ = 1.6, t = 1.964 micron = 1.964×10^{-6} m

We know, number of fringes shifted = $\frac{(\mu - 1)t}{\lambda}$

So, the corresponding shift = No.of fringes shifted $\times$ fringe width

$= \frac{(\mu - 1)t}{\lambda} \times \frac{\lambda D}{d} = \frac{(\mu - 1)tD}{d}$ … (1)

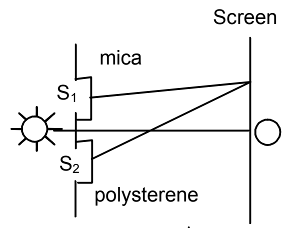
Again, when the distance between the screen and the slits is doubled,

Fringe width = $\frac{\lambda(2D)}{d}$ …(2)

From (1) and (2), $\frac{(\mu - 1)tD}{d} = \frac{\lambda(2D)}{d}$

$\Rightarrow \lambda = \frac{(\mu - 1)t}{\lambda} = \frac{(1.6 - 1) \times (1.964) \times 10^{-6}}{2}$ = 589.2×10^{-9} = 589.2 nm.

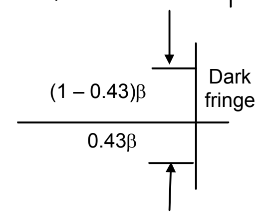
16. Given that, $t_1 = t_2 = 0.5$ mm $= 0.5 \times 10^{-3}$ m, $\mu_m = 1.58$ and $\mu_p = 1.55$,
$\lambda = 590$ nm $= 590 \times 10^{-9}$ m, $d = 0.12$ cm $= 12 \times 10^{-4}$ m, $D = 1$ m



a) Fringe width = $\dfrac{D\lambda}{d} = \dfrac{1 \times 590 \times 10^{-9}}{12 \times 10^{-4}}$ = 4.91×10^{-4} m.

b) When both the strips are fitted, the optical path changes by
$\Delta x = (\mu_m - 1)t_1 - (\mu_p - 1)t_2 = (\mu_m - \mu_p)t$
$= (1.58 - 1.55) \times (0.5)(10^{-3}) = 0.015 \times 10^{-13}$ m.

So, No. of fringes shifted = $\dfrac{0.015 \times 10^{-3}}{590 \times 10^{-3}}$ = 25.43.



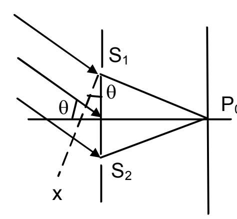
⇒ There are 25 fringes and 0.43 th of a fringe.
⇒ There are 13 bright fringes and 12 dark fringes and 0.43 th of a dark fringe.
So, position of first maximum on both sides will be given by
∴ $x = 0.43 \times 4.91 \times 10^{-4} = 0.021$ cm
$x' = (1 - 0.43) \times 4.91 \times 10^{-4} = 0.028$ cm (since, fringe width $= 4.91 \times 10^{-4}$ m)

17. The change in path difference due to the two slabs is $(\mu_1 - \mu_2)t$ (as in problem no. 16).
For having a minimum at P_0, the path difference should change by $\lambda/2$.
So, $\Rightarrow \lambda/2 = (\mu_1 - \mu_2)t \Rightarrow t = \dfrac{\lambda}{2(\mu_1 - \mu_2)}$.

18. Given that, $t = 0.02$ mm $= 0.02 \times 10^{-3}$ m, $\mu_1 = 1.45$, $\lambda = 600$ nm $= 600 \times 10^{-9}$ m
a) Let, I_1 = Intensity of source without paper = I
b) Then I_2 = Intensity of source with paper = (4/9)I
$\Rightarrow \dfrac{I_1}{I_2} = \dfrac{9}{4} \Rightarrow \dfrac{r_1}{r_2} = \dfrac{3}{2}$ [because $I \propto r^2$]
where, r_1 and r_2 are corresponding amplitudes.
So, $\dfrac{I_{max}}{I_{min}} = \dfrac{(r_1 + r_2)^2}{(r_1 - r_2)^2}$ = 25 : 1
b) No. of fringes that will cross the origin is given by,
$n = \dfrac{(\mu - 1)t}{\lambda} = \dfrac{(1.45 - 1) \times 0.02 \times 10^{-3}}{600 \times 10^{-9}}$ = 15.

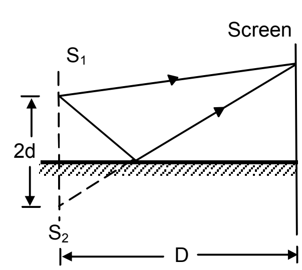
19. Given that, $d = 0.28$ mm $= 0.28 \times 10^{-3}$ m, $D = 48$ cm $= 0.48$ m, $\lambda_a = 700$ nm in vacuum
Let, λ_w = wavelength of red light in water
Since, the fringe width of the pattern is given by,
$\beta = \dfrac{\lambda_w D}{d} = \dfrac{525 \times 10^{-9} \times 0.48}{0.28 \times 10^{-3}}$ = 9×10^{-4} m = 0.90 mm.

20. It can be seen from the figure that the wavefronts reaching O from S_1 and S_2 will have a path difference of S_2X.



In the $\Delta\, S_1S_2X$,
$\sin\,\theta = \dfrac{S_2X}{S_1S_2}$
So, path difference $= S_2X = S_1S_2 \sin\theta = d \sin\theta = d \times \lambda/2d = \lambda/2$
As the path difference is an odd multiple of $\lambda/2$, there will be a dark fringe at point P_0.

21. a) Since, there is a phase difference of π between direct light and reflecting light, the intensity just above the mirror will be zero.



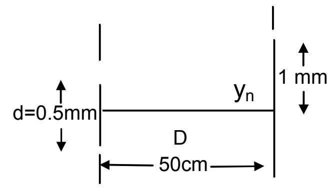
b) Here, 2d = equivalent slit separation
D = Distance between slit and screen.
We know for bright fringe, $\Delta x = \dfrac{y \times 2d}{D} = n\lambda$
But as there is a phase reversal of $\lambda/2$.
$\Rightarrow \dfrac{y \times 2d}{D} + \dfrac{\lambda}{2} = n\lambda \qquad \Rightarrow \dfrac{y \times 2d}{D} = n\lambda - \dfrac{\lambda}{2} \Rightarrow y = \dfrac{\lambda D}{4d}$

22. Given that, D = 1 m, λ = 700 nm = 700×10^{-9} m
Since, a = 2 mm, d = 2a = 2mm = 2×10^{-3} m (L loyd's mirror experiment)
Fringe width = $\frac{\lambda D}{d} = \frac{700 \times 10^{-9} m \times 1m}{2 \times 10^{-3} m}$ = 0.35 mm.

23. Given that, the mirror reflects 64% of energy (intensity) of the light.
So, $\frac{I_1}{I_2} = 0.64 = \frac{16}{25} \Rightarrow \frac{r_1}{r_2} = \frac{4}{5}$
So, $\frac{I_{max}}{I_{min}} = \frac{(r_1 + r_2)^2}{(r_1 - r_2)^2}$ = 81 : 1.

24. It can be seen from the figure that, the apparent distance of the screen from the slits is,
$D = 2D_1 + D_2$
So, Fringe width = $\frac{D\lambda}{d} = \frac{(2D_1 + D_2)\lambda}{d}$

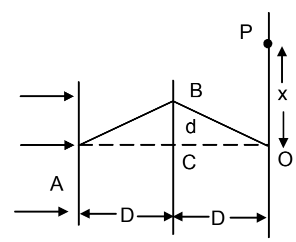
25. Given that, λ = (400 nm to 700 nm), d = 0.5 mm = 0.5×10^{-3} m,
D = 50 cm = 0.5 m and on the screen y_n = 1 mm = 1×10^{-3} m



a) We know that for zero intensity (dark fringe)
$y_n = \left(\frac{2n+1}{2}\right)\frac{\lambda_n D}{d}$ where n = 0, 1, 2, …….

$\Rightarrow \lambda_n = \frac{2}{(2n+1)}\frac{\lambda_n d}{D} = \frac{2}{2n+1} \times \frac{10^{-3} \times 0.5 \times 10^{-3}}{0.5} \Rightarrow \frac{2}{(2n+1)} \times 10^{-6} m = \frac{2}{(2n+1)} \times 10^3 nm$
If n = 1, λ_1 = (2/3) × 1000 = 667 nm
If n = 1, λ_2 = (2/5) × 1000 = 400 nm
So, the light waves of wavelengths 400 nm and 667 nm will be absent from the out coming light.
b) For strong intensity (bright fringes) at the hole
$y_n = \frac{n\lambda_n D}{d} \Rightarrow \lambda_n = \frac{y_n d}{nD}$
When, n = 1, $\lambda_1 = \frac{y_n d}{D} = \frac{10^{-3} \times 0.5 \times 10^{-3}}{0.5} = 10^{-6} m = 1000 nm$.
1000 nm is not present in the range 400 nm – 700 nm
Again, where n = 2, $\lambda_2 = \frac{y_n d}{2D}$ = 500 nm
So, the only wavelength which will have strong intensity is 500 nm.

26. From the diagram, it can be seen that at point O.
Path difference = (AB + BO) – (AC + CO)
= 2(AB – AC) [Since, AB = BO and AC = CO] = $2(\sqrt{d^2 + D^2} - D)$
For dark fringe, path difference should be odd multiple of $\lambda/2$.

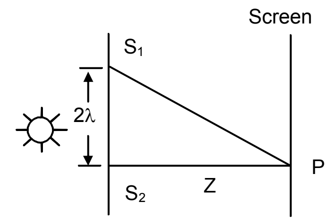


So, $2(\sqrt{d^2 + D^2} - D)$ = (2n + 1)(λ/2)
$\Rightarrow \sqrt{d^2 + D^2}$ = D + (2n + 1) λ/4
$\Rightarrow D^2 + d^2 = D^2 + (2n+1)^2 \lambda^2/16 + (2n + 1) \lambda D/2$
Neglecting, $(2n+1)^2 \lambda^2/16$, as it is very small
We get, d = $\sqrt{(2n+1)\frac{\lambda D}{2}}$
For minimum 'd', putting n = 0 $\Rightarrow d_{min} = \sqrt{\frac{\lambda D}{2}}$.

27. For minimum intensity

$\therefore$ $S_1P - S_2P = x = (2n + 1)\lambda/2$

From the figure, we get



$$\Rightarrow \sqrt{Z^2 + (2\lambda)^2} - Z = (2n+1)\frac{\lambda}{2}$$

$$\Rightarrow Z^2 + 4\lambda^2 = Z^2 + (2n+1)^2\frac{\lambda^2}{4} + Z(2n+1)\lambda$$

$$\Rightarrow Z = \frac{4\lambda^2 - (2n+1)^2(\lambda^2/4)}{(2n+1)\lambda} = \frac{16\lambda^2 - (2n+1)^2\lambda^2}{4(2n+1)\lambda} \quad \ldots(1)$$

Putting, $n = 0 \Rightarrow Z = 15\lambda/4$ $\quad$ $n = -1 \Rightarrow Z = -15\lambda/4$

$n = 1 \Rightarrow Z = 7\lambda/12$ $\quad$ $n = 2 \Rightarrow Z = -9\lambda/20$

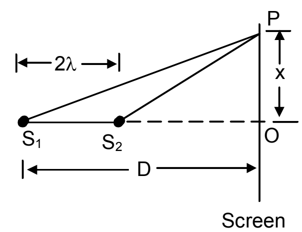
$\therefore$ $Z = 7\lambda/12$ is the smallest distance for which there will be minimum intensity.

28. Since S_1, S_2 are in same phase, at O there will be maximum intensity.

Given that, there will be a maximum intensity at P.

$\Rightarrow$ path difference $= \Delta x = n\lambda$

From the figure,



$(S_1P)^2 - (S_2P)^2 = (\sqrt{D^2 + X^2})^2 - (\sqrt{(D-2\lambda)^2 + X^2})^2$

$= 4\lambda D - 4\lambda^2 = 4\lambda D$ (λ^2 is so small and can be neglected)

$$\Rightarrow S_1P - S_2P = \frac{4\lambda D}{2\sqrt{x^2 + D^2}} = n\lambda$$

$$\Rightarrow \frac{2D}{\sqrt{x^2 + D^2}} = \nu$$

$$\Rightarrow n^2(X^2 + D^2) = 4D^2 = \Delta X = \frac{D}{n}\sqrt{4 - n^2}$$

when $n = 1$, $x = \sqrt{3}\,D$ $\quad$ (1^{st} order)

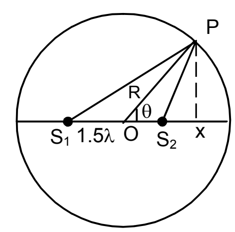
$n = 2$, $x = 0$ $\quad$ (2^{nd} order)

$\therefore$ When $X = \sqrt{3}\,D$, at P there will be maximum intensity.

29. As shown in the figure,

$(S_1P)^2 = (PX)^2 + (S_1X)^2 \quad \ldots(1)$

$(S_2P)^2 = (PX)^2 + (S_2X)^2 \quad \ldots(2)$



From (1) and (2),

$(S_1P)^2 - (S_2P)^2 = (S_1X)^2 - (S_2X)^2$

$= (1.5\,\lambda + R\cos\theta)^2 - (R\cos\theta - 15\,\lambda)^2$

$= 6\lambda\, R\cos\theta$

$$\Rightarrow (S_1P - S_2P) = \frac{6\lambda R\cos\theta}{2R} = 3\lambda\cos\theta.$$

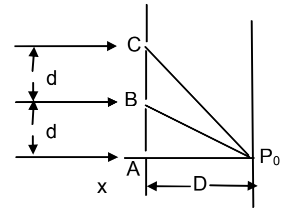
For constructive interference,

$(S_1P - S_2P)^2 = x = 3\lambda\cos\theta = n\lambda$

$\Rightarrow \cos\theta = n/3 \Rightarrow \theta = \cos^{-1}(n/3)$, where $n = 0, 1, 2, \ldots$

$\Rightarrow \theta = 0°, 48.2°, 70.5°, 90°$ and similar points in other quadrants.

30. a) As shown in the figure, $BP_0 - AP_0 = \lambda/3$



$$\Rightarrow \sqrt{(D^2 + d^2)} - D = \lambda/3$$

$\Rightarrow D^2 + d^2 = D^2 + (\lambda^2/9) + (2\lambda D)/3$

$\Rightarrow d = \sqrt{(2\lambda D)/3}$ $\quad$ (neglecting the term $\lambda^2/9$ as it is very small)

b) To find the intensity at P_0, we have to consider the interference of light waves coming from all the three slits.

Here, $CP_0 - AP_0 = \sqrt{D^2 + 4d^2} - D$

$= \sqrt{D^2 + \frac{8\lambda D}{3}} - D = D\left\{1 + \frac{8\lambda}{3D}\right\}^{1/2} - D$

$= D\left\{1 + \frac{8\lambda}{3D \times 2} + ...\right\} - D = \frac{4\lambda}{3}$ [using binomial expansion]

So, the corresponding phase difference between waves from C and A is,

$\phi_c = \frac{2\pi x}{\lambda} = \frac{2\pi \times 4\lambda}{3\lambda} = \frac{8\pi}{3} = \left(2\pi + \frac{2\pi}{3}\right) = \frac{2\pi}{3}$ …(1)

Again, $\phi_B = \frac{2\pi x}{3\lambda} = \frac{2\pi}{3}$ …(2)

So, it can be said that light from B and C are in same phase as they have some phase difference with respect to A.

So, R $= \sqrt{(2r)^2 + r^2 + 2 \times 2r \times r\cos(2\pi/3)}$ (using vector method)

$= \sqrt{4r^2 + r^2 - 2r^2} = \sqrt{3}r$

$\therefore I_{P_0} - K(\sqrt{3}r)^2 = 3Kr^2 = 3I$

As, the resulting amplitude is $\sqrt{3}$ times, the intensity will be three times the intensity due to individual slits.

31. Given that, d = 2 mm = 2×10^{-3} m, λ = 600 nm = 6×10^{-7} m, I_{max} = 0.20 W/m^2, D = 2m

For the point, y = 0.5 cm

We know, path difference = x = $\frac{yd}{D} = \frac{0.5 \times 10^{-2} \times 2 \times 10^{-3}}{2}$ = 5×10^{-6} m

So, the corresponding phase difference is,

$\phi = \frac{2\pi x}{\lambda} = \frac{2\pi \times 5 \times 10^{-6}}{6 \times 10^{-7}} \Rightarrow \frac{50\pi}{3} = 16\pi + \frac{2\pi}{3} \Rightarrow \phi = \frac{2\pi}{3}$

So, the amplitude of the resulting wave at the point y = 0.5 cm is,

$A = \sqrt{r^2 + r^2 + 2r^2\cos(2\pi/3)} = \sqrt{r^2 + r^2 - r^2} = r$

Since, $\frac{I}{I_{max}} = \frac{A^2}{(2r)^2}$ [since, maximum amplitude = 2r]

$\Rightarrow \frac{I}{0.2} = \frac{A^2}{4r^2} = \frac{r^2}{4r^2}$

$\Rightarrow I = \frac{0.2}{4} = 0.05$ W/m^2.

32. i) When intensity is half the maximum $\frac{I}{I_{max}} = \frac{1}{2}$

$\Rightarrow \frac{4a^2\cos^2(\phi/2)}{4a^2} = \frac{1}{2}$

$\Rightarrow \cos^2(\phi/2) = 1/2 \Rightarrow \cos(\phi/2) = 1/\sqrt{2}$

$\Rightarrow \phi/2 = \pi/4 \Rightarrow \phi = \pi/2$

$\Rightarrow$ Path difference, $x = \lambda/4$

$\Rightarrow y = xD/d = \lambda D/4d$

ii) When intensity is 1/4th of the maximum $\frac{I}{I_{max}} = \frac{1}{4}$

$\Rightarrow \frac{4a^2\cos^2(\phi/2)}{4a^2} = \frac{1}{4}$

$\Rightarrow \cos^2(\phi/2) = 1/4 \Rightarrow \cos(\phi/2) = 1/2$

$\Rightarrow \phi/2 = \pi/3 \Rightarrow \phi = 2\pi/3$

$\Rightarrow$ Path difference, $x = \lambda/3$

$\Rightarrow y = xD/d = \lambda D/3d$

33. Given that, D = 1 m, d = 1 mm = 10^{-3} m, λ = 500 nm = 5×10^{-7} m
For intensity to be half the maximum intensity.

$y = \frac{\lambda D}{4d}$ (As in problem no. 32)

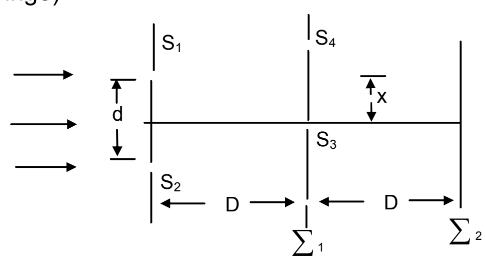
$\Rightarrow y = \frac{5 \times 10^{-7} \times 1}{4 \times 10^{-3}} \Rightarrow y = 1.25 \times 10^{-4}$ m.

34. The line width of a bright fringe is sometimes defined as the separation between the points on the two sides of the central line where the intensity falls to half the maximum.
We know that, for intensity to be half the maximum

$y = \pm \frac{\lambda D}{4d}$

$\therefore$ Line width = $\frac{\lambda D}{4d} + \frac{\lambda D}{4d} = \frac{\lambda D}{2d}$.

35. i) When, z = λD/2d, at S_4, minimum intensity occurs (dark fringe)



$\Rightarrow$ Amplitude = 0,
At S_3, path difference = 0
$\Rightarrow$ Maximum intensity occurs.
$\Rightarrow$ Amplitude = 2r.
So, on Σ2 screen,

$\frac{I_{max}}{I_{min}} = \frac{(2r+0)^2}{(2r-0)^2} = 1$

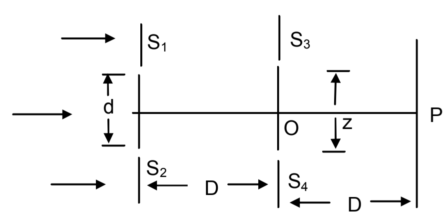
ii) When, z = λD/2d, At S_4, minimum intensity occurs. (dark fringe)
$\Rightarrow$ Amplitude = 0.
At S_3, path difference = 0
$\Rightarrow$ Maximum intensity occurs.
$\Rightarrow$ Amplitude = 2r.
So, on Σ2 screen,

$\frac{I_{max}}{I_{min}} = \frac{(2r+2r)^2}{(2r-0)^2} = \infty$

iii) When, z = λD/4d, At S_4, intensity = I_{max} / 2
$\Rightarrow$ Amplitude = $\sqrt{2}r$.
$\therefore$ At S_3, intensity is maximum.
$\Rightarrow$ Amplitude = 2r

$\therefore \frac{I_{max}}{I_{min}} = \frac{(2r+\sqrt{2}r)^2}{(2r-\sqrt{2}r)^2}$ = 34.

36. a) When, z = Dλ/d
So, $OS_3 = OS_4$ = Dλ/2d $\Rightarrow$ Dark fringe at S_3 and S_4.



$\Rightarrow$ At S_3, intensity at $S_3 = 0 \Rightarrow I_1 = 0$
At S_4, intensity at $S_4 = 0 \Rightarrow I_2 = 0$
At P, path difference = 0 $\Rightarrow$ Phase difference = 0.
$\Rightarrow I = I_1 + I_2 + \sqrt{I_1 I_2} \cos 0° = 0 + 0 + 0 = 0 \Rightarrow$ Intensity at P = 0.

b) Given that, when z = Dλ/2d, intensity at P = I
Here, $OS_3 = OS_4$ = y = Dλ/4d

$\therefore \phi = \frac{2\pi x}{\lambda} = \frac{2\pi}{\lambda} \times \frac{yd}{D} = \frac{2\pi}{\lambda} \times \frac{D\lambda}{4d} \times \frac{d}{D} = \frac{\pi}{2}$. [Since, x = path difference = yd/D]

Let, intensity at S_3 and S_4 = I′
$\therefore$ At P, phase difference = 0
So, I′ + I′ + 2I′ cos 0° = I.
$\Rightarrow$ 4I′ = I $\Rightarrow$ I′ = 1/4.

When, $z = \frac{3D\lambda}{2d}$, $\Rightarrow y = \frac{3D\lambda}{4d}$

$\therefore \phi = \frac{2\pi x}{\lambda} = \frac{2\pi}{\lambda} \times \frac{yd}{D} = \frac{2\pi}{\lambda} \times \frac{3D\lambda}{4d} \times \frac{d}{D} = \frac{3\pi}{2}$

Let, I″ be the intensity at S_3 and S_4 when, $\phi = 3\pi/2$

Now comparing,

$\frac{I''}{I} = \frac{a^2 + a^2 + 2a^2 \cos(3\pi/2)}{a^2 + a^2 + 2a^2 \cos \pi/2} = \frac{2a^2}{2a^2} = 1$ $\Rightarrow I'' = I' = I/4.$

$\therefore$ Intensity at P = I/4 + I/4 + 2 × (I/4) cos 0° = I/2 + I/2 = I.

c) When $z = 2D\lambda/d$

$\Rightarrow y = OS_3 = OS_4 = D\lambda/d$

$\therefore \phi = \frac{2\pi x}{\lambda} = \frac{2\pi}{\lambda} \times \frac{yd}{D} = \frac{2\pi}{\lambda} \times \frac{D\lambda}{d} \times \frac{d}{D} = 2\pi$.

Let, I‴ = intensity at S_3 and S_4 when, $\phi = 2\pi$.

$\frac{I'''}{I'} = \frac{a^2 + a^2 + 2a^2 \cos 2\pi}{a^2 + a^2 + 2a^2 \cos \pi/2} = \frac{4a^2}{2a^2} = 2$

$\Rightarrow I''' = 2I' = 2(I/4) = I/2$

At P, $I_{resultant}$ = I/2 + I/2 + 2(I/2) cos 0° = I + I = 2I.

So, the resultant intensity at P will be 2I.

37. Given $d = 0.0011 \times 10^{-3}$ m

For minimum reflection of light, $2\mu d = n\lambda$

$\Rightarrow \mu = \frac{n\lambda}{2d} = \frac{2n\lambda}{4d} = \frac{580 \times 10^{-9} \times 2n}{4 \times 11 \times 10^{-7}} = \frac{5.8}{44}(2n) = 0.132\ (2n)$

Given that, μ has a value in between 1.2 and 1.5.

$\Rightarrow$ When, n = 5, $\mu = 0.132 \times 10 = 1.32$.

38. Given that, $\lambda = 560 \times 10^{-9}$ m, $\mu = 1.4$.

For strong reflection, $2\mu d = (2n + 1)\lambda/2 \Rightarrow d = \frac{(2n+1)\lambda}{4d}$

For minimum thickness, putting n = 0.

$\Rightarrow d = \frac{\lambda}{4d} \Rightarrow d = \frac{560 \times 10^{-9}}{14} = 10^{-7}$ m = 100 nm.

39. For strong transmission, $2\ \mu d = n\lambda \Rightarrow \lambda = \frac{2\mu d}{n}$

Given that, $\mu = 1.33$, $d = 1 \times 10^{-4}$ cm $= 1 \times 10^{-6}$ m.

$\Rightarrow \lambda = \frac{2 \times 1.33 \times 1 \times 10^{-6}}{n} = \frac{2660 \times 10^{-9}}{n}$ m

when, n = 4, λ_1 = 665 nm

n = 5, λ_2 = 532 nm

n = 6, λ_3 = 443 nm

40. For the thin oil film,

$d = 1 \times 10^{-4}$ cm $= 10^{-6}$ m, $\mu_{oil} = 1.25$ and $\mu_x = 1.50$

$\lambda = \frac{2\mu d}{(n+1/2)} \frac{2 \times 10^{-6} \times 1.25 \times 2}{2n+1} = \frac{5 \times 10^{-6} m}{2n+1}$

$\Rightarrow \lambda = \frac{5000\ nm}{2n+1}$

For the wavelengths in the region (400 nm – 750 nm)

When, n = 3, $\lambda = \frac{5000}{2 \times 3 + 1} = \frac{5000}{7}$ = 714.3 nm

When, n = 4, $\lambda = \frac{5000}{2 \times 4 + 1} = \frac{5000}{9}$ = 555.6 nm

When, n = 5, $\lambda = \frac{5000}{2 \times 5 + 1} = \frac{5000}{11}$ = 454.5 nm

41. For first minimum diffraction, b sin $\theta = \lambda$
Here, $\theta = 30°$, b = 5 cm
$\therefore \lambda = 5 \times \sin 30° = 5/2 = 2.5$ cm.

42. $\lambda = 560$ nm $= 560 \times 10^{-9}$ m, b = 0.20 mm = 2×10^{-4} m, D = 2 m
Since, $R = 1.22\frac{\lambda D}{b} = 1.22 \times \frac{560 \times 10^{-9} \times 2}{2 \times 10^{-4}}$ = 6.832×10^{-3} M = 0.683 cm.
So, Diameter = 2R = 1.37 cm.

43. $\lambda = 620$ nm $= 620 \times 10^{-9}$ m,
D = 20 cm = 20×10^{-2} m, b = 8 cm = 8×10^{-2} m
$\therefore R = 1.22 \times \frac{620 \times 10^{-4} \times 20 \times 10^{-2}}{8 \times 10^{-2}}$ = $1891 \times 10^{-9} = 1.9 \times 10^{-6}$ m
So, diameter = 2R = 3.8×10^{-6} m

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SOLUTIONS TO CONCEPTS
CHAPTER – 18

SIGN CONVENTION :

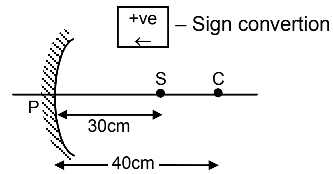
1) The direction of incident ray (from object to the mirror or lens) is taken as positive direction.
2) All measurements are taken from pole (mirror) or optical centre (lens) as the case may be.

1. u = –30 cm, R = – 40 cm

From the mirror equation,

$$\frac{1}{v}+\frac{1}{u}=\frac{2}{R}$$

$$\Rightarrow \frac{1}{v}=\frac{2}{R}-\frac{1}{u}=\frac{2}{-40}-\frac{1}{-30} = -\frac{1}{60}$$



or, v = –60 cm

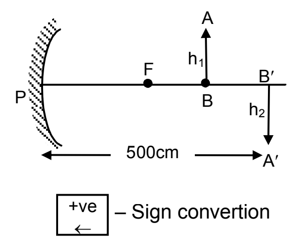
So, the image will be formed at a distance of 60 cm in front of the mirror.

2. Given that,

H_1 = 20 cm, v = –5 m = –500 cm, h_2 = 50 cm

Since, $\frac{-v}{u}=\frac{h_2}{h_1}$

or $\frac{500}{u}=-\frac{50}{20}$ (because the image in inverted)



or u = $-\frac{500\times 2}{5}$ = –200 cm = – 2 m

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f} \text{ or } \frac{1}{-5}+\frac{1}{-2}=\frac{1}{f}$$

or f = $\frac{-10}{7}$ = –1.44 m

So, the focal length is 1.44 m.

3. For the concave mirror, f = –20 cm, M = –v/u = 2

$\Rightarrow$ v = –2u

<u>1st case</u>

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

$$\Rightarrow \frac{1}{2u}-\frac{1}{u}=-\frac{1}{f}$$

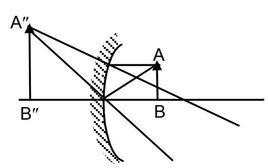
$\Rightarrow$ u = f/2 = 10 cm

<u>2nd case</u>

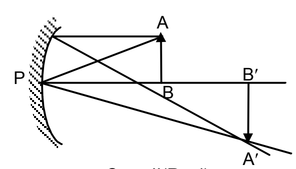
$$\frac{-1}{2u}-\frac{1}{u}=-\frac{1}{f}$$

$$\Rightarrow \frac{3}{2u}=\frac{1}{f}$$

$\Rightarrow$ u = 3f/2 = 30 cm



Case I (Virtual)



Case II(Real)

$\therefore$ The positions are 10 cm or 30 cm from the concave mirror.

4. m = –v/u = 0.6 and f = 7.5 cm = 15/2 cm

From mirror equation,

$$\Rightarrow \frac{1}{v}+\frac{1}{u}=\frac{1}{f} \Rightarrow \frac{1}{0.6u}-\frac{1}{u}=\frac{1}{f}$$

$\Rightarrow$ u = 5 cm

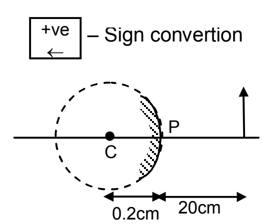
5. Height of the object AB = 1.6 cm

Diameter of the ball bearing = d = 0.4 cm

$\Rightarrow$ R = 0.2 cm

Given, u = 20 cm

We know, $\frac{1}{u}+\frac{1}{v}=\frac{2}{R}$



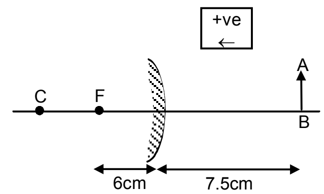
Putting the values according to sign conventions $\frac{1}{-20}+\frac{1}{v}=\frac{2}{0.2}$

$\Rightarrow \frac{1}{v}=\frac{1}{20}+10=\frac{201}{20}$ $\Rightarrow$ v = 0.1 cm = 1 mm inside the ball bearing.

Magnification = m = $\frac{A'B'}{AB}=-\frac{v}{u}=-\frac{0.1}{-20}=\frac{1}{200}$

$\Rightarrow$ A′B′ = $\frac{AB}{200}=\frac{16}{200}$ = +0.008 cm = +0.8 mm.

6. Given AB = 3 cm, u = –7.5 cm, f = 6 cm.



Using $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$ $\Rightarrow$ $\frac{1}{v}=\frac{1}{f}-\frac{1}{u}$

Putting values according to sign conventions,

$\frac{1}{v}=\frac{1}{6}-\frac{1}{-7.5}=\frac{3}{10}$

$\Rightarrow$ v = 10/3 cm

$\therefore$ magnification = m = $-\frac{v}{u}=\frac{10}{7.5\times 3}$

$\Rightarrow \frac{A'B'}{AB}=\frac{10}{7.5\times 3}\Rightarrow A'B'=\frac{100}{72}=\frac{4}{3}=1.33$ cm.

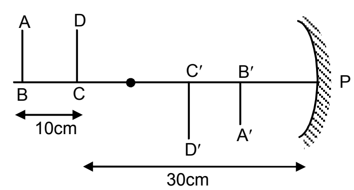
$\therefore$ Image will form at a distance of 10/3 cm. From the pole and image is 1.33 cm (virtual and erect).

7. R = 20 cm, f = R/2 = –10 cm

For part AB, PB = 30 + 10 = 40 cm

So, u = –40 cm $\Rightarrow \frac{1}{v}=\frac{1}{f}-\frac{1}{u}=-\frac{1}{10}-\left(\frac{1}{-40}\right)=-\frac{3}{40}$

$\Rightarrow$ v = $-\frac{40}{3}$ = –13.3 cm.



So, PB′ = 13.3 cm

m = $\frac{A'B'}{AB}=-\left(\frac{v}{u}\right)=-\left(\frac{-13.3}{-40}\right)=-\frac{1}{3}$

$\Rightarrow$ A′B′ = –10/3 = –3.33 cm

For part CD, PC = 30, So, u = –30 cm

$\frac{1}{v}=\frac{1}{f}-\frac{1}{u}=-\frac{1}{10}-\left(-\frac{1}{30}\right)=-\frac{1}{15}$ $\Rightarrow$ v = –15 cm = PC′

So, m = $\frac{C'D'}{CD}=-\frac{v}{u}=-\left(\frac{-15}{-30}\right)=-\frac{1}{2}$

$\Rightarrow$ C′D′ = 5 cm

B′C′ = PC′ – PB′ = 15 – 13.3 = 17 cm

So, total length A′B′ + B′C′ + C′D′ = 3.3 + 1.7 + 5 = 10 cm.

8. u = –25 cm

m = $\frac{A'B'}{AB}=-\frac{v}{u}\Rightarrow 1.4=-\left(\frac{v}{-25}\right)\Rightarrow\frac{14}{10}=\frac{v}{25}$

$\Rightarrow$ v = $\frac{25\times 14}{10}$ = 35 cm.

Now, $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$

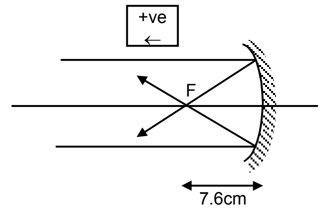
$\Rightarrow \frac{1}{f}=\frac{1}{35}-\left(\frac{1}{-25}\right)=\frac{5-7}{175}=-\frac{2}{175}$ $\Rightarrow$ f = –87.5 cm.

So, focal length of the concave mirror is 87.5 cm.

9. $u = -3.8 \times 10^5$ km

diameter of moon = 3450 km ; f = –7.6 m

$\therefore \frac{1}{v} + \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} + \left(-\frac{1}{3.8 \times 10^5}\right) = \left(-\frac{1}{7.6}\right)$



Since, distance of moon from earth is very large as compared to focal length it can be taken as ∞.

⇒ Image will be formed at focus, which is inverted.

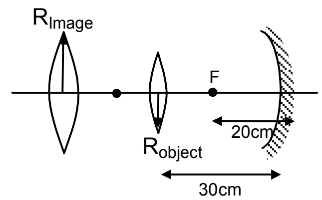
$\Rightarrow \frac{1}{v} = -\left(\frac{1}{7.6}\right) \Rightarrow v = -7.6$ m.

$m = -\frac{v}{u} = \frac{d_{image}}{d_{object}} \Rightarrow \frac{-(-7.6)}{(-3.8 \times 10^8)} = \frac{d_{image}}{3450 \times 10^3}$

$d_{image} = \frac{3450 \times 7.6 \times 10^3}{3.8 \times 10^8}$ = 0.069 m = 6.9 cm.

10. u = –30 cm, f = –20 cm

We know, $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$



$\Rightarrow \frac{1}{v} + \left(-\frac{1}{30}\right) = \left(-\frac{1}{20}\right) \Rightarrow v = -60$ cm.

Image of the circle is formed at a distance 60 cm in front of the mirror.

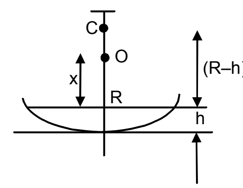
$\therefore m = -\frac{v}{u} = \frac{R_{image}}{R_{object}} \Rightarrow -\frac{-60}{-30} = \frac{R_{image}}{2}$

$\Rightarrow R_{image} = 4$ cm

Radius of image of the circle is 4 cm.

11. Let the object be placed at a height x above the surface of the water.

The apparent position of the object with respect to mirror should be at the centre of curvature so that the image is formed at the same position.



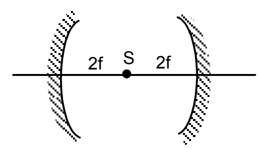
Since, $\frac{\text{Real depth}}{\text{Apparent depth}} = \frac{1}{\mu}$ (with respect to mirror)

Now, $\frac{x}{R-h} = \frac{1}{\mu} \Rightarrow x = \frac{R-h}{\mu}$.

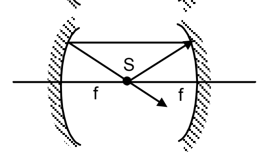
12. Both the mirrors have equal focal length f.

They will produce one image under two conditions.

Case I : When the source is at distance '2f' from each mirror i.e. the source is at centre of curvature of the mirrors, the image will be produced at the same point S. So, d = 2f + 2f = 4f.



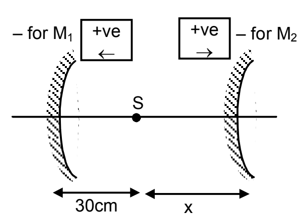
Case II : When the source S is at distance 'f' from each mirror, the rays from the source after reflecting from one mirror will become parallel and so these parallel rays after the reflection from the other mirror the object itself. So, only sine image is formed.



Here, d = f + f = 2f.

13. As shown in figure, for 1st reflection in M_1, u = –30 cm, f = –20 cm

$\Rightarrow \frac{1}{v} + \frac{1}{-30} = -\frac{1}{20} \Rightarrow v = -60$ cm.



So, for 2nd reflection in M_2

u = 60 – (30 + x) = 30 – x

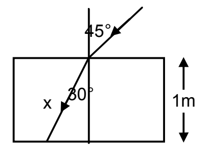
v = –x ; f = 20 cm

$\Rightarrow \frac{1}{30-x} - \frac{1}{x} = \frac{1}{20} \Rightarrow x^2 + 10x - 600 = 0$

$\Rightarrow$ x = $\frac{10 \pm 50}{2} = \frac{40}{2}$ = 20 cm or –30 cm

$\therefore$ Total distance between the two lines is 20 + 30 = 50 cm.

14. We know, $\frac{\sin i}{\sin r} = \frac{3 \times 10^8}{v} = \frac{\sin 45°}{\sin 30°} = \sqrt{2}$



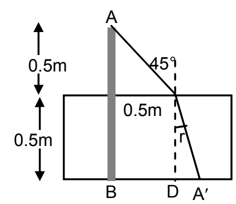
$\Rightarrow$ v = $\frac{3 \times 10^8}{\sqrt{2}}$ m/sec.

Distance travelled by light in the slab is,

x = $\frac{1\text{ m}}{\cos 30°} = \frac{2}{\sqrt{3}}$ m

So, time taken = $\frac{2 \times \sqrt{2}}{\sqrt{3} \times 3 \times 10^8}$ = 0.54 × 10^{-8} = 5.4 × 10^{-9} sec.

15. Shadow length = BA′ = BD + A′D = 0.5 + 0.5 tan r



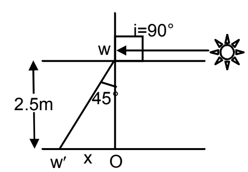
Now, 1.33 = $\frac{\sin 45°}{\sin r}$ $\Rightarrow$ sin r = 0.53.

$\Rightarrow$ cos r = $\sqrt{1 - \sin^2 r} = \sqrt{1 - (0.53)^2}$ = 0.85

So, tan r = 0.6235

So, shadow length = (0.5) (1 + 0.6235) = 81.2 cm.

16. Height of the lake = 2.5 m



When the sun is just setting, θ is approximately = 90°

$\therefore$ $\frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1} \Rightarrow \frac{1}{\sin r} = \frac{4/3}{1} \Rightarrow \sin r = \frac{3}{4} \Rightarrow$ r = 49°

As shown in the figure, x/2.5 = tan r = 1.15

$\Rightarrow$ x = 2.5 × 1.15 = 2.8 m.

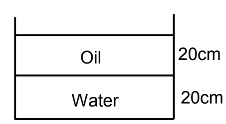
17. The thickness of the glass is d = 2.1 cm and μ =1.5

Shift due to the glass slab

$\Delta T = \left(1 - \frac{1}{\mu}\right)d = \left(1 - \frac{1}{1.5}\right)2.1$ = 0.7 CM

So, the microscope should be shifted 0.70 cm to focus the object again.

18. Shift due to water $\Delta t_w = \left(1 - \frac{1}{\mu}\right)d = \left(1 - \frac{1}{1.33}\right)20$ = 5 cm

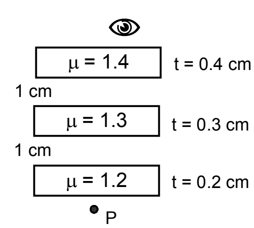


Shift due to oil, $\Delta t_o = \left(1 - \frac{1}{1.3}\right)20$ = 4.6 cm

Total shift Δt = 5 + 4.6 = 9.6 cm

Apparent depth = 40 – (9.6) = 30.4 cm below the surface.

19. The presence of air medium in between the sheets does not affect the shift.



The shift will be due to 3 sheets of different refractive index other than air.

$= \left(1 - \frac{1}{1.2}\right)(0.2) + \left(1 - \frac{1}{13}\right)(0.3) + \left(1 - \frac{1}{14}\right)(0.4)$

= 0.2 cm above point P.

20. Total no. of slabs = k, thickness = $t_1, t_2, t_3 \ldots t_k$

Refractive index = $\mu_1, \mu_2, \mu_3, \mu_4, \ldots \mu_k$

$\therefore$ The shift $\Delta t = \left(1 - \frac{1}{\mu_1}\right)t_1 + \left(1 - \frac{1}{\mu_2}\right)t_2 + \ldots\ldots + \left(1 - \frac{1}{\mu_k}\right)t_k$ …(1)

If, $\mu \rightarrow$ refractive index of combination of slabs and image is formed at same place,

$\Delta t = \left(1 - \frac{1}{\mu}\right)(t_1 + t_2 + \ldots + t_k)$ …(2)

Equation (1) and (2), we get,

$$\left(1-\frac{1}{\mu}\right)(t_1+t_2+...+t_k) = \left(1-\frac{1}{\mu_1}\right)t_1+\left(1-\frac{1}{\mu_2}\right)t_2+......+\left(1-\frac{1}{\mu_k}\right)t_k$$

$$= (t_1+t_2+...+t_k)-\left(\frac{t_1}{\mu_1}+\frac{t_2}{\mu_2}+...+\frac{t_k}{\mu_k}\right)$$

$$= -\frac{1}{\mu}\sum_{i=1}^{k} t_1 = -\sum_{i=1}^{k}\left(\frac{t_1}{\mu_1}\right) \Rightarrow \mu = \frac{\sum_{i=1}^{k} t_i}{\sum_{i=1}^{k}(t_1/\mu_1)}.$$

21. Given r = 6 cm, r_1 = 4 cm, h_1 = 8 cm

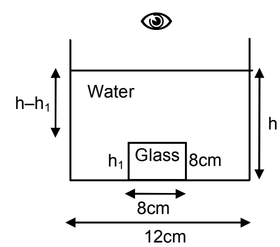
Let, h = final height of water column.

The volume of the cylindrical water column after the glass piece is put will be,

$\pi r^2 h = 800\,\pi + \pi r_1^2 h_1$

or $r^2 h = 800 + r_1^2 h_1$

or 6^2 h = 800 + $4^2 \times 8$ = 25.7 cm



There are two shifts due to glass block as well as water.

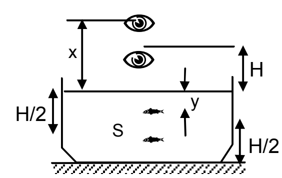
So, $\Delta t_1 = \left(1-\frac{1}{\mu_0}\right)t_0 = \left(1-\frac{1}{3/2}\right)8$ = 2.26 cm

And, $\Delta t_2 = \left(1-\frac{1}{\mu_w}\right)t_w = \left(1-\frac{1}{4/3}\right)(25.7-8)$ = 4.44 cm.

Total shift = (2.66 + 4.44) cm = 7.1 cm above the bottom.

22. a) Let x = distance of the image of the eye formed above the surface as seen by the fish

So, $\frac{H}{x} = \frac{\text{Real depth}}{\text{Apparent depth}} = \frac{1}{\mu}$ or x = μH



So, distance of the direct image = $\frac{H}{2}+\mu H = H(\mu+\frac{1}{2})$

Similarly, image through mirror = $\frac{H}{2}+(H+x) = \frac{3H}{2}+\mu H = H(\mu+\frac{3}{2})$

b) Here, $\frac{H/2}{y} = \mu$, so, y = $\frac{H}{2\mu}$

Where, y = distance of the image of fish below the surface as seen by eye.

So, Direct image = H + y = $H+\frac{H}{2\mu} = H\left(1+\frac{1}{2\mu}\right)$

Again another image of fish will be formed H/2 below the mirror.

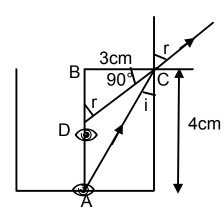
So, the real depth for that image of fish becomes H + H/2 = 3H/2

So, Apparent depth from the surface of water = 3H/2μ

So, distance of the image from the eye = $H+\frac{3H}{2\mu} = H(1+\frac{3}{2\mu})$.

23. According to the figure, x/3 = cot r …(1)

Again, $\frac{\sin i}{\sin r} = \frac{1}{1.33} = \frac{3}{4}$



$\Rightarrow$ sin r = $\frac{4}{3}\sin i = \frac{4}{3}\times\frac{3}{5} = \frac{4}{5}$ (because sin i = $\frac{BC}{AC} = \frac{3}{5}$)

$\Rightarrow$ cot r = 3/4 …(2)

From (1) and (2) $\Rightarrow$ x/3 = ¾

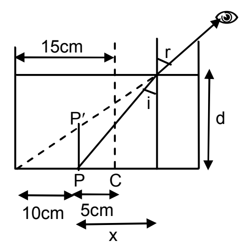
$\Rightarrow$ x = 9/4 = 2.25 cm.

$\therefore$ Ratio of real and apparent depth = 4 : (2.25) = 1.78.

24. For the given cylindrical vessel, dimetre = 30 cm

$\Rightarrow$ r = 15 cm and h = 30 cm

Now, $\frac{\sin i}{\sin r} = \frac{3}{4}\left[\mu_w = 1.33 = \frac{4}{3}\right]$



$\Rightarrow$ sin i = $3/4\sqrt{2}$ [because r = 45°]

The point P will be visible when the refracted ray makes angle 45° at point of refraction.

Let x = distance of point P from X.

Now, tan 45° = $\frac{x+10}{d}$

$\Rightarrow$ d = x + 10 …(1)

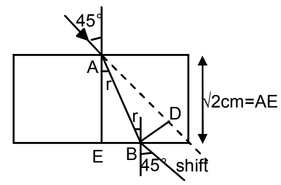
Again, tan i = x/d

$\Rightarrow \frac{3}{\sqrt{23}} = \frac{d-10}{d}$ $\left[\text{since, } \sin i = \frac{3}{4\sqrt{2}} \Rightarrow \tan i = \frac{3}{\sqrt{23}}\right]$

$\Rightarrow \frac{3}{\sqrt{23}} - 1 = -\frac{10}{d} \Rightarrow d = \frac{\sqrt{23}\times 10}{\sqrt{23}-3}$ = 26.7 cm.

25. As shown in the figure,

$\frac{\sin 45°}{\sin r} = \frac{2}{1} \Rightarrow \sin r = \frac{\sin 45°}{2} = \frac{1}{2\sqrt{2}} \Rightarrow r = 21°$



Therefore, θ = (45° – 21°) = 24°

Here, BD = shift in path = AB sin 24°

= $0.406 \times AB = \frac{AE}{\cos 21°} \times 0.406$ = 0.62 cm.

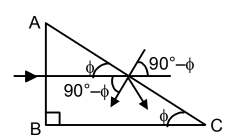
26. For calculation of critical angle,

$\frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1} \Rightarrow \frac{\sin C}{\sin 90} = \frac{15}{1.72} = \frac{75}{86}$

$\Rightarrow$ C = $\sin^{-1}\left(\frac{75}{26}\right)$.

27. Let θ_c be the critical angle for the glass

$\frac{\sin\theta_c}{\sin 90°} = \frac{1}{x} \Rightarrow \sin\theta_c = \frac{1}{1.5} = \frac{2}{3} \Rightarrow \theta_c = \sin^{-1}\left(\frac{2}{3}\right)$



From figure, for total internal reflection, 90° – $\phi > \theta_c$

$\Rightarrow \phi < 90° - \theta_c \Rightarrow \phi < \cos^{-1}(2/3)$

So, the largest angle for which light is totally reflected at the surface is $\cos^{-1}(2/3)$.

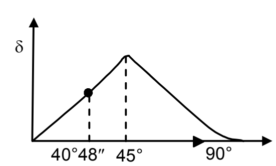
28. From the definition of critical angle, if refracted angle is more than 90°, then reflection occurs, which is known as total internal reflection.

So, maximum angle of refraction is 90°.

29. Refractive index of glass μ_g = 1.5

Given, 0° < i < 90°

Let, C $\rightarrow$ Critical angle.



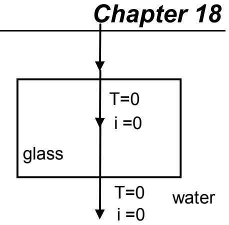
$\frac{\sin C}{\sin r} = \frac{\mu_a}{\mu_g} \Rightarrow \frac{\sin C}{\sin 90°} = \frac{1}{15}$ = 0.66

$\Rightarrow$ C = 40°48″

The angle of deviation due to refraction from glass to air increases as the angle of incidence increases from 0° to 40°48″. The angle of deviation due to total internal reflection further increases for 40°48″ to 45° and then it decreases.

30. μ_g = 1.5 = 3/2 ; μ_w = 1.33 = 4/3

For two angles of incidence,

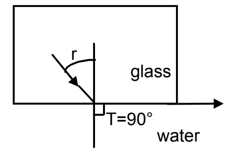


1) When light passes straight through normal,

$\Rightarrow$ Angle of incidence = 0°, angle of refraction = 0°, angle of deviation = 0

2) When light is incident at critical angle,

$$\frac{\sin C}{\sin r} = \frac{\mu_w}{\mu_g} \quad \text{(since light passing from glass to water)}$$



$\Rightarrow$ sin C = 8/9 $\Rightarrow$ C = $\sin^{-1}(8/9)$ = 62.73°.

$\therefore$ Angle of deviation = 90° – C = 90° – $\sin^{-1}(8/9)$ = $\cos^{-1}(8/9)$ = 37.27°

Here, if the angle of incidence is increased beyond critical angle, total internal reflection occurs and deviation decreases. So, the range of deviation is 0 to $\cos^{-1}(8/9)$.

31. Since, μ = 1.5, Critial angle = $\sin^{-1}(1/\mu)$ = $\sin^{-1}$ (1/1.5) = 41.8°

We know, the maximum attainable deviation in refraction is (90° – 41.8°) = 47.2°

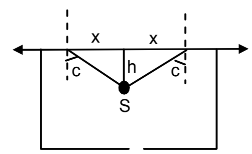
So, in this case, total internal reflection must have taken place.

In reflection,

Deviation = 180° – 2i = 90° $\Rightarrow$ 2i = 90° $\Rightarrow$ i = 45°.

32. a) Let, x = radius of the circular area

$$\frac{x}{h} = \tan C \quad \text{(where C is the critical angle)}$$



$$\Rightarrow \frac{x}{h} = \frac{\sin C}{\sqrt{1-\sin^2 C}} = \frac{1/\mu}{\sqrt{1-\frac{1}{\mu^2}}} \quad \text{(because sin C = } 1/\mu)$$

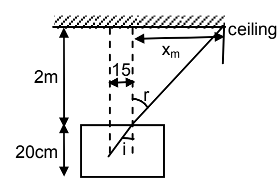
$$\Rightarrow \frac{x}{h} = \frac{1}{\sqrt{\mu^2-1}} \text{ or } x = \frac{h}{\sqrt{\mu^2-1}}$$

So, light escapes through a circular area on the water surface directly above the point source.

b) Angle subtained by a radius of the area on the source, C = $\sin^{-1}(1/\mu)$.

33. a) As shown in the figure, sin i = 15/25

So, $\frac{\sin i}{\sin r} = \frac{1}{\mu} = \frac{3}{4}$



$\Rightarrow$ sin r = 4/5

Again, x/2 = tan r (from figure)

So, sin r = $\frac{\tan r}{\sqrt{1+\tan^2 r}} = \frac{x/2}{\sqrt{1-x^2/4}}$

$$\Rightarrow \frac{x}{\sqrt{4+x^2}} = \frac{4}{5}$$

$\Rightarrow$ $25x^2 = 16(4 + x^2)$ $\Rightarrow$ $9x^2 = 64$ $\Rightarrow$ x = 8/3 m

$\therefore$ Total radius of shadow = 8/3 + 0.15 = 2.81 m

b) For maximum size of the ring, i = critical angle = C

Let, R = maximum radius

$\Rightarrow$ sin C = $\frac{\sin C}{\sin r} = \frac{R}{\sqrt{20^2+R^2}} = \frac{3}{4}$ (since, sin r = 1)

$\Rightarrow$ $16R^2 = 9R^2 + 9 \times 400$

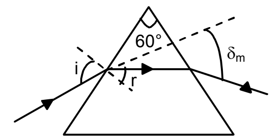
$\Rightarrow$ $7R^2 = 9 \times 400$

$\Rightarrow$ R = 22.67 cm.

34. Given, A = 60°, μ = 1.732

Since, angle of minimum deviation is given by,

$$\mu = \frac{\sin\left(\frac{A+\delta m}{2}\right)}{\sin A/2} \Rightarrow 1.732 \times ½ = \sin(30 + \delta m/2)$$



$\Rightarrow \sin^{-1}(0.866) = 30 + \delta m/2 \Rightarrow 60° = 30\ \delta m/2 \Rightarrow \delta m = 60°$

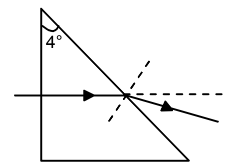
Now, $\delta m = i + i' - A$

$\Rightarrow$ 60° = i + i′ – 60° (δ = 60° minimum deviation)

$\Rightarrow$ i = 60°. So, the angle of incidence must be 60°.

35. Given μ = 1.5

And angle of prism = 4°



$$\therefore\ \mu = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin A/2} = \frac{(A+\delta_m)/2}{(A/2)} \quad \text{(for small angle } \sin\theta = \theta)$$

$$\Rightarrow\ \mu = \frac{A+\delta_m}{2} \Rightarrow 1.5 = \frac{4°+\delta_m}{4°} \Rightarrow \delta_m = 4° \times (1.5) - 4° = 2°.$$

36. Given A = 60° and δ = 30°

We know that,

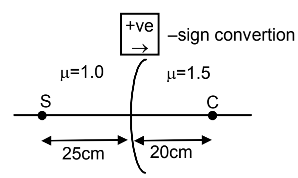
$$\mu = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin A/2} = \frac{\sin\frac{60°+\delta_m}{2}}{\sin 30°} = 2\sin\frac{60°+\delta_m}{2}$$

Since, one ray has been found out which has deviated by 30°, the angle of minimum deviation should be either equal or less than 30°. (It can not be more than 30°).

So, $\mu \le 2\sin\frac{60°+\delta_m}{2}$ (because μ will be more if δ_m will be more)

or, $\mu \le 2 \times 1/\sqrt{2}$ or, $\mu \le \sqrt{2}$.

37. μ_1 = 1, μ_2 = 1.5, R = 20 cm (Radius of curvature), u = –25 cm



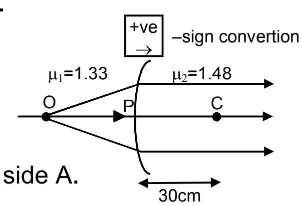
$$\therefore\ \frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2-\mu_1}{R} \Rightarrow \frac{1.5}{v} = \frac{0.5}{20} - \frac{1}{25} = \frac{1}{40} - \frac{1}{25} = \frac{-3}{200}$$

$\Rightarrow$ v = –200 × 0.5 = –100 cm.

So, the image is 100 cm from (P) the surface on the side of S.

38. Since, paraxial rays become parallel after refraction i.e. image is formed at ∞.

v = ∞, μ_1 = 1.33, u = ?, μ_2 = 1.48, R = 30 cm



$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2-\mu_1}{R} \Rightarrow \frac{1.48}{\infty} - \frac{1.33}{u} = \frac{1.48-1.33}{30} \Rightarrow -\frac{1.33}{u} - \frac{0.15}{30}$$

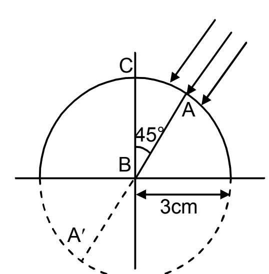
$\Rightarrow$ u = –266.0 cm

$\therefore$ Object should be placed at a distance of 266 cm from surface (convex) on side A.

39. Given, μ_2 = 2.0

So, critical angle = $\sin^{-1}\left(\frac{1}{\mu_2}\right) = \sin^{-1}\left(\frac{1}{2}\right)$ = 30°

a) As angle of incidence is greater than the critical angle, the rays are totally reflected internally.



b) Here, $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2-\mu_1}{R}$

$\Rightarrow \frac{2}{v} - \left(-\frac{1}{\infty}\right) = \frac{2-1}{3}$ [For parallel rays, u = ∞]

$\Rightarrow \frac{2}{v} = \frac{1}{3} \Rightarrow$ v = 6 cm

$\Rightarrow$ If the sphere is completed, image is formed diametrically opposite of A.

c) Image is formed at the mirror in front of A by internal reflection.

40. a) Image seen from left :

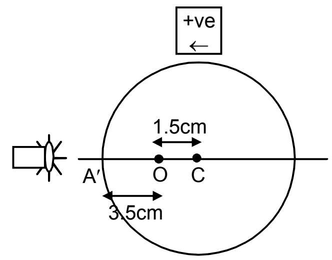
$u = (5 - 15) = -3.5$ cm

$R = -5$ cm

$\therefore \frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R} \Rightarrow \frac{1}{v} + \frac{1.5}{3.5} = -\frac{1-1.5}{5}$

$\Rightarrow \frac{1}{v} = \frac{1}{10} - \frac{3}{7} \Rightarrow v = \frac{-70}{23} = -3$ cm (inside the sphere).

$\Rightarrow$ Image will be formed, 2 cm left to centre.



b) Image seen from right :

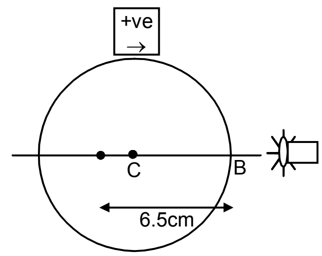
$u = -(5 + 1.5) = -6.5$ cm

$R = -5$ cm

$\therefore \frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R} \Rightarrow \frac{1}{v} + \frac{1.5}{6.5} = \frac{1-1.5}{-5}$

$\Rightarrow \frac{1}{v} = \frac{1}{10} - \frac{3}{13} \Rightarrow v = -\frac{130}{17} = -7.65$ cm (inside the sphere).

$\Rightarrow$ Image will be formed, 2.65 cm left to centre.



41. $R_1 = R_2 = 10$ cm, $t = 5$ cm, $u = -\infty$

For the first refraction, (at A)

$\frac{\mu_g}{v} - \frac{\mu_a}{u} = \frac{\mu_g - \mu_a}{R_1}$ or $\frac{1.5}{v} - 0 = \frac{1.5}{10}$

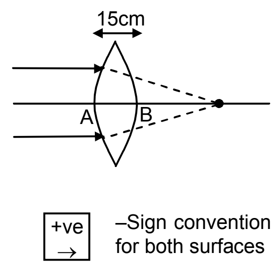
$\Rightarrow v = 30$ cm.

Again, for 2nd surface, $u = (30 - 5) = 25$ cm (virtual object)

$R_2 = -10$ cm

So, $\frac{1}{v} - \frac{15}{25} = \frac{-0.5}{-10} \Rightarrow v = 9.1$ cm.

So, the image is formed 9.1 cm further from the 2nd surface of the lens.



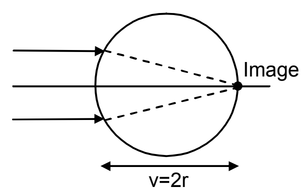
42. For the refraction at convex surface A.

$\mu = -\infty$, $\mu_1 = 1$, $\mu_2 = ?$

a) When focused on the surface, $v = 2r$, $R = r$

So, $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

$\Rightarrow \frac{\mu_2}{2r} = \frac{\mu_2 - 1}{r} \Rightarrow \mu_2 = 2\mu_2 - 2 \Rightarrow \mu_2 = 2$



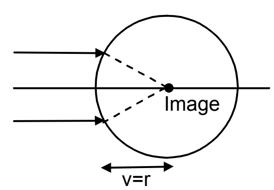
b) When focused at centre, $u = r_1$, $R = r$

So, $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

$\Rightarrow \frac{\mu_2}{R} = \frac{\mu_2 - 1}{r} \Rightarrow \mu_2 = \mu_2 - 1$.

This is not possible.

So, it cannot focus at the centre.



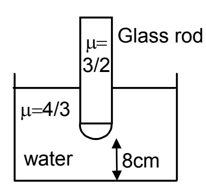
43. Radius of the cylindrical glass tube = 1 cm

We know, $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

Here, $u = -8$ cm, $\mu_2 = 3/2$, $\mu_1 = 4/3$, $R = +1$ cm

So, $\frac{3}{2v} + \frac{4}{3 \times 8} \Rightarrow \frac{3}{2v} + \frac{1}{6} = \frac{1}{6}$ $v = \infty$

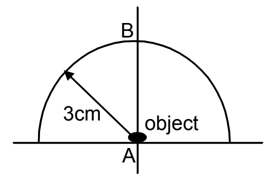
$\therefore$ The image will be formed at infinity.



44. In the first refraction at A.

$\mu_2 = 3/2$, $\mu_1 = 1$, $u = 0$, $R = \infty$

So, $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$



$\Rightarrow$ v = 0 since (R $\Rightarrow \infty$ and u = 0)

$\therefore$ The image will be formed at the point, Now for the second refraction at B,

u = –3 cm, R = –3 cm, $\mu_1 = 3/2$, $\mu_2 = 1$

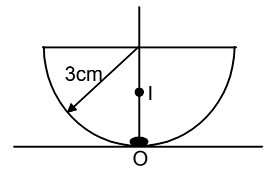
So, $\frac{1}{v} + \frac{3}{2 \times 3} = \frac{1 - 1.5}{-3} = \frac{1}{6}$

$\Rightarrow \frac{1}{v} = \frac{1}{6} - \frac{1}{2} = -\frac{1}{3}$

$\Rightarrow$ v = –3 cm, $\therefore$ There will be no shift in the final image.

45. Thickness of glass = 3 cm, $\mu_g = 1.5$

Image shift = $3\left(1 - \frac{1}{1.5}\right)$



[Treating it as a simple refraction problem because the upper surface is flat and the spherical surface is in contact with the object]

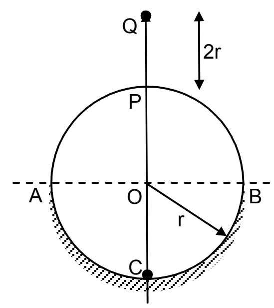
= $3 \times \frac{0.5}{1.5}$ = 1 cm.

The image will appear 1 cm above the point P.

46. As shown in the figure, OQ = 3r, OP = r

So, PQ = 2r

For refraction at APB



We know, $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

$\Rightarrow \frac{1.5}{v} - \frac{1}{-2r} = \frac{0.5}{r} = \frac{1}{2r}$ [because u = –2r]

$\Rightarrow v = \infty$

For the reflection in concave mirror

$u = \infty$

So, v = focal length of mirror = r/2

For the refraction of APB of the reflected image.

Here, u = –3r/2

$\frac{1}{v} - \frac{1.5}{-3r/2} = \frac{-0.5}{-r}$ [Here, $\mu_1 = 1.5$ and $\mu_2 = 1$ and R = –r]

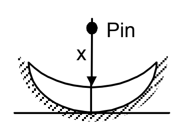
$\Rightarrow$ v = –2r

As, negative sign indicates images are formed inside APB. So, image should be at C.

So, the final image is formed on the reflecting surface of the sphere.

47. a) Let the pin is at a distance of x from the lens.

Then for 1st refraction, $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$



Here $\mu_2 = 1.5$, $\mu_1 = 1$, u = –x, R = –60 cm

$\therefore \frac{1.5}{v} - \frac{1}{-x} = \frac{0.5}{-60}$

$\Rightarrow$ 120(1.5x + v) = –vx …(1)

$\Rightarrow$ v(120 + x) = –180x

$\Rightarrow v = \frac{-180x}{120 + x}$

This image distance is again object distance for the concave mirror.

$u = \frac{-180x}{120+x}$, f = –10 cm ($\therefore$ f = R/2)

$\therefore\ \frac{1}{v}+\frac{1}{u}=\frac{1}{f} \Rightarrow \frac{1}{v_1}=\frac{1}{-10}-\frac{-(120+x)}{180x}$

$\Rightarrow\ \frac{1}{v_1}=\frac{120+x-18x}{180x} \Rightarrow v_1 = \frac{180x}{120-17x}$

Again the image formed is refracted through the lens so that the image is formed on the object taken in the 1st refraction. So, for 2nd refraction.

According to sign conversion v = –x, μ_2 = 1, μ_1 = 1.5, R = –60

Now, $\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R}$ $\left[u = \frac{180x}{120-17x}\right]$

$\Rightarrow\ \frac{1}{-x}-\frac{1.5}{180x}(120-17x)=\frac{-0.5}{-60}$

$\Rightarrow\ \frac{1}{x}+\frac{120-17x}{120x}=\frac{-1}{120}$

Multiplying both sides with 120 m, we get

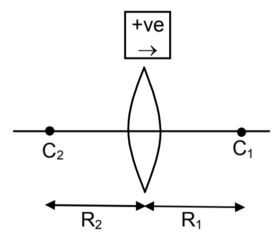
120 + 120 – 17x = –x

$\Rightarrow$ 16x = 240 $\Rightarrow$ x = 15 cm

$\therefore$ Object should be placed at 15 cm from the lens on the axis.

48. For the double convex lens

f = 25 cm, R_1 = R and R_2 = –2R (sign convention)



$\frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$

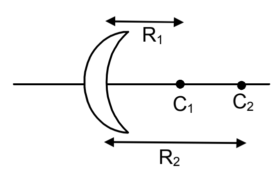
$\Rightarrow\ \frac{1}{25}=(15-1)\left(\frac{1}{R}-\frac{1}{-2R}\right) = 0.5\left(\frac{3R}{2}\right)$

$\Rightarrow\ \frac{1}{25}=\frac{3}{4}\frac{1}{R} \Rightarrow$ R = 18.75 cm

R_1 = 18.75 cm, R_2 = 2R = 37.5 cm.

49. R_1 = +20 cm ; R_2 = +30 cm ; μ = 1.6

a) If placed in air :



$\frac{1}{f}=(\mu_g-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) = \left(\frac{1.6}{1}-1\right)\left(\frac{1}{20}-\frac{1}{30}\right)$

$\Rightarrow$ f = 60/6 = 100 cm

b) If placed in water :

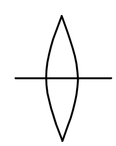
$\frac{1}{f}=(\mu_w-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) = \left(\frac{1.6}{1.33}-1\right)\left(\frac{1}{20}-\frac{1}{30}\right)$

$\Rightarrow$ f = 300 cm

50. Given μ = 1.5

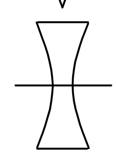
Magnitude of radii of curvatures = 20 cm and 30 cm

The 4types of possible lens are as below.



$\frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$

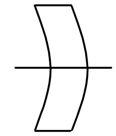
Case (1) : (Double convex) [R_1 = +ve, R_2 = –ve]



$\frac{1}{f}=(15-1)\left(\frac{1}{20}-\frac{1}{-30}\right) \Rightarrow$ f = 24 cm

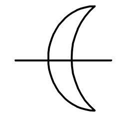
Case (2) : (Double concave) [R_1 = –ve, R_2 = +ve]

$\frac{1}{f}=(15-1)\left(\frac{-1}{20}-\frac{1}{30}\right) \Rightarrow$ f = –24 cm



Case (3) : (Concave concave) [R_1 = –ve, R_2 = –ve]

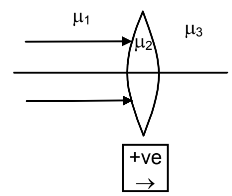
$$\frac{1}{f} = (15-1)\left(\frac{1}{-20} - \frac{1}{-30}\right) \Rightarrow f = -120 \text{ cm}$$



Case (4) : (Concave convex) [R_1 = +ve, R_2 = +ve]

$$\frac{1}{f} = (15-1)\left(\frac{1}{20} - \frac{1}{30}\right) \Rightarrow f = +120 \text{ cm}$$

51. a) When the beam is incident on the lens from medium μ_1.

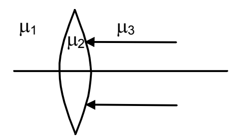


Then $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$ or $\frac{\mu_2}{v} - \frac{\mu_1}{(-\infty)} = \frac{\mu_2 - \mu_1}{R}$

or $\frac{1}{v} = \frac{\mu_2 - \mu_1}{\mu_2 R}$ or v = $\frac{\mu_2 R}{\mu_2 - \mu_1}$

Again, for 2nd refraction, $\frac{\mu_3}{v} - \frac{\mu_2}{u} = \frac{\mu_3 - \mu_2}{R}$

or, $\frac{\mu_3}{v} = -\left[\frac{\mu_3 - \mu_2}{R} - \frac{\mu_2}{\mu_2 R}(\mu_2 - \mu_1)\right] \Rightarrow -\left[\frac{\mu_3 - \mu_2 - \mu_2 + \mu_1}{R}\right]$



or, v = $-\left[\frac{\mu_3 R}{\mu_3 - 2\mu_2 + \mu_1}\right]$

So, the image will be formed at = $\frac{\mu_3 R}{2\mu_2 - \mu_1 - \mu_3}$

b) Similarly for the beam from μ_3 medium the image is formed at $\frac{\mu_1 R}{2\mu_2 - \mu_1 - \mu_3}$.

52. Given that, f = 10 cm

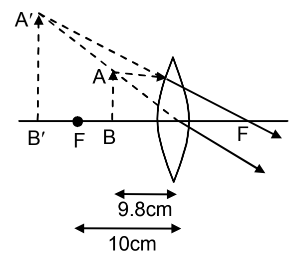
a) When u = –9.5 cm

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} = \frac{1}{10} - \frac{1}{9.8} = \frac{-0.2}{98}$$

⇒ v = – 490 cm

So, ⇒ m = $\frac{v}{u} = \frac{-490}{-9.8}$ = 50 cm

So, the image is erect and virtual.



(Virtual image)

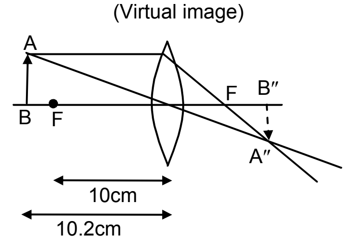
b) When u = –10.2 cm

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} = \frac{1}{10} - \frac{1}{-10.2} = \frac{102}{0.2}$$

⇒ v = 510 cm

So, m = $\frac{v}{u} = \frac{510}{-9.8}$

The image is real and inverted.



(Real image)

53. For the projector the magnification required is given by

m = $\frac{v}{u} = \frac{200}{3.5}$ ⇒ u = 17.5 cm

[35 mm > 23 mm, so the magnification is calculated taking object size 35 mm]

Now, from lens formula,

$$\Rightarrow \frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

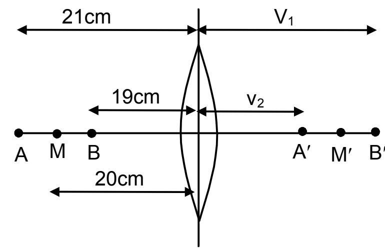
$$\Rightarrow \frac{1}{v} - \frac{1}{-u} = \frac{1}{f} \Rightarrow \frac{1}{1000} + \frac{1}{17.5} = \frac{1}{f}$$

⇒ f = 17.19 cm.

54. When the object is at 19 cm from the lens, let the image will be at, v_1.

$\Rightarrow \frac{1}{v_1} - \frac{1}{u_1} = \frac{1}{f} \Rightarrow \frac{1}{v_1} - \frac{1}{-19} = \frac{1}{12}$

$\Rightarrow v_1 = 32.57$ cm



Again, when the object is at 21 cm from the lens, let the image will be at, v_2

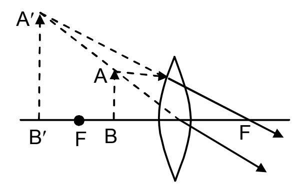
$\Rightarrow \frac{1}{v_2} - \frac{1}{u_2} = \frac{1}{f} \Rightarrow \frac{1}{v_2} + \frac{1}{21} = \frac{1}{12}$

$\Rightarrow v_2 = 28$ cm

$\therefore$ Amplitude of vibration of the image is A = $\frac{A'B'}{2} = \frac{v_1 - v_2}{2}$

$\Rightarrow$ A = $\frac{32.57 - 28}{2}$ = 2.285 cm.

55. Given, u = –5 cm, f = 8 cm



So, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{8} - \frac{1}{5} = \frac{-3}{40}$

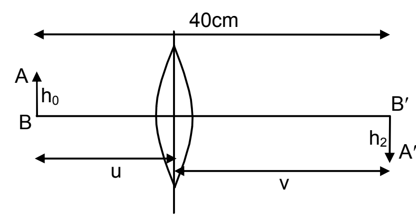
$\Rightarrow$ v = –13.3 cm (virtual image).

56. Given that,

(–u) + v = 40 cm = distance between object and image

h_o = 2 cm, h_i = 1 cm

Since $\frac{h_i}{h_o} = \frac{v}{-u}$ = magnification



$\Rightarrow \frac{1}{2} = \frac{v}{-u} \Rightarrow u = -2v$ …(1)

Now, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} + \frac{1}{2v} = \frac{1}{f}$

$\Rightarrow \frac{3}{2v} = \frac{1}{f} \Rightarrow f = \frac{2v}{3}$ …(2)

Again, (–u) + v = 40

$\Rightarrow$ 3v = 40 $\Rightarrow$ v = 40/3 cm

$\therefore$ f = $\frac{2 \times 40}{3 \times 3}$ = 8.89 cm = focal length

From eqn. (1) and (2)

u = –2v = –3f = –3(8.89) = 26.7 cm = object distance.

57. A real image is formed. So, magnification m = –2 (inverted image)

$\therefore \frac{v}{u} = -2 \Rightarrow v = -2u = (-2)(-18) = 36$

From lens formula, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{36} - \frac{1}{-18} = \frac{1}{f}$

$\Rightarrow$ f = 12 cm

Now, for triple sized image m = –3 = (v/u)

$\therefore \frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{-3u} - \frac{1}{u} = \frac{1}{12}$

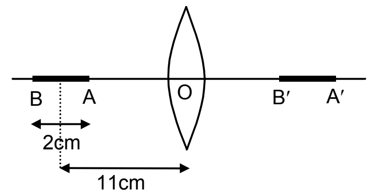
$\Rightarrow$ 3u = –48 $\Rightarrow$ u = –16 cm

So, object should be placed 16 cm from lens.

58. Now we have to calculate the image of A and B. Let the images be A′, B′. So, length of A′ B′ = size of image.

For A, u = –10 cm, f = 6 cm

Since, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} - \frac{1}{-10} = \frac{1}{6}$



$\Rightarrow$ v = 15 cm = OA′

For B, u = –12 cm, f = 6 cm

Again, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} = \frac{1}{6} - \frac{1}{12}$

$\Rightarrow$ v = 12 cm = OB′

$\therefore$ A′B′ = OA′ – OB′ = 15 – 12 = 3 cm.

So, size of image = 3 cm.

59. $u = -1.5 \times 10^{11}$ m ; $f = +20 \times 10^{-2}$ m

Since, f is very small compared to u, distance is taken as ∞. So, image will be formed at focus.

$\Rightarrow v = +20 \times 10^{-2}$ m

$\therefore$ We know, $m = \frac{v}{u} = \frac{h_{image}}{h_{object}}$

$\Rightarrow \frac{20 \times 10^{-2}}{1.5 \times 10^{11}} = \frac{D_{image}}{1.4 \times 10^{9}}$

$\Rightarrow D_{image}$ = 1.86 mm

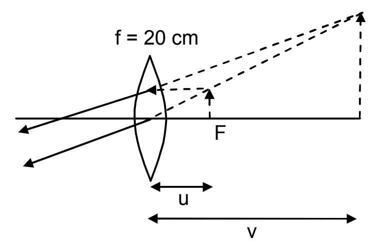
So, radius = $\frac{D_{image}}{2}$ = 0.93 mm.

60. Given, P = 5 diopter (convex lens)

$\Rightarrow$ f = 1/5 m = 20 cm

Since, a virtual image is formed, u and v both are negative.

Given, v/u = 4



$\Rightarrow$ v = 4u …(1)

From lens formula, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$\Rightarrow \frac{1}{f} = \frac{1}{4u} - \frac{1}{u} \Rightarrow \frac{1}{20} = \frac{1-4}{4u} = -\frac{3}{4u}$

$\Rightarrow$ u = –15 cm

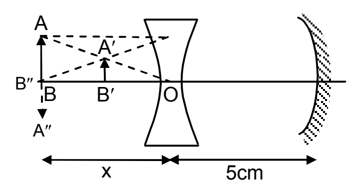
$\therefore$ Object is placed 15 cm away from the lens.

61. Let the object to placed at a distance x from the lens further away from the mirror.

For the concave lens (1st refraction)

u = –x, f = –20 cm

From lens formula,



$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} = \frac{1}{-20} + \frac{1}{-x}$

$\Rightarrow v = -\left(\frac{20x}{x+20}\right)$

So, the virtual image due to fist refraction lies on the same side as that of object. (A′B′)

This image becomes the object for the concave mirror.

For the mirror,

$u = -\left(5 + \frac{20x}{x+20}\right) = -\left(\frac{25x+100}{x+20}\right)$

f = –10 cm

From mirror equation,

$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} = \frac{1}{-10} + \frac{x+20}{25x+100}$

$\Rightarrow v = \frac{50(x+4)}{3x-20}$

So, this image is formed towards left of the mirror.

Again for second refraction in concave lens,

$u = -\left[5 - \frac{50(x+4)}{3x-20}\right]$ (assuming that image of mirror is formed between the lens and mirro)

v = +x (Since, the final image is produced on the object)

Using lens formula,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{x} + \frac{1}{5 - \frac{50(x+4}{3x-20}} = \frac{1}{-20}$$

$\Rightarrow$ x = 60 cm

The object should be placed at a distance 60 cm from the lens further away from the mirror.

So that the final image is formed on itself.

62. It can be solved in a similar manner like question no.61, by using the sign conversions properly. Left as an exercise for the student.

63. If the image in the mirror will form at the focus of the converging lens, then after transmission through the lens the rays of light will go parallel.

Let the object is at a distance x cm from the mirror

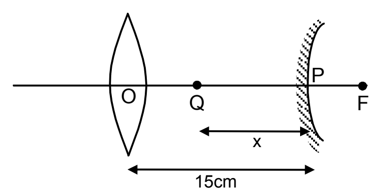
$\therefore$ u = –x cm ; v = 25 – 15 = 10 cm (because focal length of lens = 25 cm)

f = 40 cm

$\Rightarrow \frac{1}{v} + \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{x} = \frac{1}{10} - \frac{1}{40}$

$\Rightarrow$ x = 400/30 = 40/3

$\therefore$ The object is at distance $\left(15 - \frac{40}{3}\right) = \frac{5}{3}$ = 1.67 cm from the lens.



64. The object is placed in the focus of the converging mirror.

There will be two images.

a) One due to direct transmission of light through lens.

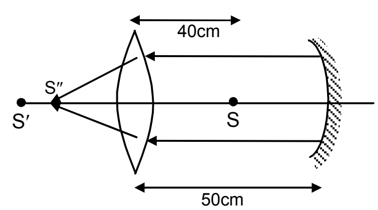
b) One due to reflection and then transmission of the rays through lens.

Case I : (S′) For the image by direct transmission,

u = –40 cm, f = 15 cm

$\Rightarrow \frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} = \frac{1}{15} + \frac{1}{-40}$

$\Rightarrow$ v = 24 cm (left of lens)



Case II : (S″) Since, the object is placed on the focus of mirror, after reflection the rays become parallel for the lens.

So, u = ∞

$\Rightarrow$ f = 15 cm

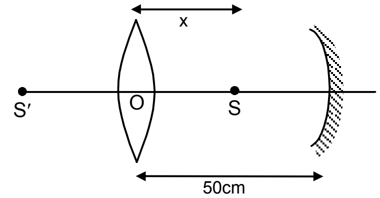
$\Rightarrow \frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow$ v = 15 cm (left of lens)

65. Let the source be placed at a distance ‘x’ from the lens as shown, so that images formed by both coincide.

For the lens, $\frac{1}{v_\ell} - \frac{1}{-x} = \frac{1}{15} \Rightarrow v_\ell = \frac{15x}{x-15}$ …(1)

Fro the mirror, u = –(50 – x), f = –10 cm

So, $\frac{1}{v_m} + \frac{1}{-(50-x)} = -\frac{1}{10}$



$\Rightarrow \dfrac{1}{v_m} = \dfrac{1}{-(50-x)} - \dfrac{1}{10}$

So, $v_m = \dfrac{10(50-x)}{x-40}$ …(2)

Since the lens and mirror are 50 cm apart,

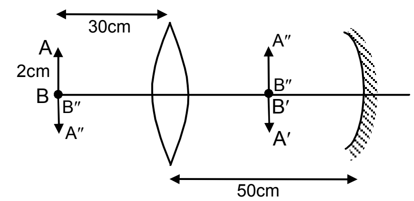
$v_\ell - v_m = 50 \Rightarrow \dfrac{15x}{x-15} - \dfrac{10(50-x)}{(x-40)} = 50$

$\Rightarrow$ x = 30 cm.

So, the source should be placed 30 cm from the lens.

66. Given that, f_1 = 15 cm, F_m = 10 cm, h_o = 2 cm

The object is placed 30 cm from lens $\dfrac{1}{v} - \dfrac{1}{u} = \dfrac{1}{f}$.



$\Rightarrow v = \dfrac{uf}{u+f}$

Since, u = –30 cm and f = 15 cm

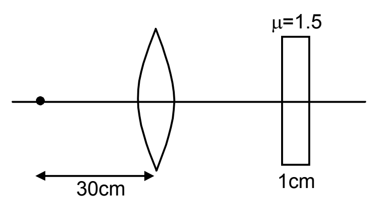
So, v = 30 cm

So, real and inverted image (A′B′) will be formed at 30 cm from the lens and it will be of same size as the object. Now, this real image is at a distance 20 cm from the concave mirror. Since, f_m = 10 cm, this real image is at the centre of curvature of the mirror. So, the mirror will form an inverted image A″B″ at the same place of same size.

Again, due to refraction in the lens the final image will be formed at AB and will be of same size as that of object. (A‴B‴)

67. For the lens, f = 15 cm, u = –30 cm

From lens formula, $\dfrac{1}{v} - \dfrac{1}{u} = \dfrac{1}{f}$



$\Rightarrow \dfrac{1}{v} = \dfrac{1}{15} - \dfrac{1}{30} = \dfrac{1}{30} \Rightarrow$ v = 30 cm

The image is formed at 30 cm of right side due to lens only.

Again, shift due to glass slab is,

$= \Delta t = \left(1 - \dfrac{1}{15}\right)1$ [since, μ_g = 1.5 and t = 1 cm]

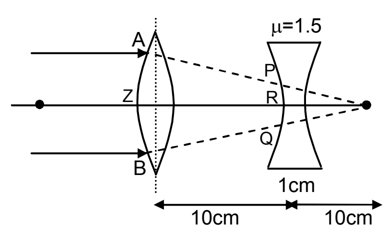
= 1 – (2/3) = 0.33 cm

$\therefore$ The image will be formed at 30 + 0.33 = 30.33 cm from the lens on right side.

68. Let, the parallel beam is first incident on convex lens.

d = diameter of the beam = 5 mm

Now, the image due to the convex lens should be formed on its focus (point B)



So, for the concave lens,

u = +10 cm (since, the virtual object is on the right of concave lens)

f = –10 cm

So, $\dfrac{1}{v} - \dfrac{1}{u} = \dfrac{1}{f} \Rightarrow \dfrac{1}{v} = \dfrac{1}{-10} + \dfrac{1}{10} = 0 \Rightarrow v = \infty$

So, the emergent beam becomes parallel after refraction in concave lens.

As shown from the triangles XYB and PQB,

$\dfrac{PQ}{XY} = \dfrac{RB}{ZB} = \dfrac{10}{20} = \dfrac{1}{2}$

So, PQ = ½ × 5 = 25 mm

So, the beam diameter becomes 2.5 mm.

Similarly, it can be proved that if the light is incident of the concave side, the beam diameter will be 1cm.

69. Given that, f_1 = focal length of converging lens = 30 cm

f_2 = focal length of diverging lens = –20 cm

and d = distance between them = 15 cm

Let, F = equivalent focal length

So, $\therefore \frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2} \Rightarrow \frac{1}{30} + \left(-\frac{1}{20}\right) - \left(\frac{15}{30(-200}\right) = \frac{1}{120}$

$\Rightarrow$ F = 120 cm

$\Rightarrow$ The equivalent lens is a converging one.

Distance from diverging lens so that emergent beam is parallel (image at infinity),

$d_1 = \frac{dF}{f_1} = \frac{15 \times 120}{30}$ = 60 cm

It should be placed 60 cm left to diverging lens

$\Rightarrow$ Object should be placed (120 – 60) = 60 cm from diverging lens.

Similarly, $d_2 = \frac{dF}{f_2} = \frac{15 \times 120}{20}$ = 90 cm

So, it should be placed 90 cm right to converging lens.

$\Rightarrow$ Object should be placed (120 + 90) = 210 cm right to converging lens.

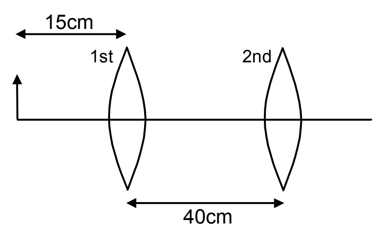
70. **a) First lens** :

u = –15 cm, f = 10 cm

$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} - \left(-\frac{1}{15}\right) = -\frac{1}{10}$

$\Rightarrow$ v = 30 cm

So, the final image is formed 10 cm right of second lens.



b) m for 1st lens :

$\frac{v}{u} = \frac{h_{image}}{h_{object}} \Rightarrow \left(\frac{30}{-15}\right) = \frac{h_{image}}{5mm}$

$\Rightarrow h_{image}$ = –10 mm (inverted)

Second lens :

u = –(40 – 30) = –10 cm ; f = 5 cm

[since, the image of 1st lens becomes the object for the second lens].

$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} - \left(-\frac{1}{10}\right) = \frac{1}{5}$

$\Rightarrow$ v = 10 cm

m for 2nd lens :

$\frac{v}{u} = \frac{h_{image}}{h_{object}} \Rightarrow \left(\frac{10}{10}\right) = \frac{h_{image}}{-10}$

$\Rightarrow h_{image}$ = 10 mm (erect, real).

c) So, size of final image = 10 mm

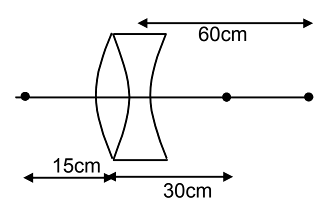
71. Let u = object distance from convex lens = –15 cm

v_1 = image distance from convex lens when alone = 30 cm

f_1 = focal length of convex lens

Now, $\therefore \frac{1}{v_1} - \frac{1}{u} = \frac{1}{f_1}$

or, $\frac{1}{f_1} = \frac{1}{30} - \frac{1}{-15} = \frac{1}{30} + \frac{1}{15}$



or f_1 = 10 cm

Again, Let v = image (final) distance from concave lens = +(30 + 30) = 60 cm

v_1 = object distance from concave lens = +30 m

f_2 = focal length of concave lens

Now, $\therefore \frac{1}{v} - \frac{1}{v_1} = \frac{1}{f_1}$

or, $\frac{1}{f_1} = \frac{1}{60} - \frac{1}{30}$ $\Rightarrow f_2 = -60$ cm.

So, the focal length of convex lens is 10 cm and that of concave lens is 60 cm.

72. a) The beam will diverge after coming out of the two convex lens system because, the image formed by the first lens lies within the focal length of the second lens.

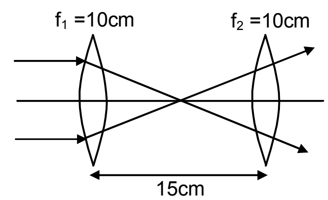
b) For 1st convex lens, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ $\Rightarrow$ $\frac{1}{v} = \frac{1}{10}$ (since, u = $-\infty$)

or, v = 10 cm

for 2nd convex lens, $\frac{1}{v'} = \frac{1}{f} + \frac{1}{u}$

or, $\frac{1}{v'} = \frac{1}{10} + \frac{1}{-(15-10)} = \frac{-1}{10}$

or, v′ = –10 cm



So, the virtual image will be at 5 cm from 1st convex lens.

c) If, F be the focal length of equivalent lens,

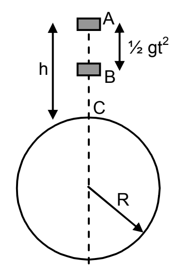
Then, $\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2} \Rightarrow \frac{1}{10} + \frac{1}{10} - \frac{15}{100} = \frac{1}{20}$

$\Rightarrow$ F = 20 cm.

73. Let us assume that it has taken time 't' from A to B.

$\therefore$ AB = $\frac{1}{2}gt^2$

$\therefore$ BC = h – $\frac{1}{2}gt^2$



This is the distance of the object from the lens at any time 't'.

Here, u = – (h – $\frac{1}{2}gt^2$)

$\mu_2 = \mu$(given) and μ_1 = i (air)

So, $\Rightarrow \frac{\mu}{v} - \frac{1}{-(h - \frac{1}{2}gf^2)} = \frac{\mu - 1}{R}$

$$\Rightarrow \frac{\mu}{v} = \frac{\mu-1}{R} - \frac{1}{(h - \frac{1}{2}gt^2)} = \frac{(\mu-1)(h-\frac{1}{2}gt^2) - R}{R(h - \frac{1}{2}gt^2)}$$

So, v = image distance at any time 't' = $\frac{\mu R(h - \frac{1}{2}gt^2)}{(\mu-1)(h-\frac{1}{2}gt^2) - R}$

So, velocity of the image = V = $\frac{dv}{dt} = \frac{d}{dt}\left[\frac{\mu R(h - \frac{1}{2}gt^2)}{(\mu-1)(h-\frac{1}{2}gt^2) - R}\right] = \frac{\mu R^2 gt}{(\mu-1)(h-\frac{1}{2}gt^2) - R}$ (can be found out).

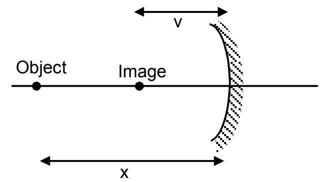
74. Given that, u = distance of the object = –x

f = focal length = –R/2

and, V = velocity of object = dx/dt

From mirror equation, $\frac{1}{-x} + \frac{1}{v} = -\frac{2}{R}$

$\frac{1}{v} = -\frac{2}{R} + \frac{1}{x} = \frac{R-2x}{Rx} \Rightarrow v = \frac{Rx}{R-2x}$ = Image distance



So, velocity of the image is given by,

$$V_1 = \frac{dv}{dt} = \frac{[\frac{d}{dt}(xR)(R-2x)] - [\frac{d}{dt}(R-2x)][xR]}{(R-2x)^2}$$

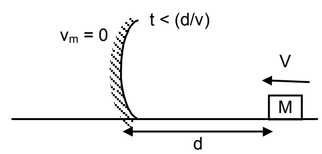
$$= \frac{R[\frac{dx}{dt}(R-2x)] - [-2\frac{dx}{dt}x]}{(R-2x)^2} = \frac{R[v(R-2x) + 2vx0}{(R-2x)^2}$$

$$= \frac{VR^2}{(2x-R)^2} = \frac{R[VR - 2xV + 2xV}{(R-2x)^2}.$$

75. a) When t < d/V, the object is approaching the mirror

As derived in the previous question,

$$V_{image} = \frac{\text{Velocity of object} \times R^2}{[2 \times \text{distance between them} - R]^2}$$



$$\Rightarrow V_{image} = \frac{VR^2}{[2(d-Vt)-R]^2}$$ [At any time, x = d – Vt]

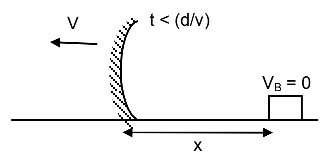
b) After a time t > d/V, there will be a collision between the mirror and the mass.

As the collision is perfectly elastic, the object (mass) will come to rest and the mirror starts to move away with same velocity V.

At any time t > d/V, the distance of the mirror from the mass will be

$$x = V\left(t - \frac{d}{V}\right) = Vt - d$$

Here, u = –(Vt – d) = d – Vt ; f = –R/2



So, $\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} = -\frac{1}{d-Vt} + \frac{1}{(-R/2)} = -\left[\frac{R+2(d-Vt)}{R(d-Vt)}\right]$

$\Rightarrow v = -\left[\frac{R(d-Vt)}{R-2(d-Vt)}\right]$ = Image distance

So, Velocity of the image will be,

$$V_{image} = \frac{d}{dt}(\text{Image distance}) = \frac{d}{dt}\left[\frac{R(d-Vt)}{R+2(d-Vt)}\right]$$

Let, y = (d – Vt)

$$\Rightarrow \frac{dy}{dt} = -V$$

So, $V_{image} = \frac{d}{dt}\left[\frac{Ry}{R+2y}\right] = \frac{(R+2y)R(-V) - Ry(+2)(-V)}{(R+2y)^2}$

$$= -Vr\left[\frac{R+2y-2y}{(R+2y)^2}\right] = \frac{-VR^2}{(R+2y)^2}$$

Since, the mirror itself moving with velocity V,

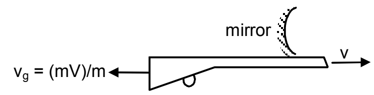
Absolute velocity of image = $V\left[1 - \frac{R^2}{(R+2y)^2}\right]$ (since, V = $V_{mirror} + V_{image}$)

$$= V\left[1 - \frac{R^2}{[2(Vt-d)-R^2}\right].$$

76. Recoil velocity of gun = $V_g = \frac{mV}{M}$.

At any time 't', position of the bullet w.r.t. mirror = $Vt + \frac{mV}{M}t = \left(1+\frac{m}{M}\right)Vt$

For the mirror, u = $-\left(1+\frac{m}{M}\right)Vt = kVt$



v = position of the image

From lens formula,

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u} \Rightarrow \frac{1}{v} = \frac{1}{-f} + \frac{1}{kVt} = \frac{1}{kVt} - \frac{1}{f} = \frac{f - kVt}{kVtf}$$

Let $\left(1+\frac{m}{M} = k\right)$,

So, v = $\frac{kVft}{-kVt + f} = \left(\frac{kVtf}{f - kVt}\right)$

So, velocity of the image with respect to mirror will be,

$$v_1 = \frac{dv}{dt} = \frac{d}{dt}\left[\frac{kVtf}{f - kVt}\right] = \frac{(f - kVt)kVf - kVtf(-kV)}{(f - kVt)^2} = \frac{kVt^2}{(f - kVt)^2}$$

Since, the mirror itself is moving at a speed of mV/M and the object is moving at 'V', the velocity of separation between the image and object at any time 't' will be,

$$v_s = V + \frac{mV}{M} + \frac{kVf^2}{(f - kVt)^2}$$

When, t = 0 (just after the gun is fired),

$$v_s = V + \frac{mV}{M} + kV = V + \frac{m}{M}V + \left(1+\frac{m}{M}\right)V = 2\left(1+\frac{m}{M}\right)V$$

77. Due to weight of the body suppose the spring is compressed by which is the mean position of oscillation.

$m = 50 \times 10^{-3}$ kg, $g = 10\ ms^{-2}$, $k = 500\ Nm^{-2}$, h = 10 cm = 0.1 m

For equilibrium, $mg = kx \Rightarrow x = mg/k = 10^{-3}$m = 0.1 cm

So, the mean position is at 30 + 0.1 = 30.1 cm from P (mirror).

Suppose, maximum compression in spring is δ.

Since, E.K.E. – I.K.E. = Work done

$\Rightarrow 0 - 0 = mg(h + \delta) - ½ k\delta^2$ (work energy principle)

$\Rightarrow mg(h + \delta) = ½ k\delta^2 \Rightarrow 50 \times 10^{-3} \times 10(0.1 + \delta) = ½\ 500\ \delta^2$

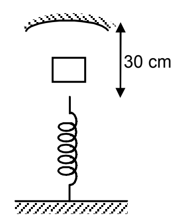


Fig-A

So, $\delta = \frac{0.5 \pm \sqrt{0.25 + 50}}{2 \times 250}$ = 0.015 m = 1.5 cm.

From figure B,

Position of B is 30 + 1.5 = 31.5 cm from pole.

Amplitude of the vibration = 31.5 – 30.1 – 1.4.

Position A is 30.1 – 1.4 = 28.7 cm from pole.

For A u = –31.5, f = –12 cm

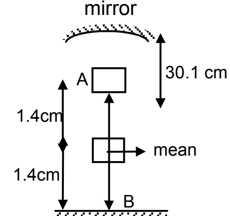


Fig-B

$$\therefore \frac{1}{v} = \frac{1}{f} - \frac{1}{u} = -\frac{1}{12} + \frac{1}{31.5}$$

$\Rightarrow v_A = -19.38$ cm

For B f = –12 cm, u = –28.7 cm

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u} = -\frac{1}{12} + \frac{1}{28.7}$$

$\Rightarrow v_B = -20.62$ cm

The image vibrates in length (20.62 – 19.38) = 1.24 cm.

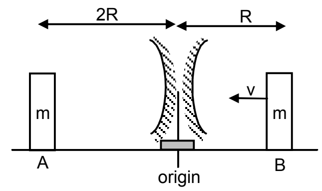
78. a) In time, t = R/V the mass B must have moved (v × R/v) = R closer to the mirror stand

So, For the block B :

u = –R, f = –R/2

$\therefore\ \frac{1}{v} = \frac{1}{f} - \frac{1}{u} = -\frac{2}{R} + \frac{1}{R} = -\frac{1}{R}$

$\Rightarrow$ v = –R at the same place.



For the block A : u = –2R, f = –R/2

$\therefore\ \frac{1}{v} = \frac{1}{f} - \frac{1}{u} = \frac{-2}{R} + \frac{1}{2R} = \frac{-3}{2R}$

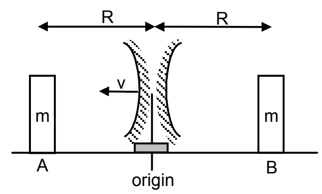
$\Rightarrow v = \frac{-2R}{3}$ image of A at $\frac{2R}{3}$ from PQ in the x-direction.

So, with respect to the given coordinate system,

$\therefore$ Position of A and B are $\frac{-2R}{3}$, R respectively from origin.

b) When t = 3R/v, the block B after colliding with mirror stand must have come to rest (elastic collision) and the mirror have travelled a distance R towards left form its initial position.

So, at this point of time,



For block A :

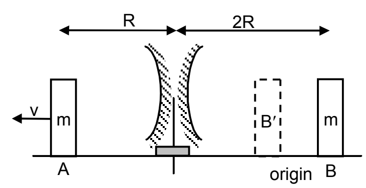
u = –R, f = –R/2

Using lens formula, v = –R (from the mirror),

So, position x_A = –2R (from origin of coordinate system)

For block B :

Image is at the same place as it is R distance from mirror. Hence, position of image is ‘0’.



Distance from PQ (coordinate system)

$\therefore$ positions of images of A and B are = –2R, 0 from origin.

c) Similarly, it can be proved that at time t = 5R/v,

the position of the blocks will be –3R and –4R/3 respectively.

79. Let a = acceleration of the masses A and B (w.r.t. elevator). From the freebody diagrams,

T – mg + ma – 2m = 0 …(1)

Similarly, T – ma = 0 …(2)

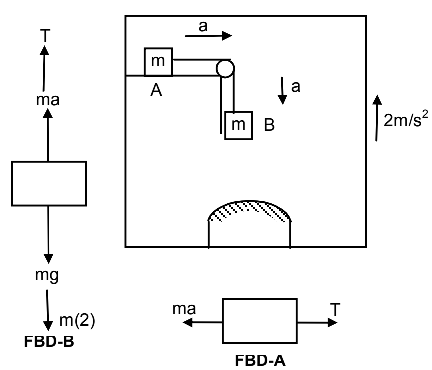
From (1) and (2), 2ma – mg – 2m = 0

$\Rightarrow$ 2ma = m(g + 2)

$\Rightarrow a = \frac{10+2}{2} = \frac{12}{2} = 6\ ms^{-2}$

so, distance travelled by B in t = 0.2 sec is,

$s = \frac{1}{2}at^2 = \frac{1}{2} \times 6 \times (0.2)^2 = 0.12$ m = 12 cm.



So, Distance from mirror, u = –(42 – 12) = –30 cm ; f = +12 cm

From mirror equation, $\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} + \left(-\frac{1}{30}\right) = \frac{1}{12}$

$\Rightarrow$ v = 8.57 cm

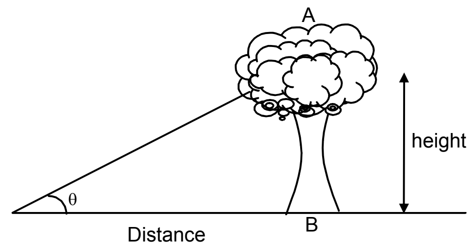
Distance between image of block B and mirror = 8.57 cm.

♠ ♠ ♠

SOLUTIONS TO CONCEPTS
CHAPTER 19

1. The visual angles made by the tree with the eyes can be calculated be below.

$$\theta = \frac{\text{Height of the tree}}{\text{Distance from the eye}} = \frac{AB}{OB} \Rightarrow \theta_A = \frac{2}{50} = 0.04$$



similarly, θ_B = 2.5 / 80 = 0.03125

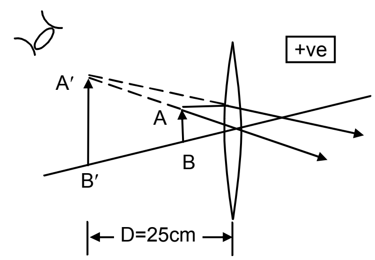
θ_C = 1.8 / 70 = 0.02571

θ_D = 2.8 / 100 = 0.028

Since, $\theta_A > \theta_B > \theta_D > \theta_C$, the arrangement in decreasing order is given by A, B, D and C.

2. For the given simple microscope,

f = 12 cm and D = 25 cm



(Simple Microscope)

For maximum angular magnification, the image should be produced at least distance of clear vision.

So, v = – D = –25 cm

Now, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$$\Rightarrow \frac{1}{u} = \frac{1}{v} - \frac{1}{f} = \frac{1}{-25} - \frac{1}{12} = -\frac{37}{300}$$

⇒ u = –8.1 cm

So, the object should be placed 8.1 cm away from the lens.

3. The simple microscope has, m = 3, when image is formed at D = 25 cm

a) $m = 1 + \frac{D}{f} \Rightarrow 3 = 1 + \frac{25}{f}$

⇒ f = 25/2 = 12.5 cm

b) When the image is formed at infinity (normal adjustment)

Magnifying power = $\frac{D}{f} = \frac{25}{12.5}$ = 2.0

4. The child has D = 10 cm and f = 10 cm

The maximum angular magnification is obtained when the image is formed at near point.

$m = 1 + \frac{D}{f} = 1 + \frac{10}{10}$ = 1 + 1 = 2

5. The simple microscope has magnification of 5 for normal relaxed eye (D = 25 cm).

Because, the eye is relaxed the image is formed at infinity (normal adjustment)

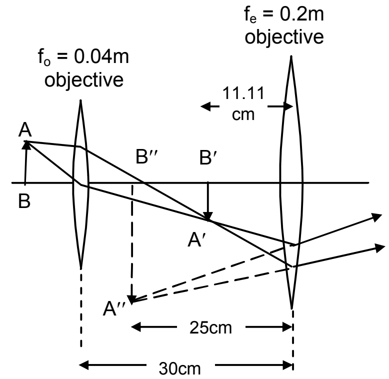
So, m = 5 = $\frac{D}{f} = \frac{25}{f}$ ⇒ f = 5 cm

For the relaxed farsighted eye, D = 40 cm

So, m = $\frac{D}{f} = \frac{40}{5}$ = 8

So, its magnifying power is 8X.

6. For the given compound microscope



$f_0 = \dfrac{1}{25 \text{ diopter}}$ = 0.04 m = 4 cm, $f_e = \dfrac{1}{5 \text{ diopter}}$ = 0.2 m = 20 cm

D = 25 cm, separation between objective and eyepiece = 30 cm

The magnifying power is maximum when the image is formed by the eye piece at least distance of clear vision i.e. D = 25 cm

for the eye piece, v_e = –25 cm, f_e = 20 cm

For lens formula, $\dfrac{1}{v_e} - \dfrac{1}{u_e} = \dfrac{1}{f_e}$

$\Rightarrow \dfrac{1}{u_e} = \dfrac{1}{v_e} - \dfrac{1}{f_e} \Rightarrow \dfrac{1}{-25} - \dfrac{1}{20}$ $\Rightarrow u_e = 11.11$ cm

So, for the objective lens, the image distance should be

v_0 = 30 – (11.11) = 18.89 cm

Now, for the objective lens,

v_0 = +18.89 cm (because real image is produced)

f_0 = 4 cm

So, $\dfrac{1}{u_o} = \dfrac{1}{v_o} - \dfrac{1}{f_o} \Rightarrow \dfrac{1}{18.89} - \dfrac{1}{4}$ = 0.053 – 0.25 = –0.197

$\Rightarrow u_o$ = –5.07 cm

So, the maximum magnificent power is given by

$m = -\dfrac{v_o}{u_o}\left[1 + \dfrac{D}{f_e}\right] = -\dfrac{18.89}{-5.07}\left[1 + \dfrac{25}{20}\right]$

= 3.7225 × 2.25 = 8.376

7. For the given compound microscope

f_o = 1 cm, f_e = 6 cm, D = 24 cm

For the eye piece, v_e = –24 cm, f_e = 6 cm

Now, $\dfrac{1}{v_e} - \dfrac{1}{u_e} = \dfrac{1}{f_e}$

$\Rightarrow \dfrac{1}{u_e} = \dfrac{1}{v_e} - \dfrac{1}{f_e} \Rightarrow -\left[\dfrac{1}{24} + \dfrac{1}{6}\right] = -\dfrac{5}{24}$

$\Rightarrow u_e$ = –4.8 cm

a) When the separation between objective and eye piece is 9.8 cm, the image distance for the objective lens must be (9.8) – (4.8) = 5.0 cm

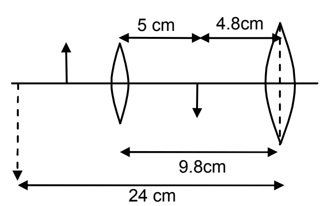


Fig-A

Now, $\dfrac{1}{v_0} - \dfrac{1}{u_0} = \dfrac{1}{f_0}$

$\Rightarrow \dfrac{1}{u_0} = \dfrac{1}{v_0} - \dfrac{1}{f_0} = \dfrac{1}{5} - \dfrac{1}{1} = -\dfrac{4}{5}$

$\Rightarrow u_0 = -\dfrac{5}{4}$ = – 1.25 cm

So, the magnifying power is given by,

$m = \dfrac{v_0}{u_o}\left[1 + \dfrac{D}{f}\right] = \dfrac{-5}{-1.25}\left[1 + \dfrac{24}{6}\right]$ = 4 × 5 = 20

(b) When the separation is 11.8 cm,

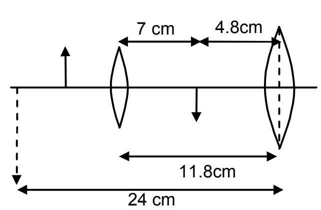


Fig-B

v_0 = 11.8 – 4.8 = 7.0 cm, f_0 = 1 cm

$\Rightarrow \dfrac{1}{u_0} = \dfrac{1}{v_0} - \dfrac{1}{f_0} = \dfrac{1}{7} - \dfrac{1}{1} = -\dfrac{6}{7}$

So, $m = -\frac{v_0}{u_o}\left[1+\frac{D}{f}\right] = \frac{-7}{-\left(\frac{7}{6}\right)}\left[1+\frac{24}{6}\right] = 6 \times 5 = 30$

So, the range of magnifying power will be 20 to 30.

8. For the given compound microscope.

$f_0 = \frac{1}{20D} = 0.05 \text{ m} = 5 \text{ cm}$, $\quad f_e = \frac{1}{10D} = 0.1 \text{ m} = 10 \text{ cm}$.

D = 25 cm, separation between objective & eyepiece= 20 cm

For the minimum separation between two points which can be distinguished by eye using the microscope, the magnifying power should be maximum.

For the eyepiece, $v_0 = -25$ cm, $f_e = 10$ cm

So, $\frac{1}{u_e} = \frac{1}{v_e} - \frac{1}{f_e} = \frac{1}{-25} - \frac{1}{10} = -\left[\frac{2+5}{50}\right] \Rightarrow u_e = -\frac{50}{7}$ cm

So, the image distance for the objective lens should be,

$V_0 = 20 - \frac{50}{7} = \frac{90}{7}$ cm

Now, for the objective lens,

$\frac{1}{u_0} = \frac{1}{v_0} - \frac{1}{f_0} = \frac{7}{90} - \frac{1}{5} = -\frac{11}{90}$

$\Rightarrow u_0 = -\frac{90}{11}$ cm

So, the maximum magnifying power is given by,

$m = \frac{-v_0}{u_0}\left[1+\frac{D}{f_e}\right]$

$= \frac{\left(\frac{90}{7}\right)}{\left(-\frac{90}{11}\right)}\left[1+\frac{25}{10}\right]$

$= \frac{11}{7} \times 3.5 = 5.5$

Thus, minimum separation eye can distinguish = $\frac{0.22}{5.5}$ mm = 0.04 mm

9. For the give compound microscope,

f_0 = 0.5cm, tube length = 6.5cm

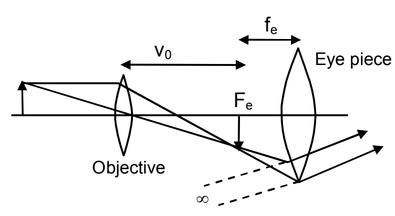
magnifying power = 100 (normal adjustment)

Since, the image is formed at infinity, the real image produced by the objective lens should lie on the focus of the eye piece.

So, $v_0 + f_e = 6.5$ cm ...(1)

Again, magnifying power= $\frac{v_0}{u_0} \times \frac{D}{f_e}$ [for normal adjustment]

$\Rightarrow m = -\left[1-\frac{v_0}{f_0}\right]\frac{D}{f_e}$ $\quad \left[\because \frac{v_0}{u_0} = 1-\frac{v_0}{f_0}\right]$



$\Rightarrow 100 = -\left[1-\frac{v_0}{0.5}\right] \times \frac{25}{f_e}$ [Taking D = 25 cm]

$\Rightarrow 100 f_e = -(1 - 2v_0) \times 25$

$\Rightarrow 2v_0 - 4f_e = 1$ ...(2)

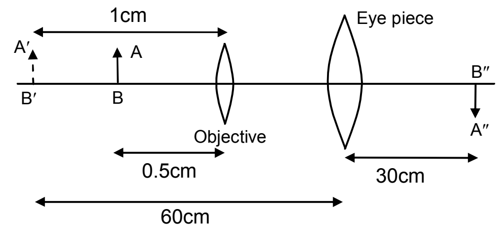
Solving equation (1) and (2) we can get,

V_0 = 4.5 cm and f_e = 2 cm

So, the focal length of the eye piece is 2cm.

10. Given that,

f_o = = 1 cm, f_e = 5 cm, u_0 = 0.5 cm, v_e = 30 cm



For the objective lens, u_0 = – 0.5 cm, f_0 = 1 cm.

From lens formula,

$$\frac{1}{v_0} - \frac{1}{u_0} = \frac{1}{f_0} \quad \Rightarrow \frac{1}{v_0} = \frac{1}{u_0} + \frac{1}{f_0} = \frac{1}{-0.5} + \frac{1}{1} = -1$$

$\Rightarrow v_0$ = – 1 cm

So, a virtual image is formed by the objective on the same side as that of the object at a distance of 1 cm from the objective lens. This image acts as a virtual object for the eyepiece.

For the eyepiece,

$$\frac{1}{v_0} - \frac{1}{u_0} = \frac{1}{f_0} \quad \Rightarrow \frac{1}{u_0} = \frac{1}{v_0} - \frac{1}{f_0} = \frac{1}{30} - \frac{1}{5} = \frac{-5}{30} = \frac{-1}{6} \quad \Rightarrow u_0 = -6 \text{ cm}$$

So, as shown in figure,

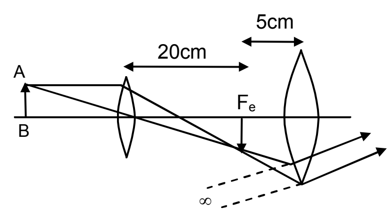
Separation between the lenses = $u_0 - v_0$ = 6 – 1 = 5 cm

11. The optical instrument has

$f_0 = \dfrac{1}{25D}$ = 0.04 m = 4 cm

$f_e = \dfrac{1}{20D}$ = 0.05 m = 5 cm

tube length = 25 cm (normal adjustment)



(a) The instrument must be a microscope as $f_0 < f_e$

(b) Since the final image is formed at infinity, the image produced by the objective should lie on the focal plane of the eye piece.

So, image distance for objective = v_0 = 25 – 5 = 20 cm

Now, using lens formula.

$$\frac{1}{v_0} - \frac{1}{u_0} = \frac{1}{f_0} \quad \Rightarrow \frac{1}{u_0} = \frac{1}{v_0} - \frac{1}{f_0} = \frac{1}{20} - \frac{1}{4} = \frac{-4}{20} = \frac{-1}{5} \quad \Rightarrow u_0 = -5 \text{ cm}$$

So, angular magnification = $m = -\dfrac{v_0}{u_0} \times \dfrac{D}{f_e}$ [Taking D = 25 cm]

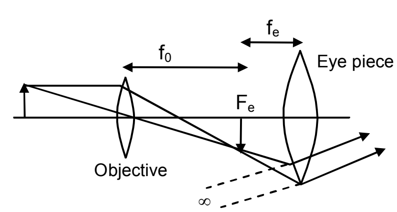
$= -\dfrac{20}{-5} \times \dfrac{25}{5}$ = 20

12. For the astronomical telescope in normal adjustment.

Magnifying power = m = 50, length of the tube = L = 102 cm

Let f_0 and f_e be the focal length of objective and eye piece respectively.

$m = \dfrac{f_0}{f_e} = 50 \Rightarrow f_0 = 50 f_e$ …(1)



and, L = $f_0 + f_e$ = 102 cm …(2)

Putting the value of f_0 from equation (1) in (2), we get,

$f_0 + f_e = 102 \Rightarrow 51 f_e = 102 \Rightarrow f_e$ = 2 cm = 0.02 m

So, f_0 = 100 cm = 1 m

$\therefore$ Power of the objective lens = $\dfrac{1}{f_0}$ = 1D

And Power of the eye piece lens = $\dfrac{1}{f_e} = \dfrac{1}{0.02}$ = 50D

13. For the given astronomical telescope in normal adjustment,

    F_e = 10 cm, L = 1 m = 100cm

    S0, $f_0 = L - f_e = 100 - 10 = 90$ cm

    and, magnifying power = $\frac{f_0}{f_e} = \frac{90}{10} = 9$

14. For the given Galilean telescope, (When the image is formed at infinity)

    f_0 = 30 cm, L = 27 cm

    Since $L = f_0 - |f_e|$

    [Since, concave eyepiece lens is used in Galilean Telescope]

    $\Rightarrow f_e = f_0 - L = 30 - 27 = 3$ cm

15. For the far sighted person,

    u = – 20 cm, v = – 50 cm

    from lens formula $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

    $\frac{1}{f} = \frac{1}{-50} - \frac{1}{-20} = \frac{1}{20} - \frac{1}{50} = \frac{3}{100}$ $\Rightarrow f = \frac{100}{3}$ cm = $\frac{1}{3}$ m

    So, power of the lens = $\frac{1}{f}$ = 3 Diopter

16. For the near sighted person,

    $u = \infty$ and v = – 200 cm = – 2m

    So, $\frac{1}{f} = \frac{1}{v} - \frac{1}{u} = \frac{1}{-2} - \frac{1}{\infty} = -\frac{1}{2} = -0.5$

    So, power of the lens is –0.5D

17. The person wears glasses of power –2.5D

    So, the person must be near sighted.

    $u = \infty$, v = far point, $f = \frac{1}{-2.5} = -0.4\text{m} = -40$ cm

    Now, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

    $\Rightarrow \frac{1}{v} = \frac{1}{u} + \frac{1}{f} = 0 + \frac{1}{-40}$ $\Rightarrow v = -40$ cm

    So, the far point of the person is 40 cm

18. On the 50^{th} birthday, he reads the card at a distance 25cm using a glass of +2.5D.

    Ten years later, his near point must have changed.

    So after ten years,

    u = – 50 cm, $f = \frac{1}{2.5D}$ = 0.4m = 40 cm v = near point

    Now, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ $\Rightarrow \frac{1}{v} = \frac{1}{u} + \frac{1}{f} = \frac{1}{-50} + \frac{1}{40} = \frac{1}{200}$

    So, near point = v = 200cm

    To read the farewell letter at a distance of 25 cm,

    U = – 25 cm

    For lens formula,

    $\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{f} = \frac{1}{200} - \frac{-}{-25} = \frac{1}{200} + \frac{1}{25} = \frac{9}{200}$ $\Rightarrow f = \frac{200}{9}$ cm = $\frac{2}{9}$ m

    $\Rightarrow$ Power of the lens = $\frac{1}{f} = \frac{9}{2}$ = 4.5D

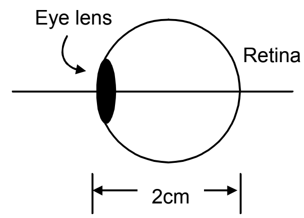
    $\therefore$ He has to use a lens of power +4.5D.

19. Since, the retina is 2 cm behind the eye-lens

v = 2cm

(a) When the eye-lens is fully relaxed

$u = \infty$, v = 2cm = 0.02 m



$\Rightarrow \frac{1}{f} = \frac{1}{v} - \frac{1}{u} = \frac{1}{0.02} - \frac{1}{\infty} = 50D$

So, in this condition power of the eye-lens is 50D

(b) When the eye-lens is most strained,

u = – 25 cm = – 0.25 m, v = +2 cm = +0.02 m

$\Rightarrow \frac{1}{f} = \frac{1}{v} - \frac{1}{u} = \frac{1}{0.02} - \frac{1}{-0.25} = 50 + 4 = 54D$

In this condition power of the eye lens is 54D.

20. The child has near point and far point 10 cm and 100 cm respectively.

Since, the retina is 2 cm behind the eye-lens, v = 2cm

For near point u = – 10 cm = – 0.1 m, v = 2 cm = 0.02 m

So, $\frac{1}{f_{near}} = \frac{1}{v} - \frac{1}{u} = \frac{1}{0.02} - \frac{1}{-0.1} = 50 + 10 = 60D$

For far point, u = – 100 cm = – 1 m, v = 2 cm = 0.02 m

So, $\frac{1}{f_{far}} = \frac{1}{v} - \frac{1}{u} = \frac{1}{0.02} - \frac{1}{-1} = 50 + 1 = 51D$

So, the rage of power of the eye-lens is +60D to +51D

21. For the near sighted person,

v = distance of image from glass

= distance of image from eye – separation between glass and eye

= 25 cm – 1cm = 24 cm = 0.24m

So, for the glass, $u = \infty$ and v = – 24 cm = –0.24m

So, $\frac{1}{f} = \frac{1}{v} - \frac{1}{u} = \frac{1}{-0.24} - \frac{1}{\infty} = -4.2\ D$

22. The person has near point 100 cm. It is needed to read at a distance of 20cm.

(a) When contact lens is used,

u = – 20 cm = – 0.2m, v = – 100 cm = –1 m

So, $\frac{1}{f} = \frac{1}{v} - \frac{1}{u} = \frac{1}{-1} - \frac{1}{-0.2} = -1 + 5 = +4D$

(b) When spectacles are used,

u = – (20 – 2) = – 18 cm = – 0.18m, v = – 100 cm = –1 m

So, $\frac{1}{f} = \frac{1}{v} - \frac{1}{u} = \frac{1}{-1} - \frac{1}{-0.18} = -1 + 5.55 = +4.5D$

23. The lady uses +1.5D glasses to have normal vision at 25 cm.

So, with the glasses, her least distance of clear vision = D = 25 cm

Focal length of the glasses = $\frac{1}{1.5}$ m = $\frac{100}{1.5}$ cm

So, without the glasses her least distance of distinct vision should be more

If, u = – 25cm, f = $\frac{100}{1.5}$ cm

Now, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f} = \frac{1.5}{100} - \frac{1}{25} = \frac{1.5 - 4}{100} = \frac{-2.5}{100}$ $\Rightarrow$ v = – 40cm = near point without glasses.

Focal length of magnifying glass = $\frac{1}{20}$ m = 0.05m = 5 cm = f

(a) The maximum magnifying power with glasses

$m = 1+\frac{D}{f} = 1+\frac{25}{5} = 6$ [$\because$ D = 25cm]

(b) Without the glasses, D = 40cm

So, $m = 1+\frac{D}{f} = 1+\frac{40}{5} = 9$

24. The lady can not see objects closer than 40 cm from the left eye and 100 cm from the right eye.

For the left glass lens,

v = – 40 cm, u = – 25 cm

$\therefore \frac{1}{f} = \frac{1}{v} - \frac{1}{u} = \frac{1}{-40} - \frac{1}{-25} = \frac{1}{25} - \frac{1}{40} = \frac{3}{200}$ $\Rightarrow f = \frac{200}{3}$ cm

For the right glass lens,

v = – 100 cm, u = – 25 cm

$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} = \frac{1}{-100} - \frac{1}{-25} = \frac{1}{25} - \frac{1}{100} = \frac{3}{100}$ $\Rightarrow f = \frac{100}{3}$ cm

(a) For an astronomical telescope, the eye piece lens should have smaller focal length. So, she should use the right lens (f = $\frac{100}{3}$ cm) as the eye piece lens.

(b) With relaxed eye, (normal adjustment)

$f_0 = \frac{200}{3}$ cm, $f_e = \frac{100}{3}$ cm

magnification = $m = \frac{f_0}{f_e} = \frac{(200/3)}{(100/3)} = 2$

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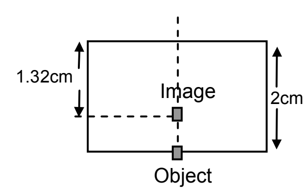
SOLUTIONS TO CONCEPTS
CHAPTER – 20

1. Given that,
Refractive index of flint glass = μ_f = 1.620
Refractive index of crown glass = μ_c = 1.518
Refracting angle of flint prism = A_f = 6.0°
For zero net deviation of mean ray
$(\mu_f - 1)A_f = (\mu_c - 1) A_c$
$\Rightarrow A_c = \frac{\mu_f - 1}{\mu_c - 1} A_f = \frac{1.620 - 1}{1.518 - 1}(6.0)^\circ = 7.2^\circ$

2. Given that
$\mu_r = 1.56$, $\mu_y = 1.60$, and $\mu_v = 1.68$
(a) Dispersive power = $\omega = \frac{\mu_v - \mu_r}{\mu_y - 1} = \frac{1.68 - 1.56}{1.60 - 1} = 0.2$
(b) Angular dispersion = $(\mu_v - \mu_r)A = 0.12 \times 6^\circ = 7.2^\circ$

3. The focal length of a lens is given by
$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$
$\Rightarrow (\mu - 1) = \frac{1}{f} \times \frac{1}{\left(\frac{1}{R_1} - \frac{1}{R_2}\right)} = \frac{K}{f}$ …(1)
So, $\mu_r - 1 = \frac{K}{100}$ …(2)
$\mu_y - 1 = \frac{K}{98}$ …(3)
And $\mu_v - 1 = \frac{K}{96}$ (4)
So, Dispersive power = $\omega = \frac{\mu_v - \mu_r}{\mu_y - 1} = \frac{(\mu_v - 1) - (\mu_r - 1)}{(\mu_y - 1)} = \frac{\frac{K}{96} - \frac{K}{100}}{\frac{K}{98}} = \frac{98 \times 4}{9600} = 0.0408$

4. Given that, $\mu_v - \mu_r = 0.014$
Again, $\mu_y = \frac{\text{Real depth}}{\text{Apparent depth}} = \frac{2.00}{1.30} = 1.515$
So, dispersive power = $\frac{\mu_v - \mu_r}{\mu_y - 1} = \frac{0.014}{1.515 - 1} = 0.027$



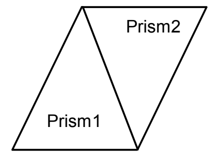
5. Given that, $\mu_r = 1.61$, $\mu_v = 1.65$, $\omega = 0.07$ and $\delta_y = 4^\circ$
Now, $\omega = \frac{\mu_v - \mu_r}{\mu_y - 1}$
$\Rightarrow 0.07 = \frac{1.65 - 1.61}{\mu_y - 1}$
$\Rightarrow \mu_y - 1 = \frac{0.04}{0.07} = \frac{4}{7}$
Again, $\delta = (\mu - 1) A$
$\Rightarrow A = \frac{\delta_y}{\mu_y - 1} = \frac{4}{(4/7)} = 7^\circ$

6. Given that, $\delta_r = 38.4°$, $\delta_y = 38.7°$ and $\delta_v = 39.2°$

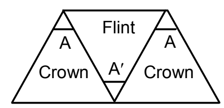
Dispersive power = $\dfrac{\mu_v - \mu_r}{\mu_y - 1} = \dfrac{(\mu_v - 1) - (\mu_r - 1)}{(\mu_y - 1)} = \dfrac{\left(\dfrac{\delta_v}{A}\right) - \left(\dfrac{\delta_r}{A}\right)}{\left(\dfrac{\delta_v}{A}\right)}$ $[\because\ \delta = (\mu - 1)\ A]$

$= \dfrac{\delta_v - \delta_r}{\delta_y} = \dfrac{39.2 - 38.4}{38.7} = 0.0204$

7. Two prisms of identical geometrical shape are combined.
Let A = Angle of the prisms
$\mu'_v = 1.52$ and $\mu_v = 1.62$, $\delta_v = 1°$
$\delta_v = (\mu_v - 1)A - (\mu'_v - 1)\ A$ [since A = A′]
$\Rightarrow \delta_v = (\mu_v - \mu'_v)A$
$\Rightarrow A = \dfrac{\delta_v}{\mu_v - \mu'_v} = \dfrac{1}{1.62 - 1.52} = 10°$

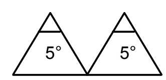


8. Total deviation for yellow ray produced by the prism combination is
$\delta_y = \delta_{cy} - \delta_{fy} + \delta_{cy} = 2\ \delta_{cy} - \delta_{fy} = 2(\mu_{cy} - 1)A - (\mu_{cy} - 1)A'$
Similarly the angular dispersion produced by the combination is
$\delta_v - \delta_r = [(\mu_{vc} - 1)A - (\mu_{vf} - 1)A' + (\mu_{vc} - 1)A] - [(\mu_{rc} - 1)\ A - (\mu_{rf} - 1)A' + (\mu_r - 1)\ A)]$
$= 2(\mu_{vc} - 1)A - (\mu_{vf} - 1)A'$
(a) For net angular dispersion to be zero,
$\delta_v - \delta_r = 0$
$\Rightarrow\ 2(\mu_{vc} - 1)A = (\mu_{vf} - 1)A'$
$\Rightarrow \dfrac{A'}{A} = \dfrac{2(\mu_{cv} - \mu_{rc})}{(\mu_{vf} - \mu_{rf})} = \dfrac{2(\mu_v - \mu_r)}{(\mu'_v - \mu'_r)}$

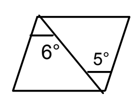


(b) For net deviation in the yellow ray to be zero,
$\delta_y = 0$
$\Rightarrow 2(\mu_{cy} - 1)A = (\mu_{fy} - 1)A'$
$\Rightarrow \dfrac{A'}{A} = \dfrac{2(\mu_{cy} - 1)}{(\mu_{fy} - 1)} = \dfrac{2(\mu_y - 1)}{(\mu'_y - 1)}$

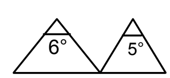
9. Given that, $\mu_{cr} = 1.515$, $\mu_{cv} = 1.525$ and $\mu_{fr} = 1.612$, $\mu_{fv} = 1.632$ and $A = 5°$
Since, they are similarly directed, the total deviation produced is given by,
$\delta = \delta_c + \delta_r = (\mu_c - 1)A + (\mu_r - 1)\ A = (\mu_c + \mu_r - 2)A$
So, angular dispersion of the combination is given by,
$\delta_v - \delta_y = (\mu_{cv} + \mu_{fv} - 2)A - (\mu_{cr} + \mu_{fr} - 2)A$
$= (\mu_{cv} + \mu_{fv} - \mu_{cr} - \mu_{fr})A = (1.525 + 1.632 - 1.515 - 1.612)\ 5 = 0.15°$



10. Given that, $A' = 6°$, $\omega' = 0.07$, $\mu'_y = 1.50$
A = ? $\omega = 0.08$, $\mu_y = 1.60$
The combination produces no deviation in the mean ray.
(a) $\delta_y = (\mu_y - 1)A - (\mu'_y - 1)A' = 0$ [Prism must be oppositely directed]
$\Rightarrow (1.60 - 1)A = ((1.50 - 1)A'$
$\Rightarrow A = \dfrac{0.50 \times 6°}{0.60} = 5°$

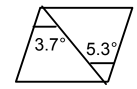


(b) When a beam of white light passes through it,
Net angular dispersion = $(\mu_y - 1)\omega A - (\mu'_y - 1)\omega' A'$
$\Rightarrow (1.60 - 1)(0.08)(5°) - (1.50 - 1)(0.07)(6°)$
$\Rightarrow 0.24° - 0.21° = 0.03°$
(c) If the prisms are similarly directed,
$\delta_y = (\mu_y - 1)A + (\mu'_y - 1)A$
$= (1.60 - 1)5° + (1.50 - 1)6° = 3° + 3° = 6°$

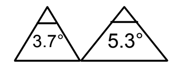


(d) Similarly, if the prisms are similarly directed, the net angular dispersion is given by,
$\delta_v - \delta_r = (\mu_y - 1)\omega A - (\mu'_y - 1)\ \omega' A' = 0.24° + 0.21° = 0.45°$

11. Given that, $\mu'_v - \mu'_r = 0.014$ and $\mu_v - \mu_r = 0.024$
    $A' = 5.3°$ and $A = 3.7°$
    (a) When the prisms are oppositely directed,
    angular dispersion = $(\mu_v - \mu_r)A - (\mu'_v - \mu'_r)A'$
    $= 0.024 \times 3.7° - 0.014 \times 5.3° = 0.0146°$



    (b) When they are similarly directed,
    angular dispersion = $(\mu_v - \mu_r)A + (\mu'_v - \mu'_r)A'$
    $= 0.024 \times 3.7° + 0.014 \times 5.3° = 0.163°$



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SOLUTIONS TO CONCEPTS
CHAPTER 21

1. In the given Fizeau'' apparatus,
   $D = 12 \text{ km} = 12 \times 10^3 \text{ m}$
   $n = 180$
   $c = 3 \times 10^8 \text{ m/sec}$
   We know, $c = \dfrac{2Dn\omega}{\pi}$
   $\Rightarrow \omega = \dfrac{\pi c}{2Dn}$ rad/sec $= \dfrac{\pi c}{2Dn} \times \dfrac{180}{\pi}$ deg/sec
   $\Rightarrow \omega = \dfrac{180 \times 3 \times 10^8}{24 \times 10^3 \times 180} = 1.25 \times 10^4$ deg/sec
2. In the given Focault experiment,
   R = Distance between fixed and rotating mirror = 16m
   ω = Angular speed = 356 rev/′ = 356 × 2π rad/sec
   b = Distance between lens and rotating mirror = 6m
   a = Distance between source and lens = 2m
   s = shift in image = 0.7 cm = 0.7×10^{-3} m
   So, speed of light is given by,
   $C = \dfrac{4R^2\omega a}{s(R+b)} = \dfrac{4 \times 16^2 \times 356 \times 2\pi \times 2}{0.7 \times 10^{-3}(16+6)} = 2.975 \times 10^8$ m/s
3. In the given Michelson experiment,
   $D = 4.8 \text{ km} = 4.8 \times 10^3 \text{ m}$
   $N = 8$
   We know, $c = \dfrac{D\omega N}{2\pi}$
   $\Rightarrow \omega = \dfrac{2\pi c}{DN}$ rad/sec $= \dfrac{c}{DN}$ rev/sec $= \dfrac{3 \times 10^8}{4.8 \times 10^3 \times 8} = 7.8 \times 10^3$ rev/sec

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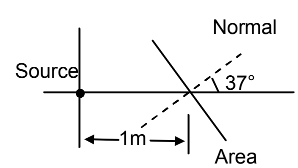
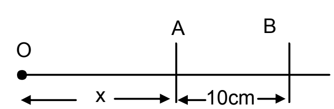
SOLUTIONS TO CONCEPTS
CHAPTER 22

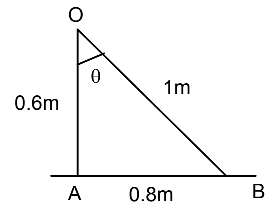
1. Radiant Flux = $\frac{\text{Total energy emitted}}{\text{Time}} = \frac{45}{15s}$ = 3W
2. To get equally intense lines on the photographic plate, the radiant flux (energy) should be same.
   S0, 10W × 12sec = 12W × t
   $\Rightarrow t = \frac{10W \times 12\,sec}{12W}$ = 10 sec.
3. it can be found out from the graph by the student.
4. Relative luminousity = $\frac{\text{Luminous flux of a source of given wavelength}}{\text{Luminous flux of a source of 555 nm of same power}}$
   Let the radiant flux needed be P watt.
   Ao, 0.6 = $\frac{\text{Luminous flux of source 'P' watt}}{685\ P}$
   $\therefore$ Luminous flux of the source = (685 P)× 0.6 = 120 × 685
   $\Rightarrow P = \frac{120}{0.6}$ = 200W
5. The luminous flux of the given source of 1W is 450 lumen/watt
   $\therefore$ Relative luminosity = $\frac{\text{Luminous flux of the source of given wavelength}}{\text{Luminous flux of 555 nm source of same power}} = \frac{450}{685}$ = 66%
   [$\therefore$ Since, luminous flux of 555nm source of 1W = 685 lumen]
6. The radiant flux of 555nm part is 40W and of the 600nm part is 30W
   (a) Total radiant flux = 40W + 30W = 70W
   (b) Luminous flux = $(L.Fllux)_{555nm} + (L.Flux)_{600nm}$
   = 1 × 40× 685 + 0.6 × 30 × 685 = 39730 lumen
   (c) Luminous efficiency = $\frac{\text{Total luminous flux}}{\text{Total radiant flux}} = \frac{39730}{70}$ = 567.6 lumen/W
7. Overall luminous efficiency = $\frac{\text{Total luminous flux}}{\text{Power input}} = \frac{35 \times 685}{100}$ = 239.75 lumen/W
8. Radiant flux = 31.4W, Solid angle = 4π
   Luminous efficiency = 60 lumen/W
   So, Luminous flux = 60 × 31.4 lumen
   And luminous intensity = $\frac{\text{Luminous Flux}}{4\pi} = \frac{60 \times 31.4}{4\pi}$ = 150 candela
9. I = luminous intensity = $\frac{628}{4\pi}$ = 50 Candela
   r = 1m, $\theta = 37°$
   So, illuminance, $E = \frac{I\cos\theta}{r^2} = \frac{50 \times \cos 37°}{1^2}$ = 40 lux



10. Let, I = Luminous intensity of source
    E_A = 900 lumen/m^2
    E_B = 400 lumen/m^2
    Now, $E_a = \frac{I\cos\theta}{x^2}$ and $E_B = \frac{I\cos\theta}{(x+10)^2}$
    So, $I = \frac{E_A x^2}{\cos\theta} = \frac{E_B (x+10)^2}{\cos\theta}$
    $\Rightarrow 900x^2 = 400(x + 10)^2 \Rightarrow \frac{x}{x+10} = \frac{2}{3} \Rightarrow 3x = 2x + 20 \Rightarrow x = 20$ cm
    So, The distance between the source and the original position is 20cm.



11. Given that, $E_a = 15 \text{ lux} = \frac{I_0}{60^2}$



$\Rightarrow I_0 = 15 \times (0.6)^2 = 5.4$ candela

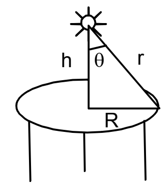
So, $E_B = \frac{I_0 \cos\theta}{(OB)^2} = \frac{5.4 \times \left(\frac{3}{5}\right)}{1^2} = 3.24$ lux

12. The illuminance will not change.

13. Let the height of the source is 'h' and the luminous intensity in the normal direction is I_0.

So, illuminance at the book is given by,

$E = \frac{I_0 \cos\theta}{r^2} = \frac{I_0 h}{r^3} = \frac{I_0 h}{(r^2 + h^2)^{3/2}}$



For maximum E, $\frac{dE}{dh} = 0 \Rightarrow \frac{I_0\left[(R^2 + h^2)^{3/2} - \frac{3}{2} h \times (R^2 + h^2)^{1/2} \times 2h\right]}{(R^2 + h^2)^3}$

$\Rightarrow (R^2 + h^2)^{1/2}[R^2 + h^2 - 3h^2] = 0$

$\Rightarrow R^2 - 2h^2 = 0 \Rightarrow h = \frac{R}{\sqrt{2}}$

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CHAPTER – 23
HEAT AND TEMPERATURE
EXERCISES

1. Ice point = 20° (L_0) L_1 = 32°
   Steam point = 80° (L_{100})
   $$T = \frac{L_1 - L_0}{L_{100} - L_0} \times 100 = \frac{32-20}{80-20} \times 100 = 20°C$$
2. P_{tr} = 1.500 × 10^4 Pa
   P = 2.050 × 10^4 Pa
   We know, For constant volume gas Thermometer
   $$T = \frac{P}{P_{tr}} \times 273.16\text{ K} = \frac{2.050 \times 10^4}{1.500 \times 10^4} \times 273.16 = 373.31$$
3. Pressure Measured at M.P = 2.2 × Pressure at Triple Point
   $$T = \frac{P}{P_{tr}} \times 273.16 = \frac{2.2 \times P_{tr}}{P_{tr}} \times 273.16 = 600.952\text{ K} \approx 601\text{ K}$$
4. P_{tr} = 40 × 10^3 Pa, P = ?
   T = 100°C = 373 K, $T = \frac{P}{P_{tr}} \times 273.16\text{ K}$
   $$\Rightarrow P = \frac{T \times P_{tr}}{273.16} = \frac{373 \times 49 \times 10^3}{273.16} = 54620 \text{ Pa} = 5.42 \times 10^3 \text{ pa} \approx 55 \text{ K Pa}$$
5. P_1 = 70 K Pa, P_2 = ?
   T_1 = 273 K, T_2 = 373K
   $$T = \frac{P_1}{P_{tr}} \times 273.16 \qquad \Rightarrow 273 = \frac{70 \times 10^3}{P_{tr}} \times 273.16 \qquad \Rightarrow P_{tr} \frac{70 \times 273.16 \times 10^3}{273}$$
   $$T_2 = \frac{P_2}{P_{tr}} \times 273.16 \qquad \Rightarrow 373 = \frac{P_2 \times 273}{70 \times 273.16 \times 10^3} \qquad \Rightarrow P_2 = \frac{373 \times 70 \times 10^3}{273} = 95.6 \text{ K Pa}$$
6. $P_{\text{ice point}} = P_{0°}$ = 80 cm of Hg
   $P_{\text{steam point}} = P_{100°}$ 90 cm of Hg
   P_0 = 100 cm
   $$t = \frac{P - P_0}{P_{100} - P_0} \times 100° = \frac{80-100}{90-100} \times 100 = 200°C$$
7. $T' = \frac{V}{V - V'} T_0$ T_0 = 273,
   V = 1800 CC, V′ = 200 CC
   $$T' = \frac{1800}{1600} \times 273 = 307.125 \approx 307$$
8. $R_t = 86\Omega$; $R_{0°} = 80\Omega$; $R_{100°} = 90\Omega$
   $$t = \frac{R_t - R_0}{R_{100} - R_0} \times 100 = \frac{86-80}{90-80} \times 100 = 60°C$$
9. R at ice point (R_0) = 20Ω
   R at steam point (R_{100}) = 27.5Ω
   R at Zinc point (R_{420}) = 50Ω
   $R_\theta = R_0 (1 + \alpha\theta + \beta\theta^2)$
   $\Rightarrow R_{100} = R_0 + R_0\,\alpha\theta + R_0\,\beta\theta^2$
   $$\Rightarrow \frac{R_{100} - R_0}{R_0} = \alpha\theta + \beta\theta^2$$

$\Rightarrow \frac{27.5-20}{20} = \alpha \times 100 + \beta \times 10000$

$\Rightarrow \frac{7.5}{20} = 100\,\alpha + 10000\,\beta$

$R_{420} = R_0\,(1+\alpha\theta+\beta\theta^2) \Rightarrow \frac{50-R_0}{R_0} = \alpha\theta + \beta\theta^2$

$\Rightarrow \frac{50-20}{20} = 420 \times \alpha + 176400 \times \beta$ $\qquad \Rightarrow \frac{3}{2} = 420\,\alpha + 176400\,\beta$

$\Rightarrow \frac{7.5}{20} = 100\,\alpha + 10000\,\beta$ $\qquad \Rightarrow \frac{3}{2} = 420\,\alpha + 176400\,\beta$

10. L_1 = ?, L_0 = 10 m, $\alpha = 1 \times 10^{-5}$/°C, t= 35

$L_1 = L_0\,(1+\alpha t) = 10(1 + 10^{-5} \times 35) = 10 + 35 \times 10^{-4} = 10.0035$m

11. t_1 = 20°C, t_2 = 10°C, L_1 = 1cm = 0.01 m, L_2 =?

$\alpha_{steel} = 1.1 \times 10^{-5}$/°C

$L_2 = L_1\,(1 + \alpha_{steel}\Delta T) = 0.01(1 + 101 \times 10^{-5} \times 10) = 0.01 + 0.01 \times 1.1 \times 10^{-4}$

$= 10^4 \times 10^{-6} + 1.1 \times 10^{-6} = 10^{-6}\,(10000 + 1.1) = 10001.1$

$=1.00011 \times 10^{-2}$ m = 1.00011 cm

12. L_0 = 12 cm, $\alpha = 11 \times 10^{-5}$ /°C

tw = 18°C ts = 48°C

$Lw = L_0(1+\alpha tw) = 12\,(1 + 11 \times 10^{-5} \times 18) = 12.002376$ m

$Ls = L_0\,(1+\alpha ts) = 12\,(1 + 11 \times 10^{-5} \times 48) = 12.006336$ m

$\Delta L = 12.006336 - 12.002376 = 0.00396$ m ≈ 0.4cm

13. $d_1 = 2$ cm $= 2 \times 10^{-2}$

t_1 = 0°C, t_2 = 100°C

$\alpha_{al} = 2.3 \times 10^{-5}$ /°C

$d_2 = d_1\,(1 + \alpha\Delta t) = 2 \times 10^{-2}\,(1 + 2.3 \times 10^{-5}\ 10^2)$

= 0.02 + 0.000046 = 0.020046 m = 2.0046 cm

14. $L_{st} = L_{Al}$ at 20°C $\qquad \alpha_{Al} = 2.3 \times 10^{-5}$/°C

So, $Lo_{st}\,(1 - \alpha_{st} \times 20) = Lo_{Al}\,(1 - \alpha_{Al} \times 20)$ $\qquad \alpha_{st} = 1.1 \times 10^{-5}$/°C

(a) $\Rightarrow \frac{Lo_{st}}{Lo_{Al}} = \frac{(1-\alpha_{Al}\times 20)}{(1-\alpha_{st}\times 20)} = \frac{1-2.3\times10^{-5}\times 20}{1-1.1\times 10^{-5}\times 20} = \frac{0.99954}{0.99978} = 0.999$

(b) $\Rightarrow \frac{Lo_{40st}}{Lo_{40Al}} = \frac{(1-\alpha_{Al}\times 40)}{(1-\alpha_{st}\times 40)} = \frac{1-2.3\times10^{-5}\times 20}{1-1.1\times 10^{-5}\times 20} = \frac{0.99954}{0.99978} = 0.999$

$= \frac{Lo_{Al}}{Lo_{st}} \times \frac{1+2.3\times10^{-5}\times 10}{273} = \frac{0.99977 \times 1.00092}{1.00044} = 1.0002496 \approx 1.00025$

$\frac{Lo_{100Al}}{Lo_{100St}} = \frac{(1+\alpha_{Al}\times 100)}{(1+\alpha_{st}\times 100)} = \frac{0.99977\times 1.00092}{1.00011} = 1.00096$

15. (a) Length at 16°C = L

L = ? T_1 =16°C, T_2 = 46°C

$\alpha = 1.1 \times 10^{-5}$/°C

$\Delta L = L\alpha\Delta\theta = L \times 1.1 \times 10^{-5} \times 30$

% of error = $\left(\frac{\Delta L}{L}\times 100\right)\% = \left(\frac{L\alpha\Delta\theta}{2}\times 100\right)\% = 1.1 \times 10^{-5} \times 30 \times 100\% = 0.033\%$

(b) T_2 = 6°C

% of error = $\left(\frac{\Delta L}{L}\times 100\right)\% = \left(\frac{L\alpha\Delta\theta}{L}\times 100\right)\% = -1.1 \times 10^{-5} \times 10 \times 100 = -0.011\%$

16. T_1 = 20°C, ΔL = 0.055mm = 0.55 × 10^{-3} m
t_2 = ? α_{st} = 11 × 10^{-6}/°C
We know,
$\Delta L = L_0 \alpha \Delta T$
In our case,
$0.055 \times 10^{-3} = 1 \times 1.1\ \mathrm{I}\ 10^{-6} \times (T_1 + T_2)$
$0.055 = 11 \times 10^{-3} \times 20 \pm 11 \times 10^{-3} \times T_2$
$T_2 = 20 + 5 = 25°C$ or $20 - 5 = 15°C$
The expt. Can be performed from 15 to 25°C

17. $f_{0°C}$=0.098 g/m^3, $f_{4°C}$ = 1 g/m^3

$$f_{0°C} = \frac{f_{4°C}}{1+\gamma\Delta T} \Rightarrow 0.998 = \frac{1}{1+\gamma \times 4} \Rightarrow 1 + 4\gamma = \frac{1}{0.998}$$

$$\Rightarrow 4 + \gamma = \frac{1}{0.998} - 1 \Rightarrow \gamma = 0.0005 \approx 5 \times 10^{-4}$$

As density decreases $\gamma = -5 \times 10^{-4}$

18. Iron rod Aluminium rod
L_{Fe} L_{Al}
$\alpha_{Fe} = 12 \times 10^{-8}$ /°C $\alpha_{Al} = 23 \times 10^{-8}$ /°C
Since the difference in length is independent of temp. Hence the different always remains constant.
$L'_{Fe} = L_{Fe}(1 + \alpha_{Fe} \times \Delta T)$ …(1)
$L'_{Al} = L_{Al}(1 + \alpha_{Al} \times \Delta T)$ …(2)
$L'_{Fe} - L'_{Al} = L_{Fe} - L_{Al} + L_{Fe} \times \alpha_{Fe} \times \Delta T - L_{Al} \times \alpha_{Al} \times \Delta T$

$$\frac{L_{Fe}}{L_{Al}} = \frac{\alpha_{Al}}{\alpha_{Fe}} = \frac{23}{12} = 23 : 12$$

19. g_1 = 9.8 m/s^2, g_2 = 9.788 m/s^2

$$T_1 = 2\pi\frac{\sqrt{l_1}}{g_1} \qquad T_2 = 2\pi\frac{\sqrt{l_2}}{g_2} = 2\pi\frac{\sqrt{l_1(1+\Delta T)}}{g}$$

$\alpha_{Steel} = 12 \times 10^{-6}$ /°C
T_1 = 20°C T_2 = ?
$T_1 = T_2$

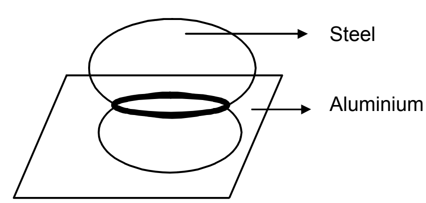
$$\Rightarrow 2\pi\frac{\sqrt{l_1}}{g_1} = 2\pi\frac{\sqrt{l_1(1+\Delta T)}}{g_2} \qquad \Rightarrow \frac{l_1}{g_1} = \frac{l_1(1+\Delta T)}{g_2}$$

$$\Rightarrow \frac{1}{9.8} = \frac{1+12\times10^{-6}\times\Delta T}{9.788} \qquad \Rightarrow \frac{9.788}{9.8} = 1 + 12 \times 10^{-6} \times \Delta T$$

$$\Rightarrow \frac{9.788}{9.8} - 1 = 12 \times 10^{-6}\,\Delta T \qquad \Rightarrow \Delta T = \frac{-0.00122}{12\times10^{-6}}$$

$$\Rightarrow T_2 - 20 = -101.6 \qquad \Rightarrow T_2 = -101.6 + 20 = -81.6 \approx -82°C$$

20. Given
d_{St} = 2.005 cm, d_{Al} = 2.000 cm
$\alpha_S = 11 \times 10^{-6}$ /°C $\alpha_{Al} = 23 \times 10^{-6}$ /°C
$d's = 2.005\,(1+ \alpha_s\,\Delta T)$ (where ΔT is change in temp.)
$\Rightarrow d's = 2.005 + 2.005 \times 11 \times 10^{-6}\,\Delta T$
$d'_{Al} = 2(1+ \alpha_{Al}\,\Delta T) = 2 + 2 \times 23 \times 10^{-6}\,\Delta T$
The two will slip i.e the steel ball with fall when both the diameters become equal.



So,
$\Rightarrow 2.005 + 2.005 \times 11 \times 10^{-6}\,\Delta T = 2 + 2 \times 23 \times 10^{-6}\,\Delta T$
$\Rightarrow (46 - 22.055)10^{-6} \times \Delta T = 0.005$

$$\Rightarrow \Delta T = \frac{0.005\times10^{6}}{23.945} = 208.81$$

Now $\Delta T = T_2 - T_1 = T_2 - 10°C$ [$\therefore$ $T_1 = 10°C$ given]

$\Rightarrow T_2 = \Delta T + T_1 = 208.81 + 10 = 281.81$

21. The final length of aluminium should be equal to final length of glass.

Let the initial length o faluminium = l

$l(1 - \alpha_{Al}\Delta T) = 20(1 - \alpha_0\Delta\theta)$

$\Rightarrow l(1 - 24 \times 10^{-6} \times 40) = 20\ (1 - 9 \times 10^{-6} \times 40)$

$\Rightarrow l(1 - 0.00096) = 20\ (1 - 0.00036)$

$\Rightarrow l = \dfrac{20 \times 0.99964}{0.99904} = 20.012$ cm

Let initial breadth of aluminium = b

$b(1 - \alpha_{Al}\Delta T) = 30(1 - \alpha_0\Delta\theta)$

$\Rightarrow b = \dfrac{30 \times (1 - 9 \times 10^{-6} \times 40)}{(1 - 24 \times 10^{-6} \times 40)} = \dfrac{30 \times 0.99964}{0.99904} = 30.018$ cm

22. $V_g = 1000$ CC, $T_1 = 20°C$

$V_{Hg} = ?$ $\gamma_{Hg} = 1.8 \times 10^{-4}$ /°C

$\gamma_g = 9 \times 10^{-6}$ /°C

ΔT remains constant

Volume of remaining space = $V'_g - V'_{Hg}$

Now

$V'_g = V_g(1 + \gamma_g\Delta T)$ …(1)

$V'_{Hg} = V_{Hg}(1 + \gamma_{Hg}\Delta T)$ …(2)

Subtracting (2) from (1)

$V'_g - V'_{Hg} = V_g - V_{Hg} + V_g\gamma_g\Delta T - V_{Hg}\gamma_{Hg}\Delta T$

$\Rightarrow \dfrac{V_g}{V_{Hg}} = \dfrac{\gamma_{Hg}}{\gamma_g} \Rightarrow \dfrac{1000}{V_{Hg}} = \dfrac{1.8 \times 10^{-4}}{9 \times 10^{-6}}$

$\Rightarrow V_{HG} = \dfrac{9 \times 10^{-3}}{1.8 \times 10^{-4}} = 500$ CC.

23. Volume of water = $500cm^3$

Area of cross section of can = 125 m^2

Final Volume of water

$= 500(1 + \gamma\Delta\theta) = 500[1 + 3.2 \times 10^{-4} \times (80 - 10)] = 511.2\ cm^3$

The aluminium vessel expands in its length only so area expansion of base cab be neglected.

Increase in volume of water = 11.2 cm^3

Considering a cylinder of volume = 11.2 cm^3

Height of water increased = $\dfrac{11.2}{125} = 0.089$ cm

24. $V_0 = 10 \times 10 \times 10 = 1000$ CC

$\Delta T = 10°C$, $V'_{HG} - V'_g = 1.6\ cm^3$

$\alpha_g = 6.5 \times 10^{-6}$/°C, $\gamma_{Hg} = ?$, $\gamma_g = 3 \times 6.5 \times 10^{-6}$/°C

$V'_{Hg} = v_{HG}(1 + \gamma_{Hg}\Delta T)$ …(1)

$V'_g = v_g(1 + \gamma_g\Delta T)$ …(2)

$V'_{Hg} - V'_g = V_{Hg} - V_g + V_{Hg}\gamma_{Hg}\,\Delta T - V_g\gamma_g\,\Delta T$

$\Rightarrow 1.6 = 1000 \times \gamma_{Hg} \times 10 - 1000 \times 6.5 \times 3 \times 10^{-6} \times 10$

$\Rightarrow \gamma_{Hg} = \dfrac{1.6 + 6.3 \times 3 \times 10^{-2}}{10000} = 1.789 \times 10^{-4} \approx 1.8 \times 10^{-4}$/°C

25. $f_\omega = 880$ Kg/m^3, $f_b = 900$ Kg/m^3

$T_1 = 0°C$, $\gamma_\omega = 1.2 \times 10^{-3}$ /°C,

$\gamma_b = 1.5 \times 10^{-3}$ /°C

The sphere begins t sink when,

$(mg)_{sphere}$ = displaced water

$\Rightarrow V f'_{\omega}\, g = V f'_{b}\, g$

$\Rightarrow \dfrac{f_{\omega}}{1+\gamma_{\omega}\Delta\theta} = \dfrac{f_{b}}{1+\gamma_{b}\Delta\theta}$

$\Rightarrow \dfrac{880}{1+1.2\times10^{-3}\Delta\theta} = \dfrac{900}{1+1.5\times10^{-3}\Delta\theta}$

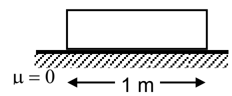
$\Rightarrow 880 + 880 \times 1.5 \times 10^{-3}\ (\Delta\theta) = 900 + 900 \times 1.2 \times 10^{-3}\ (\Delta\theta)$

$\Rightarrow (880 \times 1.5 \times 10^{-3} - 900 \times 1.2 \times 10^{-3})\ (\Delta\theta) = 20$

$\Rightarrow (1320 - 1080) \times 10^{-3}\ (\Delta\theta) = 20$

$\Rightarrow \Delta\theta = 83.3°C \approx 83°C$

26. $\Delta L = 100°C$



A longitudinal strain develops if and only if, there is an opposition to the expansion.
Since there is no opposition in this case, hence the longitudinal stain here = Zero.

27. $\theta_1 = 20°C,\ \theta_2 = 50°C$

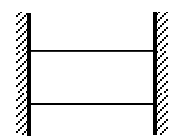
$\alpha_{steel} = 1.2 \times 10^{-5}$ /°C

Longitudinal stain = ?

Stain = $\dfrac{\Delta L}{L} = \dfrac{L\alpha\Delta\theta}{L} = \alpha\Delta\theta$

$= 1.2 \times 10^{-5} \times (50 - 20) = 3.6 \times 10^{-4}$

28. $A = 0.5mm^2 = 0.5 \times 10^{-6}\ m^2$



$T_1 = 20°C,\ T_2 = 0°C$

$\alpha_s = 1.2 \times 10^{-5}$ /°C, $\quad Y = 2 \times 2 \times 10^{11}\ N/m^2$

Decrease in length due to compression = $L\alpha\Delta\theta$ …(1)

$Y = \dfrac{\text{Stress}}{\text{Strain}} = \dfrac{F}{A}\times\dfrac{L}{\Delta L} \Rightarrow \Delta L = \dfrac{FL}{AY}$ …(2)

Tension is developed due to (1) & (2)

Equating them,

$L\alpha\Delta\theta = \dfrac{FL}{AY} \Rightarrow F = \alpha\Delta\theta AY$

$= 1.2 \times 10^{-5} \times (20 - 0) \times 0.5 \times 10^{-5}\ 2 \times 10^{11} = 24$ N

29. $\theta_1 = 20°C, \quad \theta_2 = 100°C$

$A = 2mm^2 = 2 \times 10^{-6}\ m^2$

$\alpha_{steel} = 12 \times 10^{-6}$ /°C, $\quad Y_{steel} = 2 \times 10^{11}\ N/m^2$

Force exerted on the clamps = ?

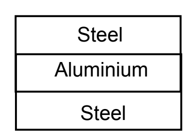
$\dfrac{\left(\frac{F}{A}\right)}{\text{Strain}} = Y \Rightarrow F = \dfrac{Y\times\Delta L}{L}\times L = \dfrac{YL\alpha\Delta\theta A}{L} = YA\alpha\Delta\theta$

$= 2 \times 10^{11} \times 2 \times 10^{-6} \times 12 \times 10^{-6} \times 80 = 384$ N

30. Let the final length of the system at system of temp. 0°C = ℓ_θ

Initial length of the system = ℓ_0

When temp. changes by θ.



Strain of the system = $\ell_1 - \dfrac{\ell_0}{\ell_\theta}$

But the total strain of the system = $\dfrac{\text{total stress of system}}{\text{total young's modulusof of system}}$

Now, total stress = Stress due to two steel rod + Stress due to Aluminium

$= \gamma_s\alpha_s\theta + \gamma_s\ ds\ \theta + \gamma_{al}\ at\ \theta = 2\%\ \alpha_s\ \theta + \gamma 2\ A\ell\ \theta$

Now young'' modulus of system = $\gamma_s + \gamma_s + \gamma_{al} = 2\gamma_s + \gamma_{al}$

$\therefore$ Strain of system = $\dfrac{2\gamma_s\alpha_s\theta + \gamma_s\alpha_{al}\theta}{2\gamma_s + \gamma_{al}}$

$\Rightarrow \dfrac{\ell_\theta - \ell_0}{\ell_0} = \dfrac{2\gamma_s\alpha_s\theta + \gamma_s\alpha_{al}\theta}{2\gamma_s + \gamma_{al}}$

$\Rightarrow \ell_\theta = \ell_0 \left[\dfrac{1 + \alpha_{al}\gamma_{al} + 2\alpha_s\gamma_s\theta}{\gamma_{al} + 2\gamma_s}\right]$

31. The ball tries to expand its volume. But it is kept in the same volume. So it is kept at a constant volume. So the stress arises

$\dfrac{P}{\left(\dfrac{\Delta V}{v}\right)} = B \Rightarrow P = B\dfrac{\Delta V}{V} = B \times \gamma\Delta\theta$

$= B \times 3\alpha\Delta\theta = 1.6 \times 10^{11} \times 10^{-6} \times 3 \times 12 \times 10^{-6} \times (120 - 20) = 57.6 \times 19^{7} \approx 5.8 \times 10^{8}$ pa.

32. Given

I_0 = Moment of Inertia at 0°C

α = Coefficient of linear expansion

To prove, $I = I_0 = (1 + 2\alpha\theta)$

Let the temp. change to θ from 0°C

$\Delta T = \theta$

Let 'R' be the radius of Gyration,

Now, $R' = R(1 + \alpha\theta)$, $I_0 = MR^2$ where M is the mass.

Now, $I' = MR'^2 = MR^2(1 + \alpha\theta)^2 \approx = MR^2(1 + 2\alpha\theta)$

[By binomial expansion or neglecting $\alpha^2\theta^2$ which given a very small value.]

So, $I = I_0(1 + 2\alpha\theta)$ (proved)

33. Let the initial m.I. at 0°C be I_0

$T = 2\pi\sqrt{\dfrac{I}{K}}$

$I = I_0(1 + 2\alpha\Delta\theta)$ (from above question)

At 5°C, $T_1 = 2\pi\sqrt{\dfrac{I_0(1+2\alpha\Delta\theta)}{K}} = 2\pi\sqrt{\dfrac{I_0(1+2\alpha 5)}{K}} = 2\pi\sqrt{\dfrac{I_0(1+10\alpha)}{K}}$

At 45°C, $T_2 = 2\pi\sqrt{\dfrac{I_0(1+2\alpha 45)}{K}} = 2\pi\sqrt{\dfrac{I_0(1+90\alpha)}{K}}$

$\dfrac{T_2}{T_1} = \sqrt{\dfrac{1+90\alpha}{1+10\alpha}} = \sqrt{\dfrac{1+90\times 2.4\times 10^{-5}}{1+10\times 2.4\times 10^{-5}}} \sqrt{\dfrac{1.00216}{1.00024}}$

% change = $\left(\dfrac{T_2}{T_1} - 1\right)\times 100 = 0.0959\% = 9.6 \times 10^{-2}\%$

34. $T_1 = 20°C$, $T_2 = 50°C$, $\Delta T = 30°C$

$\alpha = 1.2 \times 10^{5}$ /°C

ω remains constant

(I) $\omega = \dfrac{V}{R}$ (II) $\omega = \dfrac{V'}{R'}$

Now, $R' = R(1 + \alpha\Delta\theta) = R + R \times 1.2 \times 10^{-5} \times 30 = 1.00036R$

From (I) and (II)

$\dfrac{V}{R} = \dfrac{V'}{R'} = \dfrac{V'}{1.00036R}$

$\Rightarrow V' = 1.00036\, V$

% change = $\dfrac{(1.00036V - V)}{V} \times 100 = 0.00036 \times 100 = 3.6 \times 10^{-2}$

♣ ♣ ♣ ♣ ♣

CHAPTER 24
KINETIC THEORY OF GASES

1. Volume of 1 mole of gas

$PV = nRT \Rightarrow V = \frac{RT}{P} = \frac{0.082 \times 273}{1} = 22.38 \approx 22.4 \text{ L} = 22.4 \times 10^{-3} = 2.24 \times 10^{-2} \text{ m}^3$

2. $n = \frac{PV}{RT} = \frac{1 \times 1 \times 10^{-3}}{0.082 \times 273} = \frac{10^{-3}}{22.4} = \frac{1}{22400}$

No of molecules = $6.023 \times 10^{23} \times \frac{1}{22400} = 2.688 \times 10^{19}$

3. $V = 1 \text{ cm}^3$, $T = 0°C$, $P = 10^{-5}$ mm of Hg

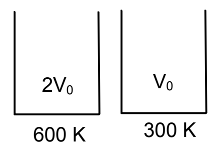
$n = \frac{PV}{RT} = \frac{f gh \times V}{RT} = \frac{1.36 \times 980 \times 10^{-6} \times 1}{8.31 \times 273} = 5.874 \times 10^{-13}$

No. of moluclues = No × n = $6.023 \times 10^{23} \times 5.874 \times 10^{-13} = 3.538 \times 10^{11}$

4. $n = \frac{PV}{RT} = \frac{1 \times 1 \times 10^{-3}}{0.082 \times 273} = \frac{10^{-3}}{22.4}$

mass = $\frac{(10^{-3} \times 32)}{22.4}$ g $= 1.428 \times 10^{-3}$ g = 1.428 mg

5. Since mass is same



$n_1 = n_2 = n$

$P_1 = \frac{nR \times 300}{V_0}$, $P_2 = \frac{nR \times 600}{2V_0}$

$\frac{P_1}{P_2} = \frac{nR \times 300}{V_0} \times \frac{2V_0}{nR \times 600} = \frac{1}{1} = 1 : 1$

6. $V = 250 \text{ cc} = 250 \times 10^{-3}$

$P = 10^{-3} \text{ mm} = 10^{-3} \times 10^{-3} \text{ m} = 10^{-6} \times 13600 \times 10 \text{ pascal} = 136 \times 10^{-3}$ pascal

$T = 27°C = 300$ K

$n = \frac{PV}{RT} = \frac{136 \times 10^{-3} \times 250}{8.3 \times 300} \times 10^{-3} = \frac{136 \times 250}{8.3 \times 300} \times 10^{-6}$

No. of molecules = $\frac{136 \times 250}{8.3 \times 300} \times 10^{-6} \times 6 \times 10^{23} = 81 \times 10^{17} \approx 0.8 \times 10^{15}$

7. $P_1 = 8.0 \times 10^5 \text{ P}_a$, $P_2 = 1 \times 10^6 \text{ P}_a$, $T_1 = 300$ K, $T_2 = ?$

Since, $V_1 = V_2 = V$

$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} \Rightarrow \frac{8 \times 10^5 \times V}{300} = \frac{1 \times 10^6 \times V}{T_2} \Rightarrow T_2 = \frac{1 \times 10^6 \times 300}{8 \times 10^5} = 375° \text{ K}$

8. m = 2 g, $V = 0.02 \text{ m}^3 = 0.02 \times 10^6 \text{ cc} = 0.02 \times 10^3$ L, T = 300 K, P = ?

M = 2 g,

$PV = nRT \Rightarrow PV = \frac{m}{M}RT \Rightarrow P \times 20 = \frac{2}{2} \times 0.082 \times 300$

$\Rightarrow P = \frac{0.082 \times 300}{20} = 1.23 \text{ atm} = 1.23 \times 10^5 \text{ pa} \approx 1.23 \times 10^5$ pa

9. $P = \frac{nRT}{V} = \frac{m}{M} \times \frac{RT}{V} = \frac{f RT}{M}$

$f \to 1.25 \times 10^{-3} \text{ g/cm}^3$

$R \to 8.31 \times 10^7$ ert/deg/mole

$T \to 273$ K

$\Rightarrow M = \frac{f RT}{P} = \frac{1.25 \times 10^{-3} \times 8.31 \times 10^7 \times 273}{13.6 \times 980 \times 76} = 0.002796 \times 10^4 \approx 28$ g/mol

10. T at Simla = 15°C = 15 + 273 = 288 K
P at Simla = 72 cm = $72 \times 10^{-2} \times 13600 \times 9.8$
T at Kalka = 35°C = 35 + 273 = 308 K
P at Kalka = 76 cm = $76 \times 10^{-2} \times 13600 \times 9.8$
PV = nRT

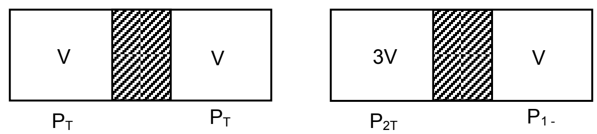
$$\Rightarrow PV = \frac{m}{M}RT \Rightarrow PM = \frac{m}{V}RT \Rightarrow f = \frac{PM}{RT}$$

$$\frac{f\text{Simla}}{f\text{Kalka}} = \frac{P_{\text{Simla}} \times M}{RT_{\text{Simla}}} \times \frac{RT_{\text{Kalka}}}{P_{\text{Kalka}} \times M}$$

$$= \frac{72 \times 10^{-2} \times 13600 \times 9.8 \times 308}{288 \times 76 \times 10^{-2} \times 13600 \times 9.8} = \frac{72 \times 308}{76 \times 288} = 1.013$$

$$\frac{f\text{Kalka}}{f\text{Simla}} = \frac{1}{1.013} = 0.987$$

11. $n_1 = n_2 = n$



$P_1 = \frac{nRT}{V}$, $\quad P_2 = \frac{nRT}{3V}$

$$\frac{P_1}{P_2} = \frac{nRT}{V} \times \frac{3V}{nRT} = 3 : 1$$

12. r.m.s velocity of hydrogen molecules = ?
T = 300 K, R = 8.3, M = 2 g = 2×10^{-3} Kg

$$C = \sqrt{\frac{3RT}{M}} \Rightarrow C = \sqrt{\frac{3 \times 8.3 \times 300}{2 \times 10^{-3}}} = 1932.6 \text{ m/s} \approx 1930 \text{ m/s}$$

Let the temp. at which the C = 2 × 1932.6 is T′

$$2 \times 1932.6 = \sqrt{\frac{3 \times 8.3 \times T'}{2 \times 10^{-3}}} \Rightarrow (2 \times 1932.6)^2 = \frac{3 \times 8.3 \times T'}{2 \times 10^{-3}}$$

$$\Rightarrow \frac{(2 \times 1932.6)^2 \times 2 \times 10^{-3}}{3 \times 8.3} = T'$$

$\Rightarrow$ T′ = 1199.98 ≈ 1200 K.

13. $V_{rms} = \sqrt{\frac{3P}{f}}$ $\quad$ P = 10^5 Pa = 1 atm, $\quad f = \frac{1.77 \times 10^{-4}}{10^{-3}}$

$$= \sqrt{\frac{3 \times 10^5 \times 10^{-3}}{1.77 \times 10^{-4}}} = 1301.8 \approx 1302 \text{ m/s.}$$

14. Agv. K.E. = 3/2 KT
3/2 KT = $0.04 \times 1.6 \times 10^{-19}$
$\Rightarrow (3/2) \times 1.38 \times 10^{-23} \times T = 0.04 \times 1.6 \times 10^{-19}$

$$\Rightarrow T = \frac{2 \times 0.04 \times 1.6 \times 10^{-19}}{3 \times 1.38 \times 10^{-23}} = 0.0309178 \times 10^4 = 309.178 \approx 310 \text{ K}$$

15. $V_{avg} = \sqrt{\frac{8RT}{\pi M}} = \sqrt{\frac{8 \times 8.3 \times 300}{3.14 \times 0.032}}$

$$T = \frac{\text{Distance}}{\text{Speed}} = \frac{6400000 \times 2}{445.25} = 445.25 \text{ m/s}$$

$$= \frac{28747.83}{3600} \text{ km} = 7.985 \approx 8 \text{ hrs.}$$

16. M = 4×10^{-3} Kg

$$V_{avg} = \sqrt{\frac{8RT}{\pi M}} = \sqrt{\frac{8 \times 8.3 \times 273}{3.14 \times 4 \times 10^{-3}}} = 1201.35$$

Momentum = M × V_{avg} = $6.64 \times 10^{-27} \times 1201.35 = 7.97 \times 10^{-24} \approx 8 \times 10^{-24}$ Kg-m/s.

17. $V_{avg} = \sqrt{\frac{8RT}{\pi M}} = \frac{8 \times 8.3 \times 300}{3.14 \times 0.032}$

Now, $\frac{8RT_1}{\pi \times 2} = \frac{8RT_2}{\pi \times 4}$ $\qquad \frac{T_1}{T_2} = \frac{1}{2}$

18. Mean speed of the molecule = $\sqrt{\frac{8RT}{\pi M}}$

Escape velocity = $\sqrt{2gr}$

$\sqrt{\frac{8RT}{\pi M}} = \sqrt{2gr} \qquad \Rightarrow \frac{8RT}{\pi M} = 2gr$

$\Rightarrow T = \frac{2gr\pi M}{8R} = \frac{2 \times 9.8 \times 6400000 \times 3.14 \times 2 \times 10^{-3}}{8 \times 8.3}$ = 11863.9 ≈ 11800 m/s.

19. $V_{avg} = \sqrt{\frac{8RT}{\pi M}}$

$\frac{V_{avg}H_2}{V_{avg}N_2} = \sqrt{\frac{8RT}{\pi \times 2}} \times \sqrt{\frac{\pi \times 28}{8RT}} = \sqrt{\frac{28}{2}} = \sqrt{14} = 3.74$

20. The left side of the container has a gas, let having molecular wt. M_1
Right part has Mol. wt = M_2
Temperature of both left and right chambers are equal as the separating wall is diathermic

$\sqrt{\frac{3RT}{M_1}} = \sqrt{\frac{8RT}{\pi M_2}} \Rightarrow \frac{3RT}{M_1} = \frac{8RT}{\pi M_2} \Rightarrow \frac{M_1}{\pi M_2} = \frac{3}{8} \Rightarrow \frac{M_1}{M_2} = \frac{3\pi}{8}$ = 1.1775 ≈ 1.18

21. $V_{mean} = \sqrt{\frac{8RT}{\pi M}} = \sqrt{\frac{8 \times 8.3 \times 273}{3.14 \times 2 \times 10^{-3}}}$ = 1698.96

Total Dist = 1698.96 m

No. of Collisions = $\frac{1698.96}{1.38 \times 10^{-7}} = 1.23 \times 10^{10}$

22. P = 1 atm = 10^5 Pascal
T = 300 K, $\qquad$ M = 2 g = 2×10^{-3} Kg

(a) $V_{avg} = \sqrt{\frac{8RT}{\pi M}} = \sqrt{\frac{8 \times 8.3 \times 300}{3.14 \times 2 \times 10^{-3}}}$ = 1781.004 ≈ 1780 m/s

(b) When the molecules strike at an angle 45°,

Force exerted = mV Cos 45° – (–mV Cos 45°) = 2 mV Cos 45° = 2 m V $\frac{1}{\sqrt{2}} = \sqrt{2}$ mV

No. of molecules striking per unit area = $\frac{\text{Force}}{\sqrt{2}mv \times \text{Area}} = \frac{\text{Pressure}}{\sqrt{2}mV}$

$= \frac{10^5}{\frac{\sqrt{2} \times 2 \times 10^{-3} \times 1780}{6 \times 10^{23}}} = \frac{3}{\sqrt{2} \times 1780} \times 10^{31} = 1.19 \times 10^{-3} \times 10^{31} = 1.19 \times 10^{28} \approx 1.2 \times 10^{28}$

23. $\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$

$P_1 \rightarrow$ 200 KPa = 2×10^5 pa $\qquad$ P_2 = ?
T_1 = 20°C = 293 K $\qquad$ T_2 = 40°C = 313 K

$V_2 = V_1 + 2\% \, V_1 = \frac{102 \times V_1}{100}$

$\Rightarrow \frac{2 \times 10^5 \times V_1}{293} = \frac{P_2 \times 102 \times V_1}{100 \times 313} \Rightarrow P_2 = \frac{2 \times 10^7 \times 313}{102 \times 293}$ = 209462 Pa = 209.462 KPa

24. $V_1 = 1 \times 10^{-3}\ m^3$, $P_1 = 1.5 \times 10^5$ Pa, $T_1 = 400$ K

$P_1V_1 = n_1R_1T_1$

$\Rightarrow n = \frac{P_1V_1}{R_1T_1} = \frac{1.5 \times 10^5 \times 1 \times 10^{-3}}{8.3 \times 400}$ $\Rightarrow n = \frac{1.5}{8.3 \times 4}$

$\Rightarrow m_1 = \frac{1.5}{8.3 \times 4} \times M = \frac{1.5}{8.3 \times 4} \times 32 = 1.4457 \approx 1.446$

$P_2 = 1 \times 10^5$ Pa, $V_2 = 1 \times 10^{-3}\ m^3$, $T_2 = 300$ K

$P_2V_2 = n_2R_2T_2$

$\Rightarrow n_2 = \frac{P_2V_2}{R_2T_2} = \frac{10^5 \times 10^{-3}}{8.3 \times 300} = \frac{1}{3 \times 8.3} = 0.040$

$\Rightarrow m_2 = 0.04 \times 32 = 1.285$

$\Delta m = m_1 - m_2 = 1.446 - 1.285 = 0.1608\ g \approx 0.16\ g$

25. $P_1 = 10^5 + \rho gh = 10^5 + 1000 \times 10 \times 3.3 = 1.33 \times 10^5$ pa

$P_2 = 10^5$, $T_1 = T_2 = T$, $V_1 = \frac{4}{3}\pi(2 \times 10^{-3})^3$

$V_2 = \frac{4}{3}\pi r^3$, $r = ?$

$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$

$\Rightarrow \frac{1.33 \times 10^5 \times \frac{4}{3} \times \pi \times (2 \times 10^{-3})^3}{T_1} = \frac{10^5 \times \frac{4}{3} \times \pi r^2}{T_2}$

$\Rightarrow 1.33 \times 8 \times 10^5 \times 10^{-9} = 10^5 \times r^3$ $\Rightarrow r = \sqrt[3]{10.64 \times 10^{-3}} = 2.19 \times 10^{-3} \approx 2.2$ mm

26. $P_1 = 2$ atm $= 2 \times 10^5$ pa

$V_1 = 0.002\ m^3$, $T_1 = 300$ K

$P_1V_1 = n_1RT_1$

$\Rightarrow n = \frac{P_1V_1}{RT_1} = \frac{2 \times 10^5 \times 0.002}{8.3 \times 300} = \frac{4}{8.3 \times 3} = 0.1606$

$P_2 = 1$ atm $= 10^5$ pa

$V_2 = 0.0005\ m^3$, $T_2 = 300$ K

$P_2V_2 = n_2RT_2$

$\Rightarrow n_2 = \frac{P_2V_2}{RT_2} = \frac{10^5 \times 0.0005}{8.3 \times 300} = \frac{5}{3 \times 8.3} \times \frac{1}{10} = 0.02$

Δn = moles leaked out $= 0.16 - 0.02 = 0.14$

27. $m = 0.040$ g, $T = 100°C$, $M_{He} = 4$ g

$U = \frac{3}{2}nRt = \frac{3}{2} \times \frac{m}{M} \times RT$ $T' = ?$

Given $\frac{3}{2} \times \frac{m}{M} \times RT + 12 = \frac{3}{2} \times \frac{m}{M} \times RT'$

$\Rightarrow 1.5 \times 0.01 \times 8.3 \times 373 + 12 = 1.5 \times 0.01 \times 8.3 \times T'$

$\Rightarrow T' = \frac{58.4385}{0.1245} = 469.3855\ K = 196.3°C \approx 196°C$

28. PV^2 = constant

$\Rightarrow P_1V_1^2 = P_2V_2^2$

$\Rightarrow \frac{nRT_1}{V_1} \times V_1^2 = \frac{nRT_2}{V_2} \times V_2^2$

$\Rightarrow T_1 V_1 = T_2 V_2 = TV = T_1 \times 2V \Rightarrow T_2 = \frac{T}{2}$

29. $P_{O_2} = \frac{n_{o_2}RT}{V}$, $P_{H_2} = \frac{n_{H_2}RT}{V}$

$n_{O_2} = \frac{m}{M_{O_2}} = \frac{1.60}{32} = 0.05$

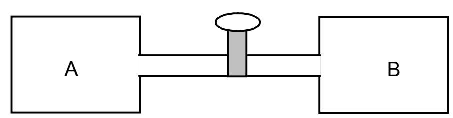
Now, $P_{mix} = \left(\frac{n_{O_2} + n_{H_2}}{V}\right)RT$

$n_{H_2} = \frac{m}{M_{H_2}} = \frac{2.80}{28} = 0.1$

$P_{mix} = \frac{(0.05 + 0.1) \times 8.3 \times 300}{0.166} = 2250 \text{ N/m}^2$

30. P_1 = Atmospheric pressure = 75 × fg
V_1 = 100 × A
P_2 = Atmospheric pressure + Mercury pessue = 75fg + hg$f$g (if h = height of mercury)
V_2 = (100 – h) A
$P_1V_1 = P_2V_2$
$\Rightarrow 75fg(100A) = (75 + h)fg(100 – h)A$
$\Rightarrow 75 \times 100 = (74 + h)(100 – h) \Rightarrow 7500 = 7500 – 75 h + 100 h – h^2$
$\Rightarrow h^2 – 25 h = 0 \Rightarrow h^2 = 25 h \Rightarrow h = 25$ cm
Height of mercury that can be poured = 25 cm

31. Now, Let the final pressure; Volume & Temp be
After connection = P_A' → Partial pressure of A
P_B' → Partial pressure of B



Now, $\frac{P_A' \times 2V}{T} = \frac{P_A \times V}{T_A}$

Or $\frac{P_A'}{T} = \frac{P_A}{2T_A}$ …(1)

Similarly, $\frac{P_B'}{T} = \frac{P_B}{2T_B}$ …(2)

Adding (1) & (2)

$\frac{P_A'}{T} + \frac{P_B'}{T} = \frac{P_A}{2T_A} + \frac{P_B}{2T_B} = \frac{1}{2}\left(\frac{P_A}{T_A} + \frac{P_B}{T_B}\right)$

$\Rightarrow \frac{P}{T} = \frac{1}{2}\left(\frac{P_A}{T_A} + \frac{P_B}{T_B}\right)$ [$\therefore P_A' + P_B' = P$]

32. V = 50 cc = 50×10^{-6} cm^3
P = 100 KPa = 10^5 Pa M = 28.8 g
(a) $PV = nrT_1$

$\Rightarrow PV = \frac{m}{M}RT_1 \Rightarrow m = \frac{PMV}{RT_1} = \frac{10^5 \times 28.8 \times 50 \times 10^{-6}}{8.3 \times 273} = \frac{50 \times 28.8 \times 10^{-1}}{8.3 \times 273} = 0.0635$ g.

(b) When the vessel is kept on boiling water

$PV = \frac{m}{M}RT_2 \Rightarrow m = \frac{PVM}{RT_2} = \frac{10^5 \times 28.8 \times 50 \times 10^{-6}}{8.3 \times 373} = \frac{50 \times 28.8 \times 10^{-1}}{8.3 \times 373} = 0.0465$

(c) When the vessel is closed

$P \times 50 \times 10^{-6} = \frac{0.0465}{28.8} \times 8.3 \times 273$

$\Rightarrow P = \frac{0.0465 \times 8.3 \times 273}{28.8 \times 50 \times 10^{-6}} = 0.07316 \times 10^6 \text{ Pa} \approx 73 \text{ KPa}$

33. Case I → Net pressure on air in volume V

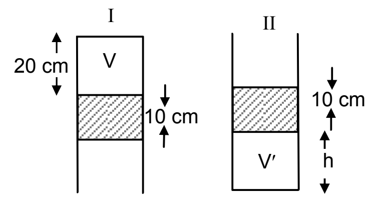
$= P_{atm} - hfg = 75 \times f_{Hg} - 10\, f_{Hg} = 65 \times f_{Hg} \times g$

Case II → Net pressure on air in volume 'V' $= P_{atm} + f_{Hg} \times g \times h$

$P_1V_1 = P_2V_2$

$\Rightarrow f_{Hg} \times g \times 65 \times A \times 20 = f_{Hg} \times g \times 75 + f_{Hg} \times g \times 10 \times A \times h$

$\Rightarrow 62 \times 20 = 85\ h \Rightarrow h = \dfrac{65 \times 20}{85} = 15.2\text{ cm} \approx 15\text{ cm}$

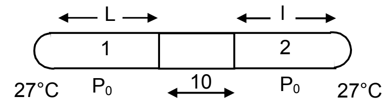


34. $2L + 10 = 100 \Rightarrow 2L = 90 \Rightarrow L = 45$ cm

Applying combined gas eqn to part 1 of the tube

$$\frac{(45A)P_0}{300} = \frac{(45-x)P_1}{273}$$

$$\Rightarrow P_1 = \frac{273 \times 45 \times P_0}{300(45-x)}$$

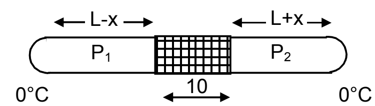


Applying combined gas eqn to part 2 of the tube

$$\frac{45AP_0}{300} = \frac{(45+x)AP_2}{400}$$

$$\Rightarrow P_2 = \frac{400 \times 45 \times P_0}{300(45+x)}$$

$P_1 = P_2$



$$\Rightarrow \frac{273 \times 45 \times P_0}{300(45-x)} = \frac{400 \times 45 \times P_0}{300(45+x)}$$

$\Rightarrow (45 - x)\ 400 = (45 + x)\ 273 \qquad \Rightarrow 18000 - 400\ x = 12285 + 273\ x$

$\Rightarrow (400 + 273)x = 18000 - 12285 \qquad \Rightarrow x = 8.49$

$P_1 = \dfrac{273 \times 46 \times 76}{300 \times 36.51} = 85$ % 25 cm of Hg

Length of air column on the cooler side = L – x = 45 – 8.49 = 36.51

35. Case I Atmospheric pressure + pressure due to mercury column

Case II Atmospheric pressure + Component of the pressure due to mercury column

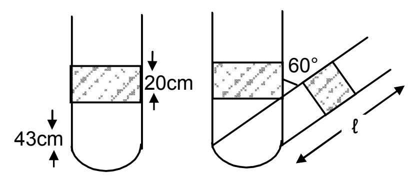
$P_1V_1 = P_2V_2$

$\Rightarrow (76 \times f_{Hg} \times g + f_{Hg} \times g \times 20) \times A \times 43$

$= (76 \times f_{Hg} \times g + f_{Hg} \times g \times 20 \times \cos 60°)\ A \times \ell$

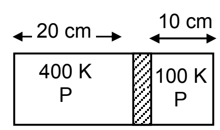
$\Rightarrow 96 \times 43 = 86 \times \ell$

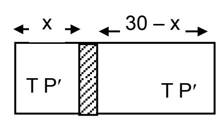
$\Rightarrow \ell = \dfrac{96 \times 43}{86} = 48$ cm



36. The middle wall is weakly conducting. Thus after a long time the temperature of both the parts will equalise.

The final position of the separating wall be at distance x from the left end. So it is at a distance 30 – x from the right end





Putting combined gas equation of one side of the separating wall,

$$\frac{P_1 \times V_1}{T_1} = \frac{P_2 \times V_2}{T_2}$$

$$\Rightarrow \frac{P \times 20A}{400} = \frac{P' \times A}{T} \qquad \ldots(1)$$

$$\Rightarrow \frac{P \times 10A}{100} = \frac{-P'(30-x)}{T} \qquad \ldots(2)$$

Equating (1) and (2)

$\Rightarrow \dfrac{1}{2} = \dfrac{x}{30-x} \qquad \Rightarrow 30 - x = 2x \Rightarrow 3x = 30 \Rightarrow x = 10$ cm

The separator will be at a distance 10 cm from left end.

37. $\frac{dV}{dt} = r \Rightarrow dV = r\,dt$

Let the pumped out gas pressure dp

Volume of container = V_0 At a pump dv amount of gas has been pumped out.

$Pdv = -V_0 df \Rightarrow P_V\, df = -V_0\, dp$

$$\Rightarrow \int_P^P \frac{dp}{p} = -\int_0^t \frac{dtr}{V_0} \Rightarrow P = P\, e^{-rt/V_0}$$

Half of the gas has been pump out, Pressure will be half = $\frac{1}{2}e^{-vt/V_0}$

$$\Rightarrow \ln 2 = \frac{rt}{V_0} \qquad \Rightarrow t = \ln^2 \frac{\gamma_0}{r}$$

38. $P = \dfrac{P_0}{1+\left(\dfrac{V}{V_0}\right)^2}$

$$\Rightarrow \frac{nRT}{V} = \frac{P_0}{1+\left(\frac{V}{V_0}\right)^2} \qquad \text{[PV = nRT according to ideal gas equation]}$$

$$\Rightarrow \frac{RT}{V} = \frac{P_0}{1+\left(\frac{V}{V_0}\right)^2} \qquad \text{[Since n = 1 mole]}$$

$$\Rightarrow \frac{RT}{V_0} = \frac{P_0}{1+\left(\frac{V}{V_0}\right)^2} \qquad [\text{At } V = V_0]$$

$$\Rightarrow P_0V_0 = RT(1+1) \Rightarrow P_0V_0 = 2\,RT \Rightarrow T = \frac{P_0V_0}{2R}$$

39. Internal energy = nRT

Now, PV = nRT

$nT = \frac{PV}{R}$ Here P & V constant

$\Rightarrow$ nT is constant

$\therefore$ Internal energy = R × Constant = Constant

40. Frictional force = μ N

Let the cork moves to a distance = dl

$\therefore$ Work done by frictional force = μNde

Before that the work will not start that means volume remains constant

$$\Rightarrow \frac{P_1}{T_1} = \frac{P_2}{T_2} \Rightarrow \frac{1}{300} = \frac{P_2}{600} \Rightarrow P_2 = 2 \text{ atm}$$

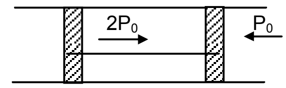
$\therefore$ Extra Pressure = 2 atm – 1 atm = 1 atm

Work done by cork = 1 atm (Adl) $\quad \mu$Ndl = [1atm][Adl]

$$N = \frac{1\times10^5 \times (5\times10^{-2})^2}{2} = \frac{1\times10^5 \times \pi \times 25 \times 10^{-5}}{2}$$

Total circumference of work = $2\pi r \;\; \frac{dN}{dl} = \frac{N}{2\pi r}$

$$= \frac{1\times10^5 \times \pi \times 25 \times 10^{-5}}{0.2 \times 2\pi r} = \frac{1\times10^5 \times 25 \times 10^{-5}}{0.2 \times 2 \times 5 \times 10^5} = 1.25 \times 10^4 \text{ N/M}$$



41. $\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$

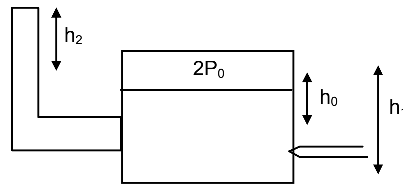
$\Rightarrow \frac{P_0V}{T_0} = \frac{P'V}{2T_0} \Rightarrow P' = 2\,P_0$

Net pressure = P_0 outwards

$\therefore$ Tension in wire = P_0 A

Where A is area of tube.

42. (a) $2P_0x = (h_2 + h_0)fg$ [$\therefore$ Since liquid at the same level have same pressure]



$\Rightarrow 2P_0 = h_2\,fg + h_0\,fg$

$\Rightarrow h_2\,fg = 2P_0 - h_0\,fg$

$h_2 = \frac{2P_0}{fg} - \frac{h_0 fg}{fg} = \frac{2P_0}{fg} - h_0$

(b) K.E. of the water = Pressure energy of the water at that layer

$\Rightarrow \frac{1}{2}mV^2 = m \times \frac{P}{f}$

$\Rightarrow V^2 = \frac{2P}{f} = \left[\frac{2}{f(P_0 + fg(h_1 - h_0))}\right]$

$\Rightarrow V = \left[\frac{2}{f(P_0 + fg(h_1 - h_0))}\right]^{1/2}$

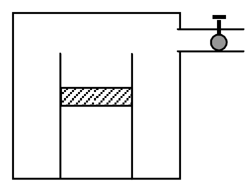
(c) $(x + P_0)fh = 2P_0$

$\therefore 2P_0 + fg\,(h - h_0) = P_0 + fgx$

$\therefore X = \frac{P_0}{fg + h_1 - h_0} = h_2 + h_1$

$\therefore$ i.e. x is h_1 meter below the top $\Rightarrow$ x is $-h_1$ above the top

43. A = 100 cm^2 = 10^{-3} m



m = 1 kg, P = 100 K Pa = 10^5 Pa

ℓ = 20 cm

<u>Case I</u> = External pressure exists

<u>Case II</u> = Internal Pressure does not exist

$P_1V_1 = P_2V_2$

$\Rightarrow \left(10^5 + \frac{1 \times 9.8}{10^{-3}}\right)V = \frac{1 \times 9.8}{10^{-3}} \times V'$

$\Rightarrow (10^5 + 9.8 \times 10^3)A \times \ell = 9.8 \times 10^3 \times A \times \ell'$

$\Rightarrow 10^5 \times 2 \times 10^{-1} + 2 \times 9.8 \times 10^2 = 9.8 \times 10^3 \times \ell'$

$\Rightarrow \ell' = \frac{2 \times 10^4 + 19.6 \times 10^2}{9.8 \times 10^3} = 2.24081$ m

44. $P_1V_1 = P_2V_2$

$\Rightarrow \left(\frac{mg}{A} + P_0\right)A\ell \;\; P_0\,A\ell$

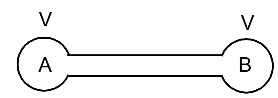
$\Rightarrow \left(\frac{1 \times 9.8}{10 \times 10^{-4}} + 10^5\right)0.2 = 10^5\,\ell'$

$\Rightarrow (9.8 \times 10^3 + 10^5) \times 0.2 = 10^5\,\ell'$

$\Rightarrow 109.8 \times 10^3 \times 0.2 = 10^5\,\ell'$

$\Rightarrow \ell' = \frac{109.8 \times 0.2}{10^2} = 0.2196 \approx 0.22$ m ≈ 22 cm

45. When the bulbs are maintained at two different temperatures.



The total heat gained by 'B' is the heat lost by 'A'

Let the final temp be x So, $m_1 S\Delta t = m_2 S\Delta t$

$\Rightarrow n_1 M \times s(x - 0) = n_2 M \times S \times (62 - x)$ $\Rightarrow n_1 x = 62n_2 - n_2 x$

$\Rightarrow x = \frac{62n_2}{n_1 + n_2} = \frac{62n_2}{2n_2} = 31°C = 304 K$

For a single ball Initial Temp = 0°C P = 76 cm of Hg

$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$ $V_1 = V_2$ Hence $n_1 = n_2$

$\Rightarrow \frac{76 \times V}{273} = \frac{P_2 \times V}{304}$ $\Rightarrow P_2 = \frac{403 \times 76}{273} = 84.630 \approx 84°C$

46. Temp is 20° Relative humidity = 100%

So the air is saturated at 20°C

Dew point is the temperature at which SVP is equal to present vapour pressure

So 20°C is the dew point.

47. T = 25°C P = 104 KPa

$RH = \frac{VP}{SVP}$ [SVP = 3.2 KPa, RH = 0.6]

$VP = 0.6 \times 3.2 \times 10^3 = 1.92 \times 10^3 \approx 2 \times 10^3$

When vapours are removed VP reduces to zero

Net pressure inside the room now = $104 \times 10^3 - 2 \times 10^3 = 102 \times 10^3$ = 102 KPa

48. Temp = 20°C Dew point = 10°C

The place is saturated at 10°C

Even if the temp drop dew point remains unaffected.

The air has V.P. which is the saturation VP at 10°C. It (SVP) does not change on temp.

49. $RH = \frac{VP}{SVP}$

The point where the vapour starts condensing, VP = SVP

We know $P_1V_1 = P_2V_2$

R_H SVP × 10 = SVP × V_2 $\Rightarrow V_2 = 10R_H \Rightarrow 10 \times 0.4 = 4\ cm^3$

50. Atm–Pressure = 76 cm of Hg

When water is introduced the water vapour exerts some pressure which counter acts the atm pressure.

The pressure drops to 75.4 cm

Pressure of Vapour = (76 – 75.4) cm = 0.6 cm

R. Humidity = $\frac{VP}{SVP} = \frac{0.6}{1} = 0.6 = 60\%$

51. From fig. 24.6, we draw ⊥r, from Y axis to meet the graphs.

Hence we find the temp. to be approximately 65°C & 45°C

52. The temp. of body is 98°F = 37°C

At 37°C from the graph SVP = Just less than 50 mm

B.P. is the temp. when atmospheric pressure equals the atmospheric pressure.

Thus min. pressure to prevent boiling is 50 mm of Hg.

53. Given

SVP at the dew point = 8.9 mm SVP at room temp = 17.5 mm

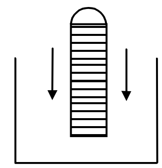
Dew point = 10°C as at this temp. the condensation starts

Room temp = 20°C

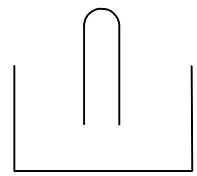
$RH = \frac{\text{SVP at dew point}}{\text{SVP at room temp}} = \frac{8.9}{17.5} = 0.508 \approx 51\%$

54. 50 cm^3 of saturated vapour is cooled 30° to 20°. The absolute humidity of saturated H_2O vapour 30 g/m^3
Absolute humidity is the mass of water vapour present in a given volume at 30°C, it contains 30 g/m^3
at 50 m^3 it contains 30 × 50 = 1500 g
at 20°C it contains 16 × 50 = 800 g
Water condense = 1500 – 800 = 700 g.

55. Pressure is minimum when the vapour present inside are at saturation vapour pressure
As this is the max. pressure which the vapours can exert.
Hence the normal level of mercury drops down by 0.80 cm
$\therefore$ The height of the Hg column = 76 – 0.80 cm = 75.2 cm of Hg.
[$\because$ Given SVP at atmospheric temp = 0.80 cm of Hg]



56. Pressure inside the tube = Atmospheric Pressure = 99.4 KPa
Pressure exerted by O_2 vapour = Atmospheric pressure – V.P.
= 99.4 KPa – 3.4 KPa = 96 KPa
No of moles of O_2 = n
$96 \times 10^3 \times 50 \times 10^{-6} = n \times 8.3 \times 300$
$\Rightarrow n = \frac{96 \times 50 \times 10^{-3}}{8.3 \times 300} = 1.9277 \times 10^{-3} \approx 1.93 \times 10^{-3}$



57. Let the barometer has a length = x
Height of air above the mercury column = (x – 74 – 1) = (x – 73)
Pressure of air = 76 – 74 – 1 = 1 cm
For 2^{nd} case height of air above = (x – 72.1 – 1 – 1) = (x – 71.1)
Pressure of air = (74 – 72.1 – 1) = 0.99
$(x-73)(1) = \frac{9}{10}(x-71.1) \qquad \Rightarrow 10(x-73) = 9(x-71.1)$
$\Rightarrow x = 10 \times 73 - 9 \times 71.1 = 730 - 639.9 = 90.1$
Height of air = 90.1
Height of barometer tube above the mercury column = 90.1 + 1 = 91.1 mm

58. Relative humidity = 40%
SVP = 4.6 mm of Hg
$0.4 = \frac{VP}{4.6} \qquad \Rightarrow VP = 0.4 \times 4.6 = 1.84$
$\frac{P_1V}{T_1} = \frac{P_2V}{T_2} \qquad \Rightarrow \frac{1.84}{273} = \frac{P_2}{293} \Rightarrow P_2 = \frac{1.84}{273} \times 293$
Relative humidity at 20°C
$= \frac{VP}{SVP} = \frac{1.84 \times 293}{273 \times 10} = 0.109 = 10.9\%$

59. $RH = \frac{VP}{SVP}$
Given, $0.50 = \frac{VP}{3600}$
$\Rightarrow VP = 3600 \times 0.5$
Let the Extra pressure needed be P
So, $P = \frac{m}{M} \times \frac{RT}{V} = \frac{m}{18} \times \frac{8.3 \times 300}{1}$
Now, $\frac{m}{18} \times 8.3 \times 300 + 3600 \times 0.50 = 3600$ [air is saturated i.e. RH = 100% = 1 or VP = SVP]
$\Rightarrow m = \left(\frac{36-18}{8.3}\right) \times 6 = 13$ g

60. T = 300 K, Rel. humidity = 20%, V = 50 m^3
SVP at 300 K = 3.3 KPa, V.P. = Relative humidity × SVP = 0.2 × 3.3 × 10^3

$PV = \frac{m}{M}RT \Rightarrow 0.2 \times 3.3 \times 10^3 \times 50 = \frac{m}{18} \times 8.3 \times 300$

$\Rightarrow m = \frac{0.2 \times 3.3 \times 50 \times 18 \times 10^3}{8.3 \times 300} = 238.55 \text{ grams} \approx 238 \text{ g}$

Mass of water present in the room = 238 g.

61. $RH = \frac{VP}{SVP} \Rightarrow 0.20 = \frac{VP}{3.3 \times 10^3} \Rightarrow VP = 0.2 \times 3.3 \times 10^3 = 660$

$PV = nRT \Rightarrow P = \frac{nRT}{V} = \frac{m}{M} \times \frac{RT}{V} = \frac{500}{18} \times \frac{8.3 \times 300}{50} = 1383.3$

Net P = 1383.3 + 660 = 2043.3 Now, $RH = \frac{2034.3}{3300} = 0.619 \approx 62\%$

62. (a) Rel. humidity = $\frac{VP}{SVP \text{ at } 15°C} \Rightarrow 0.4 = \frac{VP}{1.6 \times 10^3} \Rightarrow VP = 0.4 \times 1.6 \times 10^3$

The evaporation occurs as along as the atmosphere does not become saturated.
Net pressure change = $1.6 \times 10^3 - 0.4 \times 1.6 \times 10^3 = (1.6 - 0.4 \times 1.6)10^3 = 0.96 \times 10^3$

Net mass of water evaporated = m $\Rightarrow 0.96 \times 10^3 \times 50 = \frac{m}{18} \times 8.3 \times 288$

$\Rightarrow m = \frac{0.96 \times 50 \times 18 \times 10^3}{8.3 \times 288} = 361.45 \approx 361 \text{ g}$

(b) At 20°C SVP = 2.4 KPa, At 15°C SVP = 1.6 KPa
Net pressure charge = $(2.4 - 1.6) \times 10^3$ Pa = 0.8×10^3 Pa

Mass of water evaporated = $m' = 0.8 \times 10^3\ 50 = \frac{m'}{18} \times 8.3 \times 293$

$\Rightarrow m' = \frac{0.8 \times 50 \times 18 \times 10^3}{8.3 \times 293} = 296.06 \approx 296 \text{ grams}$

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CHAPTER – 25
CALORIMETRY

1. Mass of aluminium = 0.5kg, Mass of water = 0.2 kg
   Mass of Iron = 0.2 kg Temp. of aluminium and water = 20°C = 297°k
   Sp heat o f Iron = 100°C = 373°k. Sp heat of aluminium = 910J/kg-k
   Sp heat of Iron = 470J/kg-k Sp heat of water = 4200J/kg-k
   Heat again = 0.5 × 910(T – 293) + 0.2 × 4200 × (343 –T)
   = (T – 292) (0.5 × 910 + 0.2 × 4200) Heat lost = 0.2 × 470 × (373 – T)
   ∴ Heat gain = Heat lost
   $\Rightarrow$ (T – 292) (0.5 × 910 + 0.2 × 4200) = 0.2 × 470 × (373 – T)
   $\Rightarrow$ (T – 293) (455 + 8400) = 49(373 – T)
   $\Rightarrow (T - 293)\left(\frac{1295}{94}\right) = (373 - T)$
   $\Rightarrow$ (T – 293) × 14 = 373 – T
   $\Rightarrow T = \frac{4475}{15} = 298$ k
   ∴ T = 298 – 273 = 25°C. The final temp = 25°C.
2. mass of Iron = 100g water Eq of caloriemeter = 10g
   mass of water = 240g Let the Temp. of surface = 0°C
   S_{iron} = 470J/kg°C Total heat gained = Total heat lost.
   So, $\frac{100}{1000} \times 470 \times (\theta - 60) = \frac{250}{1000} \times 4200 \times (60 - 20)$
   $\Rightarrow 47\theta - 47 \times 60 = 25 \times 42 \times 40$
   $\Rightarrow \theta = 4200 + \frac{2820}{47} = \frac{44820}{47} = 953.61°C$
3. The temp. of A = 12°C The temp. of B = 19°C
   The temp. of C = 28°C The temp of $\Rightarrow$ A + B = 16°
   The temp. of $\Rightarrow$ B + C = 23°
   In accordance with the principle of caloriemetry when A & B are mixed
   M_{CA} (16 – 12) = M_{CB} (19 – 16) $\Rightarrow$ CA4 = CB3 $\Rightarrow CA = \frac{3}{4}CB$ …(1)
   And when B & C are mixed
   M_{CB} (23 – 19)= M_{CC} (28 – 23) $\Rightarrow$ 4CB = 5CC $\Rightarrow CC = \frac{4}{5}CB$ …(2)
   When A & c are mixed, if T is the common temperature of mixture
   M_{CA} (T – 12) = M_{CC} (28 – T)
   $\Rightarrow \left(\frac{3}{4}\right)CB(T - 12) = \left(\frac{4}{5}\right)CB(28 - T)$
   $\Rightarrow$ 15T – 180 = 448 – 16T
   $\Rightarrow T = \frac{628}{31} = 20.258°C = 20.3°C$

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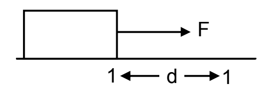
CHAPTER 26
LAWS OF THERMODYNAMICS
QUESTIONS FOR SHORT ANSWER

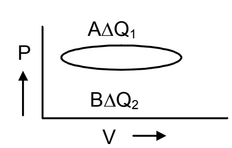
1. No in isothermal process heat is added to a system. The temperature does not increase so the internal energy does not.
2. Yes, the internal energy must increase when temp. increases; as internal energy depends upon temperature $U \propto T$
3. Work done on the gas is 0. as the P.E. of the container si increased and not of gas. Work done by the gas is 0. as the gas is not expanding.
   The temperature of the gas is decreased.
4. $W = F \times d = Fd \cos 0° = Fd$
   Change in PE is zero. Change in KE is non Zero.
   So, there may be some internal energy.



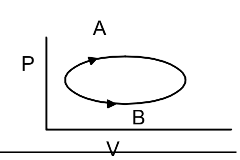
5. The outer surface of the cylinder is rubbed vigorously by a polishing machine.
   The energy given to the cylinder is work. The heat is produced on the cylinder which transferred to the gas.
6. No. work done by rubbing the hands in converted to heat and the hands become warm.
7. When the bottle is shaken the liquid in it is also shaken. Thus work is done on the liquid. But heat is not transferred to the liquid.
8. Final volume = Initial volume. So, the process is isobaric.
   Work done in an isobaric process is necessarily zero.
9. No word can be done by the system without changing its volume.
10. Internal energy = $U = nC_VT$
    Now, since gas is continuously pumped in. So $n_2 = 2n_1$ as the $p_2 = 2p_1$. Hence the internal energy is also doubled.
11. When the tyre bursts, there is adiabatic expansion of the air because the pressure of the air inside is sufficiently higher than atmospheric pressure. In expansion air does some work against surroundings. So the internal energy decreases. This leads to a fall in temperature.
12. 'No', work is done on the system during this process. No, because the object expands during the process i.e. volume increases.
13. No, it is not a reversible process.
14. Total heat input = Total heat out put i.e., the total heat energy given to the system is converted to mechanical work.
15. Yes, the entropy of the body decreases. But in order to cool down a body we need another external sink which draws out the heat the entropy of object in partly transferred to the external sink. Thus once entropy is created. It is kept by universe. And it is never destroyed. This is according to the 2^{nd} law of thermodynamics

OBJECTIVE – I

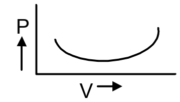
1. (d) Dq = DU + DW. This is the statement of law of conservation of energy. The energy provided is utilized to do work as well as increase the molecular K.E. and P.E.
2. (b) Since it is an isothermal process. So temp. will remain constant as a result 'U' or internal energy will also remain constant. So the system has to do positive work.
3. (a) In case of A $\Delta W_1 > \Delta W_2$ (Area under the graph is higher for A than for B).
   $\Delta Q = \Delta u + dw$.
   du for both the processes is same (as it is a state function)
   $\therefore \Delta Q_1 > \Delta Q_2$ as $\Delta W_1 > \Delta W_2$



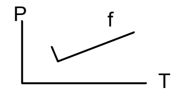
4. (b) As Internal energy is a state function and not a path function. $\Delta U_1 = \Delta U_2$



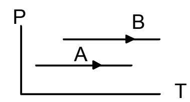
5. (a) In the process the volume of the system increases continuously. Thus, the work done increases continuously.



6. (c) for A → In a so thermal system temp remains same although heat is added.
   for B → For the work done by the system volume increase as is consumes heat.
7. (c) In this case P and T varry proportionally i.e. P/T = constant. This is possible only when volume does not change. ∴ pdv = 0 ω



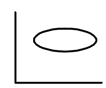
8. (c) Given : $\Delta V_A = \Delta V_B$. But $P_A < P_B$
   Now, $W_A = P_A\, \Delta V_B$; $W_B = P_B\, \Delta V_B$; So, $W_A < W_B$.



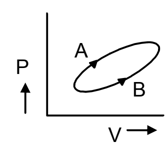
9. (b) As the volume of the gas decreases, the temperature increases as well as the pressure. But, on passage of time, the heat develops radiates through the metallic cylinder thus T decreases as well as the pressure.

OBJECTIVE – II

1. (b), (c) Pressure P and Volume V both increases. Thus work done is positive (V increases). Heat must be added to the system to follow this process. So temperature must increases.
2. (a) (b) Initial temp = Final Temp. Initial internal energy = Final internal energy.
   i.e. $\Delta U = 0$, So, this is found in case of a cyclic process.



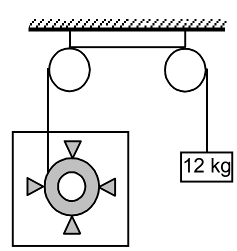
3. (d) ΔU = Heat supplied, ΔW = Work done.
   $(\Delta Q - \Delta W) = du$, du is same for both the methods since it is a state function.
4. (a) (c) Since it is a cyclic process.
   So, $\Delta U_1 = -\Delta U_2$, hence $\Delta U_1 + \Delta U_2 = 0$
   $\Delta Q - \Delta W = 0$



5. (a) (d) Internal energy decreases by the same amount as work done.
   du = dw, ∴ dQ = 0. Thus the process is adiabatic. In adiabatic process, dU = – dw. Since 'U' decreases $U_2 - U_2$ is –ve. ∴dw should be +ve ⇒ $\frac{nR}{\upsilon - 1}(T_1 - T_2)$ is +ve. $T_1 > T_2$ ∴ Temperature decreases.

EXERCISES

1. $t_1 = 15°c$ $t_2 = 17°c$
   $\Delta t = t_2 - t_1 = 17 - 15 = 2°C = 2 + 273 = 275\ K$
   $m_v = 100\ g = 0.1\ kg$ $m_w = 200\ g = 0.2\ kg$
   $cu_g = 420\ J/kg–k$ $W_g = 4200\ J/kg–k$
   (a) The heat transferred to the liquid vessel system is 0. The internal heat is shared in between the vessel and water.
   (b) Work done on the system = Heat produced unit
   $\Rightarrow dw = 100 \times 10^{-3} \times 420 \times 2 + 200 \times 10^{-3} \times 4200 \times 2 = 84 + 84 \times 20 = 84 \times 21 = 1764\ J.$
   (c)dQ = 0, dU = – dw = 1764. [since dw = –ve work done on the system]
2. (a) Heat is not given to the liquid. Instead the mechanical work done is converted to heat. So, heat given to liquid is z.
   (b) Work done on the liquid is the PE lost by the 12 kg mass = mgh = 12 × 10× 0.70 = 84 J
   (c) Rise in temp at Δt We know, $84 = ms\Delta t$
   $\Rightarrow 84 = 1 \times 4200 \times \Delta t$ (for 'm' = 1kg) $\Rightarrow \Delta t = \frac{84}{4200} = 0.02\ k$



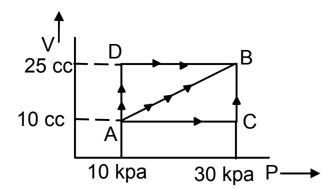
3. mass of block = 100 kg
   $u = 2$ m/s, $m = 0.2$ $v = 0$
   $dQ = du + dw$
   In this case $dQ = 0$
   $\Rightarrow -du = dw \Rightarrow du = -\left(\frac{1}{2}mv^2 - \frac{1}{2}mu^2\right) = \frac{1}{2} \times 100 \times 2 \times 2 = 200$ J

4. Q = 100 J
   We know, $\Delta U = \Delta Q - \Delta W$
   Here since the container is rigid, $\Delta V = 0$,
   Hence the $\Delta W = P\Delta V = 0$,
   So, $\Delta U = \Delta Q = 100$ J.

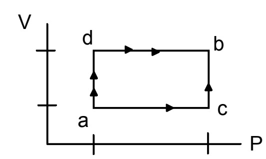
5. $P_1 = 10$ kpa $= 10 \times 10^3$ pa. $P_2 = 50 \times 10^3$ pa. $v_1 = 200$ cc. $v_2 = 50$ cc
   (i) Work done on the gas $= \frac{1}{2}(10+50) \times 10^3 \times (50-200) \times 10^{-6} = -4.5$ J
   (ii) $dQ = 0 \Rightarrow 0 = du + dw \Rightarrow du = -dw = 4.5$ J

6. initial State 'I' Final State 'f'
   Given $\frac{P_1}{T_1} = \frac{P_2}{T_2}$
   where $P_1 \rightarrow$ Initial Pressure ; $P_2 \rightarrow$ Final Pressure.
   $T_2, T_1 \rightarrow$ Absolute temp. So, $\Delta V = 0$
   Work done by gas $= P\Delta V = 0$

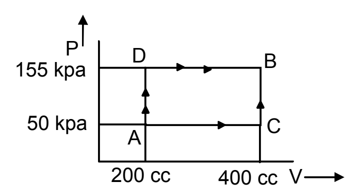
7. In path ACB,
   $W_{AC} + W_{BC} = 0 + pdv = 30 \times 10^3 (25 - 10) \times 10^{-6} = 0.45$ J
   In path AB, $W_{AB} = \frac{1}{2} \times (10 + 30) \times 10^3\ 15 \times 10^{-6} = 0.30$ J
   In path ADB, $W = W_{AD} + W_{DB} = 10 \times 10^3 (25 - 10) \times 10^{-6} + 0 = 0.15$ J



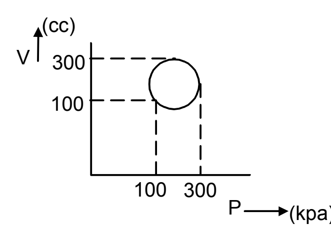
8. $\Delta Q = \Delta U + \Delta W$
   In abc, $\Delta Q = 80$ J $\Delta W = 30$ J
   So, $\Delta U = (80 - 30)$ J = 50 J
   Now in adc, $\Delta W = 10$ J
   So, $\Delta Q = 10 + 50 = 60$ J [$\therefore \Delta U = 50$ J]



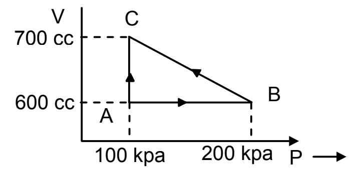
9. In path ACB,
   dQ = 50 0 50 × 4.2 = 210 J
   $dW = W_{AC} + W_{CB} = 50 \times 10^3 \times 200 \times 10^{-6} = 10$ J
   $dQ = dU + dW$
   $\Rightarrow dU = dQ - dW = 210 - 10 = 200$ J
   In path ADB, dQ = ?
   dU = 200 J (Internal energy change between 2 points is always same)
   $dW = W_{AD} + W_{DB} = 0 + 155 \times 10^3 \times 200 \times 10^{-6} = 31$ J
   $dQ = dU + dW = 200 + 31 = 231$ J = 55 cal



10. Heat absorbed = work done = Area under the graph
    In the given case heat absorbed = area of the circle
    $= \pi \times 10^4 \times 10^{-6} \times 10^3 = 3.14 \times 10 = 31.4$ J



11. $dQ = 2.4 \text{ cal} = 2.4 \text{ J Joules}$



$dw = W_{AB} + W_{BC} + W_{AC}$

$= 0 + (1/2) \times (100 + 200) \times 10^3\ 200 \times 10^{-6} - 100 \times 10^3 \times 200 \times 10^{-6}$

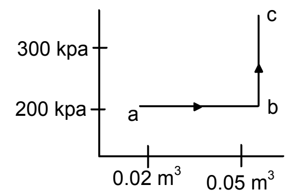
$= (1/2) \times 300 \times 10^3\ 200 \times 10^{-6} - 20 = 30 - 20 = 10$ joules.

du = 0 (in a cyclic process)

$dQ = dU + dW \Rightarrow 2.4 \text{ J} = 10$

$\Rightarrow J = \frac{10}{2.4} \approx 4.17$ J/Cal.

12. Now, $\Delta Q = (2625 \times J)$ J



$\Delta U = 5000$ J

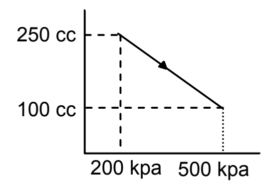
From Graph $\Delta W = 200 \times 10^3 \times 0.03 = 6000$ J.

Now, $\Delta Q = \Delta W + \Delta U$

$\Rightarrow 2625 \text{ J} = 6000 + 5000 \text{ J}$

$J = \frac{11000}{2625} = 4.19$ J/Cal

13. $dQ = 70 \text{ cal} = (70 \times 4.2)$ J



$dW = (1/2) \times (200 + 500) \times 10^3 \times 150 \times 10^{-6}$

$= (1/2) \times 500 \times 150 \times 10^{-3}$

$= 525 \times 10^{-1} = 52.5$ J

dU = ? $dQ = du + dw$

$\Rightarrow -294 = du + 52.5$

$\Rightarrow du = -294 - 52.5 = -346.5$ J

14. $U = 1.5\ pV$ $P = 1 \times 10^5$ Pa

$dV = (200 - 100) \text{ cm}^3 = 100 \text{ cm}^3 = 10^{-4} \text{ m}^3$

$dU = 1.5 \times 10^5 \times 10^{-4} = 15$

$dW = 10^5 \times 10^{-4} = 10$

$dQ = dU + dW = 10 + 15 = 25$ J

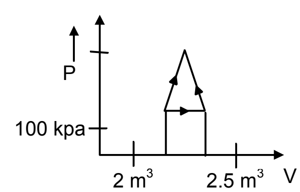
15. $dQ = 10$ J

$dV = A \times 10 \text{ cm}^3 = 4 \times 10 \text{ cm}^3 = 40 \times 10^{-6} \text{ cm}^3$

$dw = Pdv = 100 \times 10^3 \times 40 \times 10^{-6} = 4 \text{ cm}^3$

du = ? $10 = du + dw \Rightarrow 10 = du + 4 \Rightarrow du = 6$ J.

16. (a) $P_1 = 100$ KPa



$V_1 = 2 \text{ m}^3$

$\Delta V_1 = 0.5 \text{ m}^3$

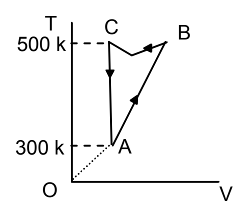
$\Delta P_1 = 100$ KPa

From the graph, We find that area under AC is greater than area under than AB. So, we see that heat is extracted from the system.

(b) Amount of heat = Area under ABC.

$= \frac{1}{2} \times \frac{5}{10} \times 10^5 = 25000$ J

17. n = 2 mole



$dQ = -1200$ J

dU = 0 (During cyclic Process)

$dQ = dU + dwc$

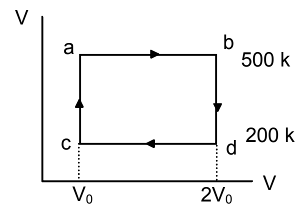
$\Rightarrow -1200 = W_{AB} + W_{BC} + W_{CA}$

$\Rightarrow -1200 = nR\Delta T + W_{BC} + 0$

$\Rightarrow -1200 = 2 \times 8.3 \times 200 + W_{BC}$

$\Rightarrow W_{BC} = -400 \times 8.3 - 1200 = -4520$ J.

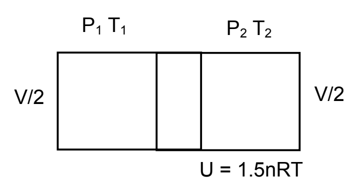
18. Given n = 2 moles
    dV = 0
    in ad and bc.
    Hence dW = dQ   dW = $dW_{ab} + dW_{cd}$
    $= nRT_1 Ln\frac{2V_0}{V_0} + nRT_2 Ln\frac{V_0}{2V_0}$
    = nR × 2.303 × log 2(500 – 300)
    = 2 × 8.314 × 2.303 × 0.301 × 200 = 2305.31 J



19. Given M = 2 kg   2t = 4°c   Sw = 4200 J/Kg–k
    $f_0 = 999.9\ kg/m^3$ $f_4 = 1000\ kg/m^3$ P = 10^5 Pa.
    Net internal energy = dv
    dQ = DU + dw ⇒ $ms\Delta Q\phi = dU + P(v_0 - v_4)$
    ⇒ 2 × 4200 × 4 = dU + 10^5(m – m)
    $\Rightarrow 33600 = dU + 10^5\left(\frac{m}{V_0} - \frac{m}{v_4}\right) = dU + 10^5(0.0020002 - 0.002) = dU + 10^5\ 0.0000002$
    ⇒ 33600 = du + 0.02 ⇒ du = (33600 – 0.02) J

20. Mass = 10g = 0.01kg.
    P = 10^5Pa
    dQ = Q_{H_2o} 0° – 100° + Q_{H_2o} – steam
    = 0.01 × 4200 × 100 + 0.01 × 2.5 × 10^6 = 4200 + 25000 = 29200
    dW = P × ΔV
    $\Delta = \frac{0.01}{0.6} - \frac{0.01}{1000} = 0.01699$
    dW = PΔV = 0.01699 × 10^5 1699J
    dQ = dW + dU or dU = dQ – dW = 29200 – 1699 = 27501 = 2.75 × 10^4 J

21. (a) Since the wall can not be moved thus dU = 0 and dQ = 0.
    Hence dW = 0.
    (b) Let final pressure in LHS = P_1
    In RHS = P_2
    (∴ no. of mole remains constant)
    $\frac{P_1 V}{2RT_1} = \frac{P_1 V}{2RT}$
    $\Rightarrow P_1 = \frac{P_1 T}{T_1} = \frac{P_1(P_1 + P_2)T_1 T_2}{\lambda}$
    As, $T = \frac{(P_1 + P_2)T_1 T_2}{\lambda}$
    Simillarly $P_2 = \frac{P_2 T_1 (P_1 + P_2)}{\lambda}$



    (c) Let $T_2 > T_1$ and 'T' be the common temp.
    Initially $\frac{P_1 V}{2} = n_1 rt_1 \Rightarrow n_1 = \frac{P_1 V}{2RT_1}$
    $n_2 = \frac{P_2 V}{2RT_2}$ Hence dQ = 0, dW = 0, Hence dU = 0.
    In case (LHS)   RHS
    $\Delta u_1 = 1.5 n_1 R(T - T_1)$ But $\Delta u_1 - \Delta u_2 = 0$ $\Delta u_2 = 1.5 n_2 R(T_2 - T)$
    $\Rightarrow 1.5\ n_1 R(T - T_1) = 1.5\ n_2 R(T_2 - T)$
    $\Rightarrow n_2 T - n_1 T_1 = n_2 T_2 - n_2 T \Rightarrow T(n_1 + n_2) = n_1 T_1 + n_2 T_2$

$\Rightarrow T = \dfrac{n_1T_1 + n_2T_2}{n_1 + n_2}$

$= \dfrac{\dfrac{P_1V}{2RT_1} \times T_1 + \dfrac{P_2V}{2RT_2} \times T_2}{\dfrac{P_1V}{2RT_1} + \dfrac{P_2V}{2RT_2}} = \dfrac{P_1 + P_2}{\dfrac{P_1T_2 + P_2T_1}{T_1T_2}}$

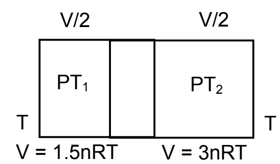
$= \dfrac{(P_1 + P_2)T_1T_2}{P_1T_2 + P_2T_1} = \dfrac{(P_1 + P_2)T_1T_2}{\lambda}$ as $P_1T_2 + P_2T_1 = \lambda$

(d) For RHS dQ = dU (As dW = 0) $= 1.5\, n_2 R(T_2 - t)$

$= \dfrac{1.5P_2V}{2RT_2} R\left[\dfrac{T_2 - (P_1 - P_2)T_1T_2}{P_1T_2 - P_2T_1}\right] = \dfrac{1.5P_2V}{2T_2}\left(\dfrac{P_1t_2^{\,2} - P_1T_1T_2}{\lambda}\right)$

$= \dfrac{1.5P_2V}{2T_2} \times \dfrac{T_2P_1(T_2 - T_1)}{\lambda} = \dfrac{3P_1P_2(T_2 - T_1)V}{4\lambda}$

22. (a) As the conducting wall is fixed the work done by the gas on the left part during the process is Zero.



(b) For left side

Pressure = P

Volume = V

No. of moles = n(1mole)

Let initial Temperature = T_1

$\dfrac{PV}{2} = nRT_1$

$\Rightarrow \dfrac{PV}{2} = (1)RT_1$

$\Rightarrow T_1 = \dfrac{PV}{2(\text{moles})R}$

For right side

Let initial Temperature = T_2

$\dfrac{PV}{2} = n_2 RT_2$

$\Rightarrow T_2 = \dfrac{PV}{2n_2R} \times 1$

$\Rightarrow T_2 = \dfrac{PV}{4(\text{moles})R}$

(c) Let the final Temperature = T

Final Pressure = R

No. of mole = 1 mole + 2 moles = 3 moles

$\therefore PV = nRT \Rightarrow T = \dfrac{PV}{nR} = \dfrac{PV}{3(\text{mole})R}$

(d) For RHS dQ = dU [as, dW = 0]

$= 1.5\, n_2 R(T - T_2) = 1.5 \times 2 \times R \times \left[\dfrac{PV}{3(\text{mole})R} - \dfrac{PV}{4(\text{mole})R}\right]$

$= 1.5 \times 2 \times R \times \dfrac{4PV - 3PV}{4 \times 3(\text{mole}} = \dfrac{3 \times R \times PV}{3 \times 4 \times R} = \dfrac{PV}{4}$

(e) As, dQ = –dU

$\Rightarrow dU = -\,dQ = \dfrac{-PV}{4}$

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CHAPTER – 27
SPECIFIC HEAT CAPACITIES OF GASES

1. N = 1 mole, W = 20 g/mol, V = 50 m/s

K.E. of the vessel = Internal energy of the gas

= (1/2) mv^2 = (1/2) × 20 × 10^{-3} × 50 × 50 = 25 J

$25 = n\frac{3}{2}r(\Delta T) \Rightarrow 25 = 1 \times \frac{3}{2} \times 8.31 \times \Delta T \Rightarrow \Delta T = \frac{50}{3 \times 8.3} \approx 2$ k.

2. m = 5 g, Δt = 25 – 15 = 10°C

C_V = 0.172 cal/g-°CJ = 4.2 J/Cal.

dQ = du + dw

Now, V = 0 (for a rigid body)

So, dw = 0.

So dQ = du.

Q = msdt = 5 × 0.172 × 10 = 8.6 cal = 8.6 × 4.2 = 36.12 Joule.

3. γ = 1.4, w or piston = 50 kg., A of piston = 100 cm^2

Po = 100 kpa, g = 10 m/s^2, x = 20 cm.

$dw = pdv = \left(\frac{mg}{A} + Po\right)Adx = \left(\frac{50 \times 10}{100 \times 10^{-4}} + 10^5\right)100 \times 10^{-4} \times 20 \times 10^{-2} = 1.5 \times 10^5 \times 20 \times 10^{-4} = 300$ J.

$nRdt = 300 \Rightarrow dT = \frac{300}{nR}$

$dQ = nCpdT = nCp \times \frac{300}{nR} = \frac{n\gamma R300}{(\gamma - 1)nR} = \frac{300 \times 1.4}{0.4} = 1050$ J.

4. C_VH_2 = 2.4 Cal/g°C, C_PH^2 = 3.4 Cal/g°C

M = 2 g/ Mol, R = 8.3 × 10^7 erg/mol-°C

We know, $C_P - C_V$ = 1 Cal/g°C

So, difference of molar specific heats

= C_P × M – C_V × M = 1 × 2 = 2 Cal/g°C

Now, 2 × J = R $\Rightarrow$ 2 × J = 8.3 × 10^7 erg/mol-°C $\Rightarrow$ J = 4.15 × 10^7 erg/cal.

5. $\frac{C_P}{C_V}$ = 7.6, n = 1 mole, ΔT = 50K

(a) Keeping the pressure constant, dQ = du + dw,

ΔT = 50 K, γ = 7/6, m = 1 mole,

dQ = du + dw $\Rightarrow$ nC_VdT = du + RdT $\Rightarrow$ du = nCpdT – RdT

$= 1 \times \frac{R\gamma}{\gamma - 1} \times dT - RdT = \frac{R \times \frac{7}{6}}{\frac{7}{6} - 1}dT - RdT$

= DT – RdT = 7RdT – RdT = 6 RdT = 6 × 8.3 × 50 = 2490 J.

(b) Kipping Volume constant, dv = nC_VdT

$= 1 \times \frac{R}{\gamma - 1} \times dt = \frac{1 \times 8.3}{\frac{7}{6} - 1} \times 50$

= 8.3 × 50 × 6 = 2490 J

(c) Adiabetically dQ = 0, du = – dw

$= \left[\frac{n \times R}{\gamma - 1}(T_1 - T_2)\right] = \frac{1 \times 8.3}{\frac{7}{6} - 1}(T_2 - T_1)$ = 8.3 × 50 × 6 = 2490 J

6. m = 1.18 g, $V = 1 \times 10^3\ cm^3 = 1\ L$ T = 300 k, $P = 10^5$ Pa

$PV = nRT$ or $n = \frac{PV}{RT} = 10^5 = atm.$

$N = \frac{PV}{RT} = \frac{1}{8.2 \times 10^{-2} \times 3 \times 10^2} = \frac{1}{8.2 \times 3} = \frac{1}{24.6}$

Now, $C_v = \frac{1}{n} \times \frac{Q}{dt} = 24.6 \times 2 = 49.2$

$C_p = R + C_v = 1.987 + 49.2 = 51.187$

$Q = nC_p dT = \frac{1}{24.6} \times 51.187 \times 1 = 2.08$ Cal.

7. $V_1 = 100\ cm^3$, $V_2 = 200\ cm^3$ $P = 2 \times 10^5$ Pa, $\Delta Q = 50J$

(a) $\Delta Q = du + dw \Rightarrow 50 = du + 20 \times 10^5 (200 - 100 \times 10^{-6}) \Rightarrow 50 = du + 20 \Rightarrow du = 30$ J

(b) $30 = n \times \frac{3}{2} \times 8.3 \times 300$ [$U = \frac{3}{2} nRT$ for monoatomic]

$\Rightarrow n = \frac{2}{3 \times 83} = \frac{2}{249} = 0.008$

(c) $du = nC_v dT \Rightarrow C_v = \frac{dndTu}{} = \frac{30}{0.008 \times 300} = 12.5$

$C_p = C_v + R = 12.5 + 8.3 = 20.3$

(d) $C_v = 12.5$ (Proved above)

8. Q = Amt of heat given

Work done = $\frac{Q}{2}$, $\Delta Q = W + \Delta U$

for monoatomic gas $\Rightarrow \Delta U = Q - \frac{Q}{2} = \frac{Q}{2}$

$V = n\frac{3}{2}RT = \frac{Q}{2} = nT \times \frac{3}{2}R = 3R \times nT$

Again Q = n CpdT Where C_P > Molar heat capacity at const. pressure.

$3RnT = ndTC_P \Rightarrow C_P = 3R$

9. $P = KV \Rightarrow \frac{nRT}{V} = KV \Rightarrow RT = KV^2 \Rightarrow R\,\Delta T = 2KV\,\Delta U \Rightarrow \frac{R\Delta T}{2KV} = dv$

$dQ = du + dw \Rightarrow mcdT = C_V dT + pdv \Rightarrow msdT = C_V\,dT + \frac{PRdF}{2KV}$

$\Rightarrow ms = C_V + \frac{RKV}{2KV} \Rightarrow C_P + \frac{R}{2}$

10. $\frac{C_P}{C_V} = \gamma$, $C_P - C_V = R$, $C_V = \frac{r}{\gamma - 1}$, $C_P = \frac{\gamma R}{\gamma - 1}$

$Pdv = \frac{1}{b+1}(Rdt)$

$\Rightarrow 0 = C_V dT + \frac{1}{b+1}(Rdt) \Rightarrow \frac{1}{b+1} = \frac{-C_V}{R}$

$\Rightarrow b + 1 = \frac{-R}{C_V} = \frac{-(C_P - C_V)}{C_V} = -\gamma + 1 \Rightarrow b = -\gamma$

11. Considering two gases, in Gas(1) we have,

γ, Cp_1 (Sp. Heat at const. 'P'), Cv_1 (Sp. Heat at const. 'V'), n_1 (No. of moles)

$\frac{Cp_1}{Cv_1} = \gamma$ & $Cp_1 - Cv_1 = R$

$\Rightarrow \gamma Cv_1 - Cv_1 = R \Rightarrow Cv_1 (\gamma - 1) = R$

$\Rightarrow Cv_1 = \frac{R}{\gamma - 1}$ & $Cp_1 = \frac{\gamma R}{\gamma - 1}$

In Gas(2) we have, γ, Cp_2 (Sp. Heat at const. 'P'), Cv_2 (Sp. Heat at const. 'V'), n_2 (No. of moles)

$\frac{Cp_2}{Cv_2} = \gamma$ & $Cp_2 - Cv_2 = R \Rightarrow \gamma Cv_2 - Cv_2 = R \Rightarrow Cv_2 (\gamma - 1) = R \Rightarrow Cv_2 = \frac{R}{\gamma - 1}$ & $Cp_2 = \frac{\gamma R}{\gamma - 1}$

Given $n_1 : n_2 = 1 : 2$

$dU_1 = nCv_1\, dT$ & $dU_2 = 2nCv_2\, dT = 3nCvdT$

$$\Rightarrow C_V = \frac{Cv_1 + 2Cv_2}{3} = \frac{\frac{R}{\gamma - 1} + \frac{2R}{\gamma - 1}}{3} = \frac{3R}{3(\gamma - 1)} = \frac{R}{\gamma - 1} \quad \ldots(1)$$

$$\&Cp = \gamma Cv = \frac{\gamma r}{\gamma - 1} \quad \ldots(2)$$

So, $\frac{Cp}{Cv} = \gamma$ [from (1) & (2)]

12. $Cp' = 2.5\ RCp'' = 3.5\ R$

$Cv' = 1.5\ R \qquad Cv'' = 2.5\ R$

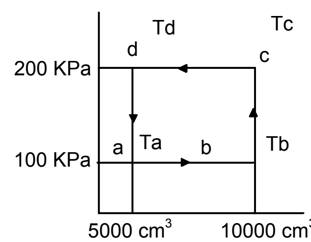
$n_1 = n_2 = 1$ mol $\qquad (n_1 + n_2)C_VdT = n_1\ C_vdT + n_2\ C_vdT$

$$\Rightarrow C_V = \frac{n_1Cv' + n_2Cv''}{n_1 + n_2} = \frac{1.5R + 2.5R}{2}\ 2R$$

$C_P = C_V + R = 2R + R = 3R$

$$\gamma = \frac{C_p}{C_V} = \frac{3R}{2R} = 1.5$$

13. $n = \frac{1}{2}$ mole, $\qquad R = \frac{25}{3}$ J/mol-k, $\qquad \gamma = \frac{5}{3}$



(a) Temp at A = T_a, $P_aV_a = nRT_a$

$$\Rightarrow T_a = \frac{P_aV_a}{nR} = \frac{5000 \times 10^{-6} \times 100 \times 10^3}{\frac{1}{2} \times \frac{25}{3}} = 120\text{ k.}$$

Similarly temperatures at point b = 240 k at C it is 480 k and at D it is 240 k.

(b) For ab process,

dQ = nCpdT $\qquad$ [since ab is isobaric]

$$= \frac{1}{2} \times \frac{R\gamma}{\gamma - 1}(T_b - T_a) = \frac{1}{2} \times \frac{\frac{35}{3} \times \frac{5}{3}}{\frac{5}{3} - 1} \times (240 - 120) = \frac{1}{2} \times \frac{125}{9} \times \frac{3}{2} \times 120 = 1250\text{ J}$$

For bc, $\qquad$ dQ = du + dw $\quad$ [dq = 0, Isochorie process]

$$\Rightarrow dQ = du = nC_vdT = \frac{nR}{\gamma - 1}(T_c - T_a) = \frac{1}{2} \times \frac{\frac{25}{3}}{\left(\frac{5}{3} - 1\right)}(240) = \frac{1}{2} \times \frac{25}{3} \times \frac{3}{2} \times 240 = 1500\text{ J}$$

(c) Heat liberated in cd = $-\,nC_pdT$

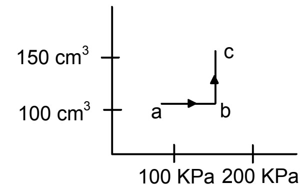
$$= \frac{-1}{2} \times \frac{nR}{\gamma - 1}(T_d - T_c) = \frac{-1}{2} \times \frac{125}{3} \times \frac{3}{2} \times 240 = 2500\text{ J}$$

Heat liberated in da = $-\,nC_vdT$

$$= \frac{-1}{2} \times \frac{R}{\gamma - 1}(T_a - T_d) = \frac{-1}{2} \times \frac{25}{2} \times (120 - 240) = 750\text{ J}$$

14. (a) For a, b 'V' is constant

So, $\frac{P_1}{T_1} = \frac{P_2}{T_2} \Rightarrow \frac{100}{300} = \frac{200}{T_2} \Rightarrow T_2 = \frac{200 \times 300}{100}$ = 600 k



For b,c 'P' is constant

So, $\frac{V_1}{T_1} = \frac{V_2}{T_2} \Rightarrow \frac{100}{600} = \frac{150}{T_2} \Rightarrow T_2 = \frac{600 \times 150}{100}$ = 900 k

(b) Work done = Area enclosed under the graph 50 cc × 200 kpa = $50 \times 10^{-6} \times 200 \times 10^3$ J = 10 J

(c) 'Q' Supplied = $nC_v dT$

Now, n = $\frac{PV}{RT}$ considering at pt. 'b'

$C_v = \frac{R}{\gamma - 1} dT$ = 300 a, b.

$Q_{bc} = \frac{PV}{RT} \times \frac{R}{\gamma - 1} dT = \frac{200 \times 10^3 \times 100 \times 10^{-6}}{600 \times 0.67} \times 300 = 14.925$ $(\therefore \gamma = 1.67)$

Q supplied to be $nC_p dT$ $[\therefore C_p = \frac{\gamma R}{\gamma - 1}]$

$= \frac{PV}{RT} \times \frac{\gamma R}{\gamma - 1} dT = \frac{200 \times 10^3 \times 150 \times 10^{-6}}{8.3 \times 900} \times \frac{1.67 \times 8.3}{0.67} \times 300 = 24.925$

(d) Q = ΔU + w

Now, ΔU = Q – w = Heat supplied – Work done = (24.925 + 14.925) – 1 = 29.850

15. In Joly's differential steam calorimeter

$C_v = \frac{m_2 L}{m_1(\theta_2 - \theta_1)}$

m_2 = Mass of steam condensed = 0.095 g, L = 540 Cal/g = 540 × 4.2 J/g

m_1 = Mass of gas present = 3 g, θ_1 = 20°C, θ_2 = 100°C

$\Rightarrow C_v = \frac{0.095 \times 540 \times 4.2}{3(100 - 20)}$ = 0.89 ≈ 0.9 J/g-K

16. γ = 1.5

Since it is an adiabatic process, So PV^γ = const.

(a) $P_1 V_1^\gamma = P_2 V_2^\gamma$ Given V_1 = 4 L, V_2 = 3 L, $\frac{P_2}{P_1}$ = ?

$\Rightarrow \frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^\gamma = \left(\frac{4}{3}\right)^{1.5}$ = 1.5396 ≈ 1.54

(b) $TV^{\gamma-1}$ = Const.

$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1} \Rightarrow \frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} = \left(\frac{4}{3}\right)^{0.5}$ = 1.154

17. $P_1 = 2.5 \times 10^5$ Pa, V_1 = 100 cc, T_1 = 300 k

(a) $P_1 V_1^\gamma = P_2 V_2^\gamma$

$\Rightarrow 2.5 \times 10^5 \times V^{1.5} = \left(\frac{V}{2}\right)^{1.5} \times P_2$

$\Rightarrow P_2 = 2^{1.5} \times 2.5 \times 10^5 = 7.07 \times 10^5 \approx 7.1 \times 10^5$

(b) $T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1} \Rightarrow 300 \times (100)^{1.5-1} = T_2 \times (50)^{1.5-1}$

$\Rightarrow T_2 = \frac{3000}{7.07}$ = 424.32 k ≈ 424 k

(c) Work done by the gas in the process

$W = \frac{mR}{\gamma - 1}\left[T_2 - T_1\right] = \frac{P_1V_1}{T(\gamma - 1)}\left[T_2 - T_1\right]$

$= \frac{2.5 \times 10^5 \times 100 \times 10^{-6}}{300(1{,}5 - 1)}[424 - 300] = \frac{2.5 \times 10}{300 \times 0.5} \times 124 = 20.72 \approx 21$ J

18. $\gamma = 1.4$, $T_1 = 20°C = 293$ k, $P_1 = 2$ atm, $p_2 = 1$ atm

We know for adiabatic process,

$P_1^{1-\gamma} \times T_1^{\gamma} = P_2^{1-\gamma} \times T_2^{\gamma}$ or $(2)^{1-1.4} \times (293)^{1.4} = (1)^{1-1.4} \times T_2^{1.4}$

$\Rightarrow (2)^{0.4} \times (293)^{1.4} = T_2^{1.4} \Rightarrow 2153.78 = T_2^{1.4} \Rightarrow T_2 = (2153.78)^{1/1.4} = 240.3$ K

19. $P_1 = 100$ KPa $= 10^5$ Pa, $V_1 = 400\ cm^3 = 400 \times 10^{-6}\ m^3$, $T_1 = 300$ k,

$\gamma = \frac{C_P}{C_V} = 1.5$

(a) Suddenly compressed to $V_2 = 100\ cm^3$

$P_1V_1^{\gamma} = P_2V_2^{\gamma} \Rightarrow 10^5\ (400)^{1.5} = P_2 \times (100)^{1.5}$

$\Rightarrow P_2 = 10^5 \times (4)^{1.5} = 800$ KPa

$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1} \Rightarrow 300 \times (400)^{1.5-1} = T_2 \times (100)^{1.5-1} \Rightarrow T_2 = \frac{300 \times 20}{10} = 600$ K

(b) Even if the container is slowly compressed the walls are adiabatic so heat transferred is 0.

Thus the values remain, $P_2 = 800$ KPa, $T_2 = 600$ K.

20. Given $\frac{C_P}{C_V} = \gamma$ P_0 (Initial Pressure), V_0 (Initial Volume)

(a) (i) Isothermal compression, $P_1V_1 = P_2V_2$ or, $P_0V_0 = \frac{P_2V_0}{2} \Rightarrow P_2 = 2P_0$

(ii) Adiabatic Compression $P_1V_1^{\gamma} = P_2V_2^{\gamma}$ or $2P_0\left(\frac{V_0}{2}\right)^{\gamma} = P1\left(\frac{V_0}{4}\right)^{\gamma}$

$\Rightarrow P' = \frac{V_0^{\ \gamma}}{2^{\gamma}} \times 2P_0 \times \frac{4^{\gamma}}{V_0^{\ \gamma}} = 2^{\gamma} \times 2\ P_0 \Rightarrow P_0 2^{\gamma+1}$

(b) (i) Adiabatic compression $P_1V_1^{\gamma} = P_2V_2^{\gamma}$ or $P_0V_0^{\gamma} = P'\left(\frac{V_0}{2}\right)^{\gamma} \Rightarrow P' = P_0 2^{\gamma}$

(ii) Isothermal compression $P_1V_1 = P_2V_2$ or $2^{\gamma}\ P_0 \times \frac{V_0}{2} = P_2 \times \frac{V_0}{4} \Rightarrow P_2 = P_0 2^{\gamma+1}$

21. Initial pressure = P_0

Initial Volume = V_0

$\gamma = \frac{C_P}{C_V}$

(a) Isothermally to pressure $\frac{P_0}{2}$

$P_0V_0 = \frac{P_0}{2}V_1 \Rightarrow V_1 = 2\ V_0$

Adiabetically to pressure = $\frac{P_0}{4}$

$\frac{P_0}{2}(V_1)^{\gamma} = \frac{P_0}{4}(V_2)^{\gamma} \Rightarrow \frac{P_0}{2}(2V_0)^{\gamma} = \frac{P_0}{4}(V_2)^{\gamma}$

$\Rightarrow 2^{\gamma+1}\ V_0^{\gamma} = V_2^{\gamma} \Rightarrow V_2 = 2^{(\gamma+1)/\gamma}\ V_0$

$\therefore$ Final Volume = $2^{(\gamma+1)/\gamma}\ V_0$

(b) Adiabetically to pressure $\frac{P_0}{2}$ to P_0

$P_0 \times (2^{\gamma+1} V_0^{\gamma}) = \frac{P_0}{2} \times (V')^{\gamma}$

Isothermal to pressure $\frac{P_0}{4}$

$\frac{P_0}{2} \times 2^{1/\gamma} V_0 = \frac{P_0}{4} \times V'' \Rightarrow V'' = 2^{(\gamma+1)/\gamma} V_0$

$\therefore$ Final Volume = $2^{(\gamma+1)/\gamma} V_0$

22. PV = nRT

Given P = 150 KPa = 150×10^3 Pa, V = 150 cm^3 = 150×10^{-6} m^3, T = 300 k

(a) $n = \frac{PV}{RT} = \frac{150 \times 10^3 \times 150 \times 10^{-6}}{8.3 \times 300} = 9.036 \times 10^{-3} = 0.009$ moles.

(b) $\frac{C_P}{C_V} = \gamma \Rightarrow \frac{\gamma R}{(\gamma - 1)C_V} = \gamma$ $\quad \left[\therefore C_P = \frac{\gamma R}{\gamma - 1}\right]$

$\Rightarrow C_V = \frac{R}{\gamma - 1} = \frac{8.3}{1.5 - 1} = \frac{8.3}{0.5} = 2R = 16.6$ J/mole

(c) Given P_1 = 150 KPa = 150×10^3 Pa, P_2 =?

V_1 = 150 cm^3 = 150×10^{-6} m^3, $\gamma = 1.5$

V_2 = 50 cm^3 = 50×10^{-6} m^3, T_1 = 300 k, T_2 = ?

Since the process is adiabatic Hence – $P_1V_1^{\gamma} = P_2V_2^{\gamma}$

$\Rightarrow 150 \times 10^3 (150 \times 10^{-6})^{\gamma} = P_2 \times (50 \times 10^{-6})^{\gamma}$

$\Rightarrow P_2 = 150 \times 10^3 \times \left(\frac{150 \times 10^{-6}}{50 \times 10^{-6}}\right)^{1.5} = 150000 \times 3^{1.5} = 779.422 \times 10^3 \text{ Pa} \approx 780$ KPa

(d) $\Delta Q = W + \Delta U$ or $W = -\Delta U$ $\quad [\therefore \Delta U = 0$, in adiabatic]

$= -nC_VdT = -0.009 \times 16.6 \times (520 - 300) = -0.009 \times 16.6 \times 220 = -32.8 \text{ J} \approx -33$ J

(e) $\Delta U = nC_VdT = 0.009 \times 16.6 \times 220 \approx 33$ J

23. $V_A = V_B = V_C$

For A, the process is isothermal

$P_AV_A = P_A'V_A' \Rightarrow P_A' = P_A \frac{V_A}{V_A'} = P_A \times \frac{1}{2}$

For B, the process is adiabatic,

$P_A(V_B)^{\gamma} = P_A'(V_B)^{\gamma} = P_B' = P_B\left(\frac{V_B}{V_B'}\right)^{\gamma} = P_B \times \left(\frac{1}{2}\right)^{1.5} = \frac{P_B}{2^{1.5}}$

For, C, the process is isobaric

$\frac{V_C}{T_C} = \frac{V_C'}{T_C'} \Rightarrow \frac{V_C}{T_C} = \frac{2V_C'}{T_C'} \Rightarrow T_C' = \frac{2}{T_C}$

Final pressures are equal.

$= \frac{p_A}{2} = \frac{P_B}{2^{1.5}} = P_C \Rightarrow P_A : P_B : P_C = 2 : 2^{1.5} : 1 = 2 : 2\sqrt{2} : 1$

24. P_1 = Initial Pressure $\quad V_1$ = Initial Volume $\quad P_2$ = Final Pressure $\quad V_2$ = Final Volume

Given, $V_2 = 2V_1$, Isothermal workdone = $nRT_1 \text{ Ln}\left(\frac{V_2}{V_1}\right)$

Adiabatic workdone = $\frac{P_1V_1 - P_2V_2}{\gamma - 1}$

Given that workdone in both cases is same.

Hence $nRT_1 \, Ln\left(\frac{V_2}{V_1}\right) = \frac{P_1V_1 - P_2V_2}{\gamma - 1} \Rightarrow (\gamma - 1) \ln \left(\frac{V_2}{V_1}\right) = \frac{P_1V_1 - P_2V_2}{nRT_1}$

$\Rightarrow (\gamma - 1) \ln \left(\frac{V_2}{V_1}\right) = \frac{nRT_1 - nRT_2}{nRT_1} \Rightarrow (\gamma - 1) \ln 2 = \frac{T_1 - T_1}{T_1}$ …(i) [$\therefore V_2 = 2V_1$]

We know $TV^{\gamma-1}$ = const. in adiabatic Process.

$T_1V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$, or $T_1 (V_2)^{\gamma-1} = T_2 \times (2)^{\gamma-1} \times (V_1)^{\gamma-1}$

Or, $T_1 = 2^{\gamma-1} \times T_2$ or $T_2 = T_1^{1-\gamma}$ …(ii)

From (i) & (ii)

$(\gamma - 1) \ln 2 = \frac{T_1 - T_1 \times 2^{1-\gamma}}{T_1} \Rightarrow (\gamma - 1) \ln 2 = 1 - 2^{1-\gamma}$

25. $\gamma = 1.5$, T = 300 k, V = 1Lv = $\frac{1}{2}$l

(a) The process is adiabatic as it is sudden,

$P_1 V_1^{\gamma} = P_2 V_2^{\gamma} \Rightarrow P_1 (V_0)^{\gamma} = P_2 \left(\frac{V_0}{2}\right)^{\gamma} \Rightarrow P_2 = P_1 \left(\frac{1}{1/2}\right)^{1.5} = P_1 (2)^{1.5} \Rightarrow \frac{P_2}{P_1} = 2^{1.5} = 2\sqrt{2}$

(b) P_1 = 100 KPa = 10^5 Pa $W = \frac{nR}{\gamma - 1}[T_1 - T_2]$

$T_1 V_1^{\gamma-1} = P_2 V_2^{\gamma-1} \Rightarrow 300 \times (1)^{1.5-1} = T_2 (0.5)^{1.5-1} \Rightarrow 300 \times 1 = T_2 \sqrt{0.5}$

$T_2 = 300 \times \sqrt{\frac{1}{0.5}} = 300\sqrt{2}$ K

$P_1 V_1 = nRT_1 \Rightarrow n = \frac{P_1V_1}{RT_1} = \frac{10^5 \times 10^{-3}}{R \times 300} = \frac{1}{3R}$ (V in m^3)

$w = \frac{nR}{\gamma - 1}[T_1 - T_2] = \frac{1R}{3R(1.5 - 1)}\left[300 - 300\sqrt{2}\right] = \frac{300}{3 \times 0.5}\left(1 - \sqrt{2}\right) = -82.8 \text{ J} \approx -82 \text{ J}.$

(c) Internal Energy,

dQ = 0, $\Rightarrow$ du = – dw = –(–82.8)J = 82.8 J ≈ 82 J.

(d) Final Temp = $300\sqrt{2}$ = 300 × 1.414 × 100 = 424.2 k ≈ 424 k.

(e) The pressure is kept constant. $\therefore$ The process is isobaric.

Work done = nRdT = $\frac{1}{3R} \times R \times (300 - 300\sqrt{2})$ Final Temp = 300 K

$= -\frac{1}{3} \times 300 (0.414) = -41.4$ J. Initial Temp = $300\sqrt{2}$

(f) Initial volume $\Rightarrow \frac{V_1}{T_1} = \frac{V_1'}{T_1'} = V_1' = \frac{V_1}{T_1} \times T_1' = \frac{1}{2 \times 300 \times \sqrt{2}} \times 300 = \frac{1}{2\sqrt{2}}$ L.

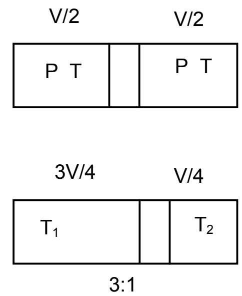
Final volume = 1L

Work done in isothermal = $nRT \ln \frac{V_2}{V_1}$

$= \frac{1}{3R} \times R \times 300 \ln\left(\frac{1}{1/2\sqrt{2}}\right) = 100 \times \ln\left(2\sqrt{2}\right) = 100 \times 1.039 \approx 103$

(g) Net work done = $W_A + W_B + W_C$ = – 82 – 41.4 + 103 = – 20.4 J.

26. Given $\gamma = 1.5$



We know fro adiabatic process $TV^{\gamma-1}$ = Const.

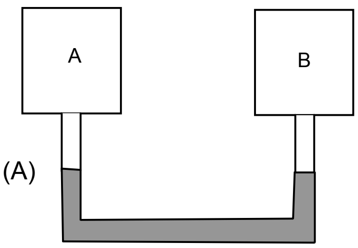
So, $T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$ …(eq)

As, it is an adiabatic process and all the other conditions are same. Hence the above equation can be applied.

So, $T_1 \times \left(\frac{3V}{4}\right)^{1.5-1} = T_2 \times \left(\frac{V}{4}\right)^{1.5-1} \Rightarrow T_1 \times \left(\frac{3V}{4}\right)^{0.5} = T_2 \times \left(\frac{V}{4}\right)^{0.5}$

$\Rightarrow \frac{T_1}{T_2} = \left(\frac{V}{4}\right)^{0.5} \times \left(\frac{4}{3V}\right)^{0.5} = \frac{1}{\sqrt{3}}$ So, $T_1 : T_2 = 1 : \sqrt{3}$

27. V = 200 cm^3, C = 12.5 J/mol-k, T = 300 k, P = 75 cm



(a) No. of moles of gas in each vessel,

$\frac{PV}{RT} = \frac{75 \times 13.6 \times 980 \times 200}{8.3 \times 10^7 \times 300} = 0.008$

(b) Heat is supplied to the gas but dv = 0

$dQ = du \Rightarrow 5 = nC_V dT \Rightarrow 5 = 0.008 \times 12.5 \times dT \Rightarrow dT = \frac{5}{0.008 \times 12.5}$ for (A)

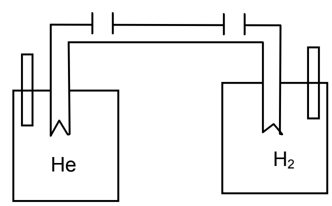
For (B) $dT = \frac{10}{0.008 \times 12.5}$ $\because \frac{P}{T} = \frac{P_A}{T_A}$ [For container A]

$\Rightarrow \frac{75}{300} = \frac{P_A \times 0.008 \times 12.5}{5} \Rightarrow P_A = \frac{75 \times 5}{300 \times 0.008 \times 12.5}$ = 12.5 cm of Hg.

$\because \frac{P}{T} = \frac{P_B}{T_B}$ [For Container B] $\Rightarrow \frac{75}{300} = \frac{P_B \times 0.008 \times 12.5}{10} \Rightarrow P_B = 2 P_A$ = 25 cm of Hg.

Mercury moves by a distance $P_B - P_A$ = 25 – 12.5 = 12.5 Cm.

28. mHe = 0.1 g, $\gamma = 1.67$, μ = 4 g/mol, mH_2 =?



μ = 28/mol $\gamma_2 = 1.4$

Since it is an adiabatic surrounding

He $dQ = nC_V dT = \frac{0.1}{4} \times \frac{R}{\gamma - 1} \times dT = \frac{0.1}{4} \times \frac{R}{(1.67 - 1)} \times dT$ …(i)

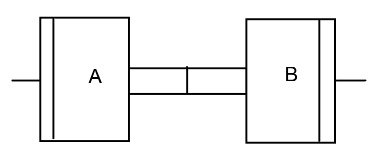
$H_2 = nC_V dT = \frac{m}{2} \times \frac{R}{\gamma - 1} \times dT = \frac{m}{2} \times \frac{R}{1.4 - 1} \times dT$ [Where m is the rqd. Mass of H_2]

Since equal amount of heat is given to both and ΔT is same in both.

Equating (i) & (ii) we get

$\frac{0.1}{4} \times \frac{R}{0.67} \times dT = \frac{m}{2} \times \frac{R}{0.4} \times dT \Rightarrow m = \frac{0.1}{2} \times \frac{0.4}{0.67} = 0.0298 \approx 0.03$ g

29. Initial pressure = P_0, Initial Temperature = T_0



Initial Volume = V_0

$\frac{C_P}{C_V} = \gamma$

(a) For the diathermic vessel the temperature inside remains constant

$P_1 V_1 - P_2 V_2 \Rightarrow P_0 V_0 = P_2 \times 2V_0 \Rightarrow P_2 = \frac{P_0}{2}$, Temperature = T_o

For adiabatic vessel the temperature does not remains constant. The process is adiabatic

$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1} \Rightarrow T_0 V_0^{\gamma-1} = T_2 \times (2V_0)^{\gamma-1} \Rightarrow T_2 = T_0 \left(\frac{V_0}{2V_0}\right)^{\gamma-1} = T_0 \times \left(\frac{1}{2}\right)^{\gamma-1} = \frac{T_0}{2^{\gamma-1}}$

$P_1 V_1^{\gamma} = P_2 V_2^{\gamma} \Rightarrow P_0 V_0^{\gamma} = p_1 (2V_0)^{\gamma} \Rightarrow P_1 = P_0\left(\frac{V_0}{2V_0}\right)^{\gamma} = \frac{P_0}{2^{\gamma}}$

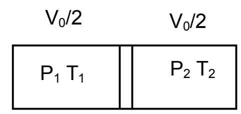
(b) When the values are opened, the temperature remains T_0 through out

$P_1 = \frac{n_1RT_0}{4V_0}$, $P_2 = \frac{n_2RT_0}{4V_0}$ [Total value after the expt = $2V_0 + 2V_0 = 4V_0$]

$P = P_1 + P_2 = \frac{(n_1+n_2)RT_0}{4V_0} = \frac{2nRT_0}{4V_0} = \frac{nRT_0}{2V} = \frac{P_0}{2}$

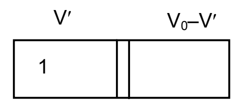
30. For an adiabatic process, Pv^{γ} = Const.

There will be a common pressure 'P' when the equilibrium is reached



Hence $P_1\left(\frac{V_0}{2}\right)^{\gamma} = P(V')^{\gamma}$

For left $P = P_1\left(\frac{V_0}{2}\right)^{\gamma} (V')^{\gamma}$ …(1)



For Right $P = P_2\left(\frac{V_0}{2}\right)^{\gamma} (V_0 - V')^{\gamma}$ …(2)

Equating 'P' for both left & right

$= \frac{P_1}{(V')^{\gamma}} = \frac{P_2}{(V_0 - V')^{\gamma}}$ or $\frac{V_0 - V'}{V'} = \left(\frac{P_2}{P_1}\right)^{1/\gamma}$

$\Rightarrow \frac{V_0}{V'} - 1 = \frac{P_2^{1/\gamma}}{P_1^{1/\gamma}} \Rightarrow \frac{V_0}{V'} = \frac{P_2^{1/\gamma} + P_1^{1/\gamma}}{P_1^{1/\gamma}} \Rightarrow V' = \frac{V_0P_1^{1/\gamma}}{P_1^{1/\gamma} + P_2^{1/\gamma}}$ For left …….(3)

Similarly $V_0 - V' = \frac{V_0P_2^{1/\gamma}}{P_1^{1/\gamma} + P_2^{1/\gamma}}$ For right ……(4)

(b) Since the whole process takes place in adiabatic surroundings. The separator is adiabatic. Hence heat given to the gas in the left part = Zero.

(c) From (1) Final pressure $P = \frac{P_1\left(\frac{V_0}{2}\right)^{y}}{(V')^{\gamma}}$

Again from (3) $V' = \frac{V_0P_1^{1/\gamma}}{P_1^{1/\gamma} + P_2^{1/\gamma}}$ or $P = \frac{P_1\frac{(V_0)^{\gamma}}{2^{\gamma}}}{\left(\frac{V_0P_1^{1/\gamma}}{P_1^{1/\gamma} + P_2^{1/\gamma}}\right)^{\gamma}} = \frac{P_1(V_0)^{\gamma}}{2^{\gamma}} \times \frac{\left(P_1^{1/\gamma} + P_2^{1/\gamma}\right)^{\gamma}}{(V_0)^{\gamma}P_1} = \left(\frac{P_1^{1/\gamma} + P_2^{1/\gamma}}{2}\right)^{\gamma}$

31. $A = 1\ cm^2 = 1 \times 10^{-4}\ m^2$, $M = 0.03\ g = 0.03 \times 10^{-3}$ kg,

P = 1 atm = 10^5 pascal, L= 40 cm = 0.4 m.

L_1 = 80 cm = 0.8 m, P = 0.355 atm

The process is adiabatic

$P(V)^{\gamma} = P(V')^{\gamma} = \Rightarrow 1 \times (AL)^{\gamma} = 0.355 \times (A2L)^{\gamma} \Rightarrow 1$ *1 = 0.355* $2^{\gamma} \Rightarrow \frac{1}{0.355} = 2^{\gamma}$

$= \gamma \log 2 = \log\left(\frac{1}{0.355}\right) = 1.4941$

$V = \sqrt{\frac{\gamma P}{f}} = \sqrt{\frac{1.4941 \times 10^5}{m/v}} = \sqrt{\frac{1.4941 \times 10^5}{\left(\frac{0.03 \times 10^{-3}}{10^{-4} \times 1 \times 0.4}\right)}} = \sqrt{\frac{1.441 \times 10^5 \times 4 \times 10^{-5}}{3 \times 10^{-5}}} = 446.33 \approx 447$ m/s

32. V = 1280 m/s, T = 0°C, $f oH_2$ = 0.089 kg/m^3, rR = 8.3 J/mol-k,
At STP, P = 10^5 Pa, We know

$V_{sound} = \sqrt{\frac{\gamma P}{f o}} \Rightarrow 1280 = \sqrt{\frac{\gamma \times 10^5}{0.089}} \Rightarrow (1280)^2 = \frac{\gamma \times 10^5}{0.089} \Rightarrow \gamma = \frac{0.089 \times (1280)^2}{10^5} \approx 1.458$

Again,

$C_V = \frac{R}{\gamma - 1} = \frac{8.3}{1.458 - 1}$ = 18.1 J/mol-k

Again, $\frac{C_P}{C_V} = \gamma$ or $C_P = \gamma C_V$ = 1.458 × 18.1 = 26.3 J/mol-k

33. μ = 4g = 4 × 10^{-3} kg, V = 22400 cm^3 = 22400 × 10^{-6} m^3
C_P = 5 cal/mol-ki = 5 × 4.2 J/mol-k = 21 J/mol-k

$C_P = \frac{\gamma R}{\gamma - 1} = \frac{\gamma \times 8.3}{\gamma - 1}$

$\Rightarrow 21(\gamma - 1) = \gamma\,(8.3) \Rightarrow 21\,\gamma - 21 = 8.3\,\gamma \Rightarrow \gamma = \frac{21}{12.7}$

Since the condition is STP, P = 1 atm = 10^5 pa

$V = \sqrt{\frac{\gamma f}{f}} = \sqrt{\frac{\frac{21}{12.7} \times 10^5}{\frac{4 \times 10^{-3}}{22400 \times 10^{-6}}}} = \sqrt{\frac{21 \times 10^5 \times 22400 \times 10^{-6}}{12.7 \times 4 \times 10^{-3}}}$ = 962.28 m/s

34. Given fo = 1.7 ×10^{-3} g/cm^3 = 1.7 kg/m^3, P = 1.5 × 10^5 Pa, R = 8.3 J/mol-k,
f = 3.0 KHz.

Node separation in a Kundt'' tube = $\frac{\lambda}{2}$ = 6 cm, $\Rightarrow \lambda$ = 12 cm = 12 × 10^{-3} m

So, V = $f\lambda$ = 3 × 10^3 × 12 × 10^{-2} = 360 m/s

We know, Speed of sound = $\sqrt{\frac{\gamma P}{f o}} \Rightarrow (360)^2 = \frac{\gamma \times 1.5 \times 10^5}{1.7} \Rightarrow \gamma = \frac{(360)^2 \times 1.7}{1.5 \times 10^5}$ = 1.4688

But $C_V = \frac{R}{\gamma - 1} = \frac{8.3}{1.488 - 1}$ = 17.72 J/mol-k

Again $\frac{C_P}{C_V} = \gamma$ So, $C_P = \gamma C_V$ = 17.72 × 1.468 = 26.01 ≈ 26 J/mol-k

35. f = 5 × 10^3 Hz, T = 300 Hz, $\frac{\lambda}{2}$ = 3.3 cm $\Rightarrow \lambda$ = 6.6 × 10^{-2} m

V = $f\lambda$ = 5 × 10^3 × 6.6 × 10^{-2} = (66 × 5) m/s

$V = \frac{\lambda P}{f}$ [Pv = nRT $\Rightarrow P = \frac{m}{mV}$ ×Rt $\Rightarrow$ PM = foRT $\Rightarrow \frac{P}{f o} = \frac{RT}{m}$]

$= \sqrt{\frac{\gamma RT}{m}} (66 \times 5) = \sqrt{\frac{\gamma \times 8.3 \times 300}{32 \times 10^{-3}}} \Rightarrow (66 \times 5)^2 = \frac{\gamma \times 8.3 \times 300}{32 \times 10^{-3}} \Rightarrow \gamma = \frac{(66 \times 5)^2 \times 32 \times 10^{-3}}{8.3 \times 300}$ = 1.3995

$C_v = \frac{R}{\gamma - 1} = \frac{8.3}{0.3995}$ = 20.7 J/mol-k,

$C_P = C_V + R$ = 20.77 + 8.3 = 29.07 J/mol-k.

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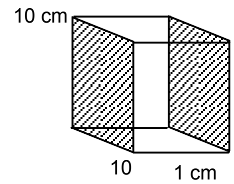
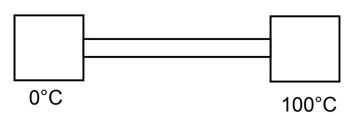
CHAPTER 28
HEAT TRANSFER



1. t_1 = 90°C, t_2 = 10°C
   l = 1 cm = 1 × 10^{-3} m
   A = 10 cm × 10 cm = 0.1 × 0.1 m^2 = 1 × 10^{-2} m^2
   K = 0.80 w/m-°C
   $\frac{Q}{t} = \frac{KA(\theta_1 - \theta_2)}{l} = \frac{8 \times 10^{-1} \times 1 \times 10^{-2} \times 80}{1 \times 10^{-2}}$ = 64 J/s = 64 × 60 3840 J.
2. t = 1 cm = 0.01 m, A = 0.8 m^2
   θ_1 = 300, θ_2 = 80
   K = 0.025,
   $\frac{Q}{t} = \frac{KA(\theta_1 - \theta_2)}{l} = \frac{0.025 \times 0.8 \times (30030)}{0.01}$ = 440 watt.
3. K = 0.04 J/m-5°C, A = 1.6 m^2
   t_1 = 97°F = 36.1°C t_2 = 47°F = 8.33°C
   l = 0.5 cm = 0.005 m
   $\frac{Q}{t} = \frac{KA(\theta_1 - \theta_2)}{l} = \frac{4 \times 10^{-2} \times 1.6 \times 27.78}{5 \times 10^{-3}}$ = 356 J/s
4. A = 25 cm^2 = 25 × 10^{-4} m^2
   l = 1 mm = 10^{-3} m
   K = 50 w/m-°C
   $\frac{Q}{t}$ = Rate of conversion of water into steam
   $= \frac{100 \times 10^{-3} \times 2.26 \times 10^{6}}{1\ \text{min}} = \frac{10^{-1} \times 2.26 \times 10^{6}}{60}$ = 0.376 × 10^4
   $\frac{Q}{t} = \frac{KA(\theta_1 - \theta_2)}{l} \Rightarrow 0.376 \times 10^4 = \frac{50 \times 25 \times 10^{-4} \times (\theta - 100)}{10^{-3}}$
   $\Rightarrow \theta = \frac{10^{-3} \times 0.376 \times 10^{4}}{50 \times 25 \times 10^{-4}} = \frac{10^{5} \times 0.376}{50 \times 25}$ = 30.1 ≈ 30
5. K = 46 w/m-s°C
   l = 1 m
   A = 0.04 cm^2 = 4 × 10^{-6} m^2
   $L_{\text{fussion ice}}$ = 3.36 × 10^5 j/Kg
   $\frac{Q}{t} = \frac{46 \times 4 \times 10^{-6} \times 100}{1}$ = 5.4 × 10^{-8} kg ≈ 5.4 × 10^{-5} g.



6. A = 2400 cm^2 = 2400 × 10^{-4} m^2
   ℓ = 2 mm = 2 × 10^{-3} m
   K = 0.06 w/m-°C
   θ_1 = 20°C
   θ_2 = 0°C
   $\frac{Q}{t} = \frac{KA(\theta_1 - \theta_2)}{\ell} = \frac{0.06 \times 2400 \times 10^{-4} \times 20}{2 \times 10^{-3}}$ = 24 × 6 × 10^{-1} × 10 = 24 × 6 = 144 J/sec
   Rate in which ice melts = $\frac{m}{t} = \frac{Q}{t \times L} = \frac{144}{3.4 \times 10^{5}}$ Kg/h = $\frac{144 \times 3600}{3.4 \times 10^{5}}$ Kg/s = 1.52 kg/s.
7. ℓ = 1 mm = 10^{-3} m m = 10 kg
   A = 200 cm^2 = 2 × 10^{-2} m^2
   L_{vap} = 2.27 × 10^6 J/kg
   K = 0.80 J/m-s-°C

$dQ = 2.27 \times 10^6 \times 10$,

$$\frac{dQ}{dt} = \frac{2.27 \times 10^7}{10^5} = 2.27 \times 10^2 \text{ J/s}$$

Again we know

$$\frac{dQ}{dt} = \frac{0.80 \times 2 \times 10^{-2} \times (42 - T)}{1 \times 10^{-3}}$$

So, $\frac{8 \times 2 \times 10^{-3}(42 - T)}{10^{-3}} = 2.27 \times 10^2$

$\Rightarrow 16 \times 42 - 16T = 227 \Rightarrow T = 27.8 \approx 28°C$

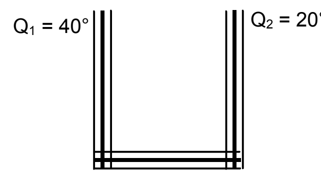
8. $K = 45$ w/m-°C

$\ell = 60 \text{ cm} = 60 \times 10^{-2}$ m

$A = 0.2 \text{ cm}^2 = 0.2 \times 10^{-4} \text{ m}^2$

Rate of heat flow,

$= \frac{KA(\theta_1 - \theta_2)}{\ell} = \frac{45 \times 0.2 \times 10^{-4} \times 20}{60 \times 10^{-2}} = 30 \times 10^{-3}$ 0.03 w



9. $A = 10 \text{ cm}^2$, $h = 10$ cm

$$\frac{\Delta Q}{\Delta t} = \frac{KA(\theta_1 - \theta_2)}{\ell} = \frac{200 \times 10^{-3} \times 30}{1 \times 10^{-3}} = 6000$$

Since heat goes out from both surfaces. Hence net heat coming out.

$= \frac{\Delta Q}{\Delta t} = 6000 \times 2 = 12000$, $\frac{\Delta Q}{\Delta t} = MS\frac{\Delta\theta}{\Delta t}$

$\Rightarrow 6000 \times 2 = 10^{-3} \times 10^{-1} \times 1000 \times 4200 \times \frac{\Delta\theta}{\Delta t}$

$\Rightarrow \frac{\Delta\theta}{\Delta t} = \frac{72000}{420} = 28.57$

So, in 1 Sec. 28.57°C is dropped

Hence for drop of 1°C $\frac{1}{28.57}$ sec. = 0.035 sec. is required

10. $\ell = 20 \text{ cm} = 20 \times 10^{-2}$ m

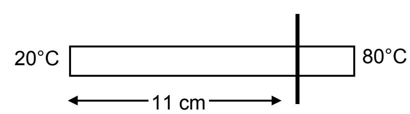
$A = 0.2 \text{ cm}^2 = 0.2 \times 10^{-4} \text{ m}^2$

$\theta_1 = 80°C$, $\theta_2 = 20°C$, $K = 385$

(a) $\frac{Q}{t} = \frac{KA(\theta_1 - \theta_2)}{\ell} = \frac{385 \times 0.2 \times 10^{-4}(80 - 20)}{20 \times 10^{-2}} = 385 \times 6 \times 10^{-4} \times 10 = 2310 \times 10^{-3} = 2.31$

(b) Let the temp of the 11 cm point be θ

$$\frac{\Delta\theta}{\Delta l} = \frac{Q}{tKA}$$



$$\Rightarrow \frac{\Delta\theta}{\Delta l} = \frac{2.31}{385 \times 0.2 \times 10^{-4}}$$

$$\Rightarrow \frac{\theta - 20}{11 \times 10^{-2}} = \frac{2.31}{385 \times 0.2 \times 10^{-4}}$$

$$\Rightarrow \theta - 20 = \frac{2.31 \times 10^4}{385 \times 0.2} \times 11 \times 10^{-2} = 33$$

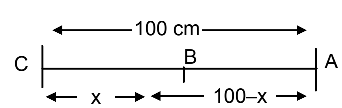
$\Rightarrow \theta = 33 + 20 = 53$

11. Let the point to be touched be 'B'

No heat will flow when, the temp at that point is also 25°C

i.e. $Q_{AB} = Q_{BC}$

So, $\frac{KA(100 - 25)}{100 - x} = \frac{KA(25 - 0)}{x}$

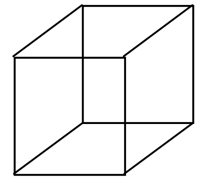


$\Rightarrow 75x = 2500 - 25x \Rightarrow 100x = 2500$ $\Rightarrow x = 25$ cm from the end with 0°C

12. $V = 216\ cm^3$

$a = 6$ cm, Surface area $= 6\ a^2 = 6 \times 36\ m^2$

$t = 0.1$ cm $\frac{Q}{t} = 100$ W,



$$\frac{Q}{t} = \frac{KA(\theta_1 - \theta_2)}{\ell}$$

$$\Rightarrow 100 = \frac{K \times 6 \times 36 \times 10^{-4} \times 5}{0.1 \times 10^{-2}}$$

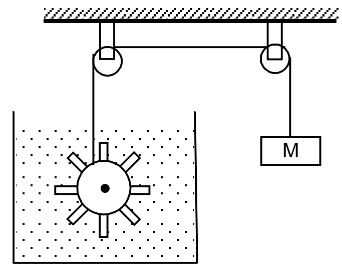
$$\Rightarrow K = \frac{100}{6 \times 36 \times 5 \times 10^{-1}} = 0.9259\ W/m°C \approx 0.92\ W/m°C$$

13. Given $\theta_1 = 1°C$, $\theta_2 = 0°C$

$K = 0.50$ w/m-°C, $d = 2\ mm = 2 \times 10^{-3}$ m

$A = 5 \times 10^{-2}\ m^2$, $v = 10$ cm/s $= 0.1$ m/s

Power = Force × Velocity = Mg × v



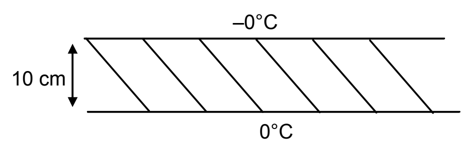
Again Power $= \frac{dQ}{dt} = \frac{KA(\theta_1 - \theta_2)}{d}$

So, $Mgv = \frac{KA(\theta_1 - \theta_2)}{d}$

$$\Rightarrow M = \frac{KA(\theta_1 - \theta_2)}{dvg} = \frac{5 \times 10^{-1} \times 5 \times^{-2} \times 1}{2 \times 10^{-3} \times 10^{-1} \times 10} = 12.5\ kg.$$

14. $K = 1.7$ W/m-°C $f_w = 1000\ Kg/m^3$

$L_{ice} = 3.36 \times 10^5$ J/kg $T = 10\ cm = 10 \times 10^{-2}$ m



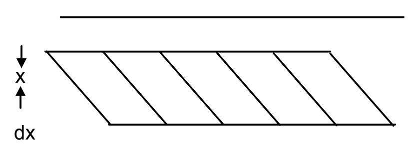
(a) $\frac{Q}{t} = \frac{KA(\theta_1 - \theta_2)}{\ell} \Rightarrow \frac{\ell}{t} = \frac{KA(\theta_1 - \theta_2)}{Q} = \frac{KA(\theta_1 - \theta_2)}{mL}$

$$= \frac{KA(\theta_1 - \theta_2)}{Atf_w L} = \frac{1.7 \times [0 - (-10)]}{10 \times 10^{-2} \times 1000 \times 3.36 \times 10^5}$$

$$= \frac{17}{3.36} \times 10^{-7} = 5.059 \times 10^{-7} \approx 5 \times 10^{-7}\ m/sec$$

(b) let us assume that x length of ice has become formed to form a small strip of ice of length dx, dt time is required.

$$\frac{dQ}{dt} = \frac{KA(\Delta\theta)}{x} \Rightarrow \frac{dmL}{dt} = \frac{KA(\Delta\theta)}{x} \Rightarrow \frac{Adx f\omega L}{dt} = \frac{KA(\Delta\theta)}{x}$$



$$\Rightarrow \frac{dx f\omega L}{dt} = \frac{K(\Delta\theta)}{x} \Rightarrow dt = \frac{xdx f\omega L}{K(\Delta\theta)}$$

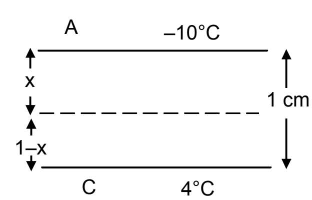
$$\Rightarrow \int_0^t dt = \frac{f\omega L}{K(\Delta\theta)} \int_0^t xdx \qquad \Rightarrow t = \frac{f\omega L}{K(\Delta\theta)} \left[\frac{x^2}{2}\right]_o^l = \frac{f\omega L}{K\Delta\theta} \frac{l^2}{2}$$

Putting values

$$\Rightarrow t = \frac{1000 \times 3.36 \times 10^5 \times (10 \times 10^{-2})^2}{1.7 \times 10 \times 2} = \frac{3.36}{2 \times 17} \times 10^6\ sec. = \frac{3.36 \times 10^6}{2 \times 17 \times 3600}\ hrs = 27.45\ hrs \approx 27.5\ hrs.$$

15. let 'B' be the maximum level upto which ice is formed. Hence the heat conducted at that point from both the levels is the same.

Let AB = x

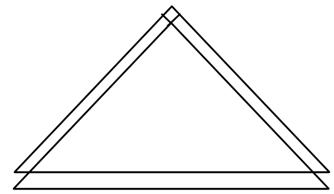


i.e. $\frac{Q}{t}ice = \frac{Q}{t}water \qquad \Rightarrow \frac{K_{ice} \times A \times 10}{x} = \frac{K_{water} \times A \times 4}{(1-x)}$

$$\Rightarrow \frac{1.7 \times 10}{x} = \frac{5 \times 10^{-1} \times 4}{1-x} \Rightarrow \frac{17}{x} = \frac{2}{1-x}$$

$$\Rightarrow 17 - 17x = 2x \Rightarrow 19x = 17 \Rightarrow x = \frac{17}{19} = 0.894 \approx 89\ cm$$

16. K_{AB} = 50 j/m-s-°c θ_A = 40°C
K_{BC} = 200 j/m-s-°c θ_B = 80°C
K_{AC} = 400 j/m-s-°c θ_C = 80°C
Length = 20 cm = 20 × 10^{-2} m
A = 1 cm^2 = 1 × 10^{-4} m^2

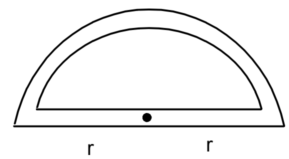


(a) $\frac{Q_{AB}}{t} = \frac{K_{AB} \times A(\theta_B - \theta_A)}{l} = \frac{50 \times 1 \times 10^{-4} \times 40}{20 \times 10^{-2}}$ = 1 W.

(b) $\frac{Q_{AC}}{t} = \frac{K_{AC} \times A(\theta_C - \theta_A)}{l} = \frac{400 \times 1 \times 10^{-4} \times 40}{20 \times 10^{-2}}$ = 800 × 10^{-2} = 8

(c) $\frac{Q_{BC}}{t} = \frac{K_{BC} \times A(\theta_B - \theta_C)}{l} = \frac{200 \times 1 \times 10^{-4} \times 0}{20 \times 10^{-2}}$ = 0

17. We know $Q = \frac{KA(\theta_1 - \theta_2)}{d}$



$Q_1 = \frac{KA(\theta_1 - \theta_2)}{d_1}$, $Q_2 = \frac{KA(\theta_1 - \theta_2)}{d_2}$

$\frac{Q_1}{Q_2} = \frac{\frac{KA(\theta_1 - \theta_1)}{\pi r}}{\frac{KA(\theta_1 - \theta_1)}{2r}} = \frac{2r}{\pi r} = \frac{2}{\pi}$ [$d_1 = \pi r$, d_2 = 2r]

18. The rate of heat flow per sec.
$= \frac{dQ_A}{dt} = KA\frac{d\theta}{dt}$
The rate of heat flow per sec.
$= \frac{dQ_B}{dt} = KA\frac{d\theta_B}{dt}$
This part of heat is absorbed by the red.
$\frac{Q}{t} = \frac{ms\Delta\theta}{dt}$ where $\frac{d\theta}{dt}$ = Rate of net temp. variation

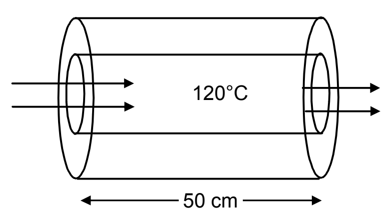
$\Rightarrow \frac{msd\theta}{dt} = KA\frac{d\theta_A}{dt} - KA\frac{d\theta_B}{dt}$ $\Rightarrow ms\frac{d\theta}{dt} = KA\left(\frac{d\theta_A}{dt} - \frac{d\theta_B}{dt}\right)$

$\Rightarrow 0.4 \times \frac{d\theta}{dt}$ = 200 × 1 × 10^{-4} (5 – 2.5) °C/cm

$\Rightarrow 0.4 \times \frac{d\theta}{dt}$ = 200 × 10^{-4} × 2.5

$\Rightarrow \frac{d\theta}{dt} = \frac{200 \times 2.5 \times 10^{-4}}{0.4 \times 10^{-2}}$ °C/m = 1250 × 10^{-2} = 12.5 °C/m

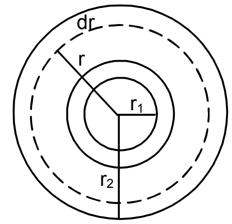
19. Given
K_{rubber} = 0.15 J/m-s-°C $T_2 - T_1$ = 90°C
We know for radial conduction in a Cylinder



$\frac{Q}{t} = \frac{2\pi Kl(T_2 - T_1)}{\ln(R_2/R_1)}$

$= \frac{2 \times 3.14 \times 15 \times 10^{-2} \times 50 \times 10^{-1} \times 90}{\ln(1.2/1)}$ = 232.5 ≈ 233 j/s.

20. $\frac{dQ}{dt}$ = Rate of flow of heat
Let us consider a strip at a distance r from the center of thickness dr.
$\frac{dQ}{dt} = \frac{K \times 2\pi rd \times d\theta}{dr}$ [$d\theta$ = Temperature diff across the thickness dr]



$$\Rightarrow C = \frac{K \times 2\pi rd \times d\theta}{dr} \qquad \left[c = \frac{d\theta}{dr}\right]$$

$$\Rightarrow C\frac{dr}{r} = K2\pi d\, d\theta$$

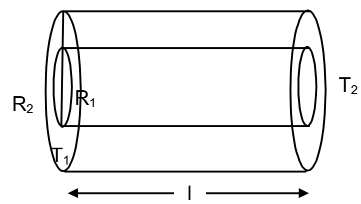
Integrating

$$\Rightarrow C\int_{r_1}^{r_2}\frac{dr}{r} = K2\pi d\int_{\theta_1}^{\theta_2} d\theta \qquad \Rightarrow C[\log r]_{r_1}^{r_2} = K2\pi d\ (\theta_2 - \theta_1)$$

$$\Rightarrow C(\log r_2 - \log r_1) = K2\pi d\ (\theta_2 - \theta_1) \Rightarrow C \log\left(\frac{r_2}{r_1}\right) = K2\pi d\ (\theta_2 - \theta_1)$$

$$\Rightarrow C = \frac{K2\pi d(\theta_2 - \theta_1)}{\log(r_2 / r_1)}$$

21. $T_1 > T_2$



$A = \pi(R_2^2 - R_1^2)$

$$\text{So, } Q = \frac{KA(T_2 - T_1)}{l} = \frac{KA(R_2^2 - R_1^2)(T_2 - T_1)}{l}$$

Considering a concentric cylindrical shell of radius ‘r’ and thickness ‘dr’. The radial heat flow through the shell

$$H = \frac{dQ}{dt} = -KA\frac{d\theta}{dt}$$ [(-)ve because as r – increases θ decreases]

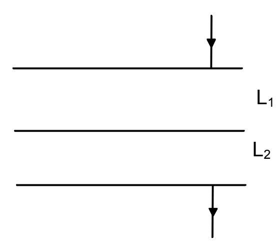
$A = 2\pi rl \qquad H = -2\pi rl\, K\dfrac{d\theta}{dt}$

$$\text{or } \int_{R_1}^{R_2}\frac{dr}{r} = -\frac{2\pi LK}{H}\int_{T_1}^{T_2} d\theta$$

Integrating and simplifying we get

$$H = \frac{dQ}{dt} = \frac{2\pi KL(T_2 - T_1)}{\text{Loge}(R_2 / R_1)} = \frac{2\pi KL(T_2 - T_1)}{\ln(R_2 / R_1)}$$

22. Here the thermal conductivities are in series,



$$\therefore \frac{\dfrac{K_1A(\theta_1 - \theta_2)}{l_1} \times \dfrac{K_2A(\theta_1 - \theta_2)}{l_2}}{\dfrac{K_1A(\theta_1 - \theta_2)}{l_1} + \dfrac{K_2A(\theta_1 - \theta_2)}{l_2}} = \frac{KA(\theta_1 - \theta_2)}{l_1 + l_2}$$

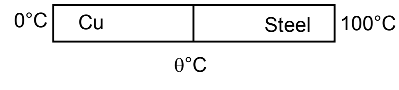
$$\Rightarrow \frac{\dfrac{K_1}{l_1} \times \dfrac{K_2}{l_2}}{\dfrac{K_1}{l_1} + \dfrac{K_2}{l_2}} = \frac{K}{l_1 + l_2}$$

$$\Rightarrow \frac{K_1K_2}{K_1l_2 + K_2l_1} = \frac{K}{l_1 + l_2} \Rightarrow K = \frac{(K_1K_2)(l_1 + l_2)}{K_1l_2 + K_2l_1}$$

23. K_{Cu} = 390 w/m-°C $\qquad K_{St}$ = 46 w/m-°C

Now, Since they are in series connection,

So, the heat passed through the crossections in the same.



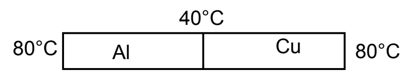
So, $Q_1 = Q_2$

$$\text{Or } \frac{K_{Cu} \times A \times (\theta - 0)}{l} = \frac{K_{St} \times A \times (100 - \theta)}{l}$$

$\Rightarrow 390(\theta - 0) = 46 \times 100 - 46\,\theta \Rightarrow 436\,\theta = 4600$

$$\Rightarrow \theta = \frac{4600}{436} = 10.55 \approx 10.6°C$$

24. As the Aluminum rod and Copper rod joined are in parallel



$$\frac{Q}{t} = \left(\frac{Q}{t_1}\right)_{Al} + \left(\frac{Q}{t}\right)_{Cu}$$

$$\Rightarrow \frac{KA(\theta_1 - \theta_2)}{l} = \frac{K_1A(\theta_1 - \theta_2)}{l} + \frac{K_2A(\theta_1 - \theta_2)}{l}$$

$\Rightarrow K = K_1 + K_2 = (390 + 200) = 590$

$$\frac{Q}{t} = \frac{KA(\theta_1 - \theta_2)}{l} = \frac{590 \times 1 \times 10^{-4} \times (60 - 20)}{1} = 590 \times 10^{-4} \times 40 = 2.36 \text{ Watt}$$

25. $K_{Al} = 200$ w/m-°C $\quad K_{Cu} = 400$ w/m-°C

$A = 0.2 \text{ cm}^2 = 2 \times 10^{-5} \text{ m}^2$

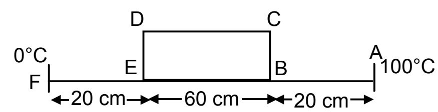
$l = 20 \text{ cm} = 2 \times 10^{-1}$ m

Heat drawn per second

$$= Q_{Al} + Q_{Cu} = \frac{K_{Al} \times A(80 - 40)}{l} + \frac{K_{Cu} \times A(80 - 40)}{l} = \frac{2 \times 10^{-5} \times 40}{2 \times 10^{-1}}[200 + 400] = 2.4 \text{ J}$$

Heat drawn per min = 2.4 × 60 = 144 J

26. $(Q/t)_{AB} = (Q/t)_{BE\ bent} + (Q/t)_{BE}$



$$(Q/t)_{BE\ bent} = \frac{KA(\theta_1 - \theta_2)}{70} \qquad (Q/t)_{BE} = \frac{KA(\theta_1 - \theta_2)}{60}$$

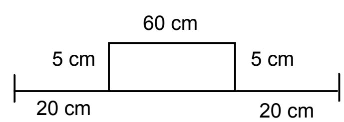
$$\frac{(Q/t)_{BE\ bent}}{(Q/t)_{BE}} = \frac{60}{70} = \frac{6}{7}$$

$(Q/t)_{BE\ bent} + (Q/t)_{BE} = 130$

$\Rightarrow (Q/t)_{BE\ bent} + (Q/t)_{BE}\ 7/6 = 130$

$$\Rightarrow \left(\frac{7}{6} + 1\right)(Q/t)_{BE\ bent} = 130 \qquad \Rightarrow (Q/t)_{BE\ bent} = \frac{130 \times 6}{13} = 60$$

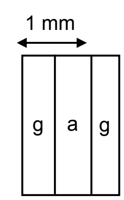
27. $$\frac{Q}{t}\text{bent} = \frac{780 \times A \times 100}{70}$$



$$\frac{Q}{t}\text{str} = \frac{390 \times A \times 100}{60}$$

$$\frac{(Q/t)\text{bent}}{(Q/t)\text{ str}} = \frac{780 \times A \times 100}{70} \times \frac{60}{390 \times A \times 100} = \frac{12}{7}$$

28. (a) $$\frac{Q}{t} = \frac{KA(\theta_1 - \theta_2)}{\ell} = \frac{1 \times 2 \times 1(40 - 32)}{2 \times 10^{-3}} = 8000 \text{ J/sec.}$$



(b) Resistance of glass = $\frac{\ell}{ak_g} + \frac{\ell}{ak_g}$

Resistance of air = $\frac{\ell}{ak_a}$

Net resistance = $\frac{\ell}{ak_g} + \frac{\ell}{ak_g} + \frac{\ell}{ak_a}$

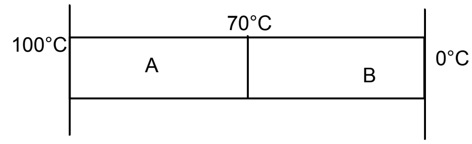
$$= \frac{\ell}{a}\left(\frac{2}{k_g} + \frac{1}{k_a}\right) = \frac{\ell}{a}\left(\frac{2k_a + k_g}{K_g k_a}\right)$$

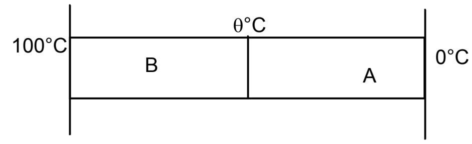
$$= \frac{1 \times 10^{-3}}{2}\left(\frac{2 \times 0.025 + 1}{0.025}\right)$$

$$= \frac{1 \times 10^{-3} \times 1.05}{0.05}$$

$$\frac{Q}{t} = \frac{\theta_1 - \theta_2}{R} = \frac{8 \times 0.05}{1 \times 10^{-3} \times 1.05} = 380.9 \approx 381 \text{ W}$$

29. Now; Q/t remains same in both cases





In Case I : $\frac{K_A \times A \times (100-70)}{\ell} = \frac{K_B \times A \times (70-0)}{\ell}$

$\Rightarrow 30\ K_A = 70\ K_B$

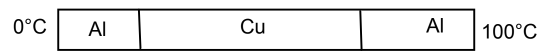
In Case II : $\frac{K_B \times A \times (100-\theta)}{\ell} = \frac{K_A \times A \times (\theta-0)}{\ell}$

$\Rightarrow 100K_B - K_B\ \theta = K_A\ \theta$

$\Rightarrow 100K_B - K_B\ \theta = \frac{70}{30} K_B\ \theta$

$\Rightarrow 100 = \frac{7}{3}\theta + \theta$ $\qquad \Rightarrow \theta = \frac{300}{10} = 30°C$

30. $\theta_1 - \theta_2 = 100$



$\frac{Q}{t} = \frac{\theta_1 - \theta_2}{R}$

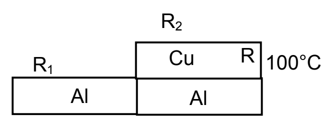
$R = R_1 + R_2 + R_3 = \frac{\ell}{aK_{Al}} + \frac{\ell}{aK_{Cu}} + \frac{\ell}{aK_{Al}} = \frac{\ell}{a}\left(\frac{2}{200} + \frac{1}{400}\right) = \frac{\ell}{a}\left(\frac{4+1}{400}\right) = \frac{\ell}{a}\frac{1}{80}$

$\frac{Q}{t} = \frac{100}{(\ell/a)(1/80)} \Rightarrow 40 = 80 \times 100 \times \frac{a}{\ell}$

$\Rightarrow \frac{a}{\ell} = \frac{1}{200}$

For (b)

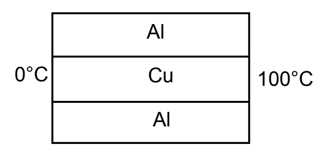
$R = R_1 + R_2 = R_1 + \frac{R_{Cu}R_{Al}}{R_{Cu} + R_{Al}} = R_{Al} + \frac{R_{Cu}R_{Al}}{R_{Cu} + R_{Al}} = \frac{\frac{l}{AK_{Al}} + \frac{l}{AK_{Cu}} + \frac{l}{AK_{Al}}}{\frac{l}{A_{Cu}} + \frac{l}{A_{Al}}}$



$= \frac{l}{AK_{Al}} + \frac{l}{A} + \frac{l}{K_{Al} + K_{Cu}} = \frac{l}{A}\left(\frac{1}{200} + \frac{1}{200+400}\right) = \frac{l}{A} \times \frac{4}{600}$

$\frac{Q}{t} = \frac{\theta_1 - \theta_2}{R} = \frac{100}{(l/A)(4/600)} = \frac{100 \times 600}{4}\frac{A}{l} = \frac{100 \times 600}{4} \times \frac{1}{200} = 75$

For (c)



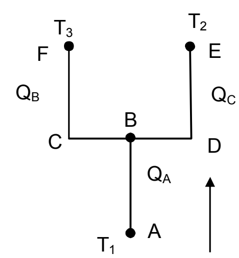
$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{\frac{l}{aK_{Al}}} + \frac{1}{\frac{l}{aK_{Cu}}} + \frac{1}{\frac{l}{aK_{Al}}}$

$= \frac{a}{l}(K_{Al} + K_{Cu} + K_{Al}) = \frac{a}{l}(2 \times 200 + 400) = \frac{a}{l}(800)$

$\Rightarrow R = \frac{l}{a} \times \frac{1}{800}$

$\Rightarrow \frac{Q}{t} = \frac{\theta_1 - \theta_2}{R} = \frac{100 \times 800 \times a}{l}$

$= \frac{100 \times 800}{200} = 400\ W$

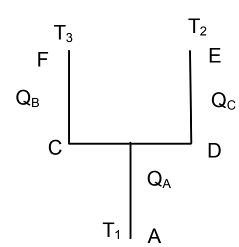


31. Let the temp. at B be T

$\frac{Q_A}{t} = \frac{Q_B}{t} + \frac{Q_C}{t}$ $\qquad \Rightarrow \frac{KA(T_1 - T)}{l} = \frac{KA(T - T_3)}{l + (l/2)} + \frac{KA(T - T_2)}{l + (l/2)}$

$\Rightarrow \frac{T_1 - T}{l} = \frac{T - T_3}{3l/2} + \frac{T - T_2}{3l/2}$ $\qquad \Rightarrow 3T_1 - 3T = 4T - 2(T_2 + T_3)$

$\Rightarrow -7T = -3T_1 - 2(T_2 + T_3)$ $\qquad \Rightarrow T = \frac{3T_1 + 2(T_2 + T_3)}{7}$



32. The temp at the both ends of bar F is same

Rate of Heat flow to right = Rate of heat flow through left

$\Rightarrow (Q/t)_A + (Q/t)_C = (Q/t)_B + (Q/t)_D$

$\Rightarrow \frac{K_A(T_1 - T)A}{l} + \frac{K_C(T_1 - T)A}{l} = \frac{K_B(T - T_2)A}{l} + \frac{K_D(T - T_2)A}{l}$

$\Rightarrow 2K_0(T_1 - T) = 2 \times 2K_0(T - T_2)$

$\Rightarrow T_1 - T = 2T - 2T_2$

$\Rightarrow T = \frac{T_1 + 2T_2}{3}$

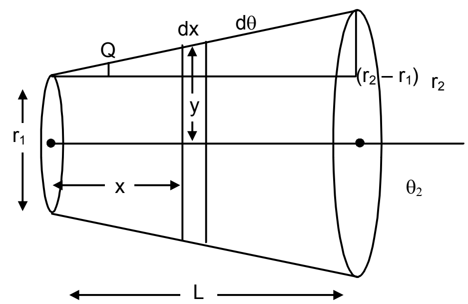
33. $\text{Tan } \phi = \frac{r_2 - r_1}{L} = \frac{(y - r_1)}{x}$

$\Rightarrow xr_2 - xr_1 = yL - r_1L$

Differentiating wr to 'x'

$\Rightarrow r_2 - r_1 = \frac{Ldy}{dx} - 0$

$\Rightarrow \frac{dy}{dx} = \frac{r_2 - r_1}{L} \Rightarrow dx = \frac{dyL}{(r_2 - r_1)}$ ...(1)



Now $\frac{Q}{T} = \frac{K\pi y^2 d\theta}{dx} \Rightarrow \frac{\theta dx}{T} = k\pi y^2 d\theta$

$\Rightarrow \frac{\theta Ldy}{r_2 r_1} = K\pi y^2 d\theta$ from(1)

$\Rightarrow d\theta \frac{QLdy}{(r_2 - r_1)K\pi y^2}$

Integrating both side

$\Rightarrow \int_{\theta_1}^{\theta_2} d\theta = \frac{QL}{(r_2 - r_1)k\pi} \int_{r_1}^{r_2} \frac{dy}{y}$

$\Rightarrow (\theta_2 - \theta_1) = \frac{QL}{(r_2 - r_1)K\pi} \times \left[\frac{-1}{y}\right]_{r_1}^{r_2}$

$\Rightarrow (\theta_2 - \theta_1) = \frac{QL}{(r_2 - r_1)K\pi} \times \left[\frac{1}{r_1} - \frac{1}{r_2}\right]$

$\Rightarrow (\theta_2 - \theta_1) = \frac{QL}{(r_2 - r_1)K\pi} \times \left[\frac{r_2 - r_1}{r_1 + r_2}\right]$

$\Rightarrow Q = \frac{K\pi r_1 r_2(\theta_2 - \theta_1)}{L}$

34. $\frac{d\theta}{dt} = \frac{60}{10 \times 60} = 0.1°\text{C/sec}$

$\frac{dQ}{dt} = \frac{KA}{d}(\theta_1 - \theta_2)$

$= \frac{KA \times 0.1}{d} + \frac{KA \times 0.2}{d} + ... + \frac{KA \times 60}{d}$

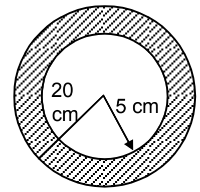
$= \frac{KA}{d}(0.1 + 0.2 + ... + 60) = \frac{KA}{d} \times \frac{600}{2} \times (2 \times 0.1 + 599 \times 0.1)$

[$\therefore$ a + 2a +.........+ na = n/2{2a + (n – 1)a}]

$= \frac{200 \times 1 \times 10^{-4}}{20 \times 10^{-2}} \times 300 \times (0.2 + 59.9) = \frac{200 \times 10^{-2} \times 300 \times 60.1}{20}$

= 3 × 10 × 60.1 = 1803 w ≈ 1800 w

35. $a = r_1 = 5$ cm = 0.05 m



$b = r_2 = 20$ cm = 0.2 m

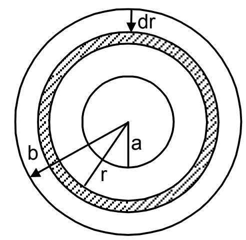
$\theta_1 = T_1 = 50°C$ $\qquad \theta_2 = T_2 = 10°C$

Now, considering a small strip of thickness 'dr' at a distance 'r'.

$A = 4\pi r^2$

$H = -4\pi r^2 K \dfrac{d\theta}{dr}$ [(–)ve because with increase of r, θ decreases]

$= \int_a^b \dfrac{dr}{r^2} = \dfrac{-4\pi K}{H} \int_{\theta_1}^{\theta_2} d\theta$ On integration,



$H = \dfrac{dQ}{dt} = K \dfrac{4\pi ab(\theta_1 - \theta_2)}{(b-a)}$

Putting the values we get

$\dfrac{K \times 4 \times 3.14 \times 5 \times 20 \times 40 \times 10^{-3}}{15 \times 10^{-2}} = 100$

$\Rightarrow K = \dfrac{15}{4 \times 3.14 \times 4 \times 10^{-1}} = 2.985 \approx 3$ w/m-°C

36. $\dfrac{Q}{t} = \dfrac{KA(T_1 - T_2)}{L}$ Rise in Temp. in $T_2 \Rightarrow \dfrac{KA(T_1 - T_2)}{Lms}$

Fall in Temp in $T_1 = \dfrac{KA(T_1 - T_2)}{Lms}$ Final Temp. $T_1 \Rightarrow T_1 - \dfrac{KA(T_1 - T_2)}{Lms}$

Final Temp. $T_2 = T_2 + \dfrac{KA(T_1 - T_2)}{Lms}$

Final $\dfrac{\Delta T}{dt} = T_1 - \dfrac{KA(T_1 - T_2)}{Lms} - T_2 - \dfrac{KA(T_1 - T_2)}{Lms}$

$= (T_1 - T_2) - \dfrac{2KA(T_1 - T_2)}{Lms} = \dfrac{dT}{dt} = -\dfrac{2KA(T_1 - T_2)}{Lms} \Rightarrow \int_{(T_1 - T_2)}^{(T_1 - T_2)} \dfrac{dt}{(T_1 - T_2)} = \dfrac{-2KA}{Lms} dt$

$\Rightarrow Ln \dfrac{(T_1 - T_2)/2}{(T_1 - T_2)} = \dfrac{-2KAt}{Lms}$ $\Rightarrow \ln(1/2) = \dfrac{-2KAt}{Lms}$ $\Rightarrow \ln_2 = \dfrac{2KAt}{Lms}$ $\Rightarrow t = \ln_2 \dfrac{Lms}{2KA}$

37. $\dfrac{Q}{t} = \dfrac{KA(T_1 - T_2)}{L}$ Rise in Temp. in $T_2 \Rightarrow \dfrac{KA(T_1 - T_2)}{Lm_1s_1}$

Fall in Temp in $T_1 \Rightarrow \dfrac{KA(T_1 - T_2)}{Lm_2s_2}$ Final Temp. $T_1 = T_1 - \dfrac{KA(T_1 - T_2)}{Lm_1s_1}$

Final Temp. $T_2 = T_2 + \dfrac{KA(T_1 - T_2)}{Lm_1s_1}$

$\dfrac{\Delta T}{dt} = T_1 - \dfrac{KA(T_1 - T_2)}{Lm_1s_1} - T_2 - \dfrac{KA(T_1 - T_2)}{Lm_2s_2} = (T_1 - T_2) - \left[\dfrac{KA(T_1 - T_2)}{Lm_1s_1} + \dfrac{KA(T_1 - T_2)}{Lm_2s_2}\right]$

$\Rightarrow \dfrac{dT}{dt} = -\dfrac{KA(T_1 - T_2)}{L}\left(\dfrac{1}{m_1s_1} + \dfrac{1}{m_2s_2}\right)$ $\qquad \Rightarrow \dfrac{dT}{(T_1 - T_2)} = -\dfrac{KA}{L}\left(\dfrac{m_2s_2 + m_1s_1}{m_1s_1m_2s_2}\right)dt$

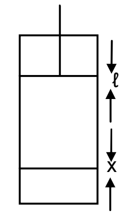
$\Rightarrow \ln\Delta t = -\dfrac{KA}{L}\left(\dfrac{m_2s_2 + m_1s_1}{m_1s_1m_2s_2}\right)t + C$

At time $t = 0$, $T = T_0$, $\qquad \Delta T = \Delta T_0$ $\qquad \Rightarrow C = \ln\Delta T_0$

$\Rightarrow \ln \dfrac{\Delta T}{\Delta T_0} = -\dfrac{KA}{L}\left(\dfrac{m_2s_2 + m_1s_1}{m_1s_1m_2s_2}\right)t \Rightarrow \dfrac{\Delta T}{\Delta T_0} = e^{-\frac{KA}{L}\left(\frac{m_1s_1 + m_2s_2}{m_1s_1m_2s_2}\right)t}$

$\Rightarrow \Delta T = \Delta T_0 \, e^{-\frac{KA}{L}\left(\frac{m_1s_1 + m_2s_2}{m_1s_1m_2s_2}\right)t} = (T_2 - T_1)\, e^{-\frac{KA}{L}\left(\frac{m_1s_1 + m_2s_2}{m_1s_1m_2s_2}\right)t}$

38. $\frac{Q}{t} = \frac{KA(T_s - T_0)}{x} \Rightarrow \frac{nC_P dT}{dt} = \frac{KA(T_s - T_0)}{x}$



$\Rightarrow \frac{n(5/2)RdT}{dt} = \frac{KA(T_s - T_0)}{x} \Rightarrow \frac{dT}{dt} = \frac{-2LA}{5nRx}(T_S - T_0)$

$\Rightarrow \frac{dT}{(T_S - T_0)} = -\frac{2KAdt}{5nRx} \Rightarrow \ln(T_S - T_0)_{T_0}^{T} = -\frac{2KAdt}{5nRx}$

$\Rightarrow \ln\frac{T_S - T}{T_S - T_0} = -\frac{2KAdt}{5nRx} \Rightarrow T_S - T = (T_S - T_0)e^{-\frac{2KAt}{5nRx}}$

$\Rightarrow T = T_S - (T_S - T_0)e^{-\frac{2KAt}{5nRx}} = T_S + (T_S + T_0)e^{+\frac{2KAt}{5nRx}}$

$\Rightarrow \Delta T = T - T_0 = (T_S - T_0) + (T_S - T_0)e^{+\frac{2KAt}{5nRx}} = (T_S - T_0) + \left(1 + e^{+\frac{2KAt}{5nRx}}\right)$

$\Rightarrow \frac{P_a AL}{nR} = (T_S - T_0) + \left(1 + e^{+\frac{2KAt}{5nRx}}\right)$ [$p_a dv = nRdt$ $P_a Al = nRdt$ $dT = \frac{P_a AL}{nR}$]

$\Rightarrow L = \frac{nR}{P_a A}(T_S - T_0) + \left(1 - e^{-\frac{2KAt}{5nRx}}\right)$

39. $A = 1.6\ m^2$, $T = 37°C = 310\ K$, $\sigma = 6.0 \times 10^{-8}\ w/m^2\text{-}K^4$

Energy radiated per second

$= A\sigma T^4 = 1.6 \times 6 \times 10^{-8} \times (310)^4 = 8865801 \times 10^{-4} = 886.58 \approx 887\ J$

40. $A = 12\ cm^2 = 12 \times 10^{-4}\ m^2$ $T = 20°C = 293\ K$

$e = 0.8$ $\sigma = 6 \times 10^{-8}\ w/m^2\text{-}k^4$

$\frac{Q}{t} = Ae\ \sigma T^4 = 12 \times 10^{-4}\ 0.8 \times 6 \times 10^{-8}\ (293)^4 = 4.245 \times 10^{12} \times 10^{-13} = 0.4245 \approx 0.42$

41. $E \rightarrow$ Energy radiated per unit area per unit time

Rate of heat flow $\rightarrow$ Energy radiated

(a) Per time = E × A

So, $E_{Al} = \frac{e\sigma T^4 \times A}{e\sigma T^4 \times A} = \frac{4\pi r^2}{4\pi(2r)^2} = \frac{1}{4}$ $\therefore$ 1 : 4

(b) Emissivity of both are same

$= \frac{m_1 S_1 dT_1}{m_2 S_2 dT_2} = 1$

$\Rightarrow \frac{dT_1}{dT_2} = \frac{m_2 S_2}{m_1 S_1} = \frac{s_1 4\pi r_1^3 \times S_2}{s_2 4\pi r_2^3 \times S_1} = \frac{1 \times \pi \times 900}{3.4 \times 8\pi \times 390} = 1 : 2 : 9$

42. $\frac{Q}{t} = Ae\ \sigma T^4$

$\Rightarrow T^4 = \frac{\theta}{teA\sigma} \Rightarrow T^4 = \frac{100}{0.8 \times 2 \times 3.14 \times 4 \times 10^{-5} \times 1 \times 6 \times 10^{-8}}$

$\Rightarrow T = 1697.0 \approx 1700\ K$

43. (a) $A = 20\ cm^2 = 20 \times 10^{-4}\ m^2$, $T = 57°C = 330\ K$

$E = A\ \sigma T^4 = 20 \times 10^{-4} \times 6 \times 10^{-8} \times (330)^4 \times 10^4 = 1.42\ J$

(b) $\frac{E}{t} = A\sigma e(T_1^4 - T_2^4)$, $A = 20\ cm^2 = 20 \times 10^{-4}\ m^2$

$\sigma = 6 \times 10^{-8}$ $T_1 = 473\ K$, $T_2 = 330\ K$

$= 20 \times 10^{-4} \times 6 \times 10^{-8} \times 1[(473)^4 - (330)^4]$

$= 20 \times 6 \times [5.005 \times 10^{10} - 1.185 \times 10^{10}]$

$= 20 \times 6 \times 3.82 \times 10^{-2} = 4.58\ w$ from the ball.

44. $r = 1\text{ cm} = 1 \times 10^{-3}\text{ m}$

$A = 4\pi(10^{-2})^2 = 4\pi \times 10^{-4}\text{ m}^2$

$E = 0.3,\qquad \sigma = 6 \times 10^{-8}$

$\frac{E}{t} = A\sigma e(T_1^{\,4} - T_2^{\,4})$

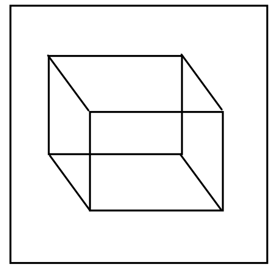
$= 0.3 \times 6 \times 10^{-8} \times 4\pi \times 10^{-4} \times [(100)^4 - (300)^4]$

$= 0.3 \times 6 \times 4\pi \times 10^{-12} \times [1 - 0.0081] \times 10^{12}$

$= 0.3 \times 6 \times 4 \times 3.14 \times 9919 \times 10^{-4}$

$= 4 \times 18 \times 3.14 \times 9919 \times 10^{-5} = 22.4 \approx 22\text{ W}$

45. Since the Cube can be assumed as black body



$e = \ell$

$\sigma = 6 \times 10^{-8}\text{ w/m}^2\text{-k}^4$

$A = 6 \times 25 \times 10^{-4}\text{ m}^2$

$m = 1\text{ kg}$

$s = 400\text{ J/kg-°K}$

$T_1 = 227°C = 500\text{ K}$

$T_2 = 27°C = 300\text{ K}$

$\Rightarrow ms\frac{d\theta}{dt} = e\sigma A(T_1^{\,4} - T_2^{\,4})$

$\Rightarrow \frac{d\theta}{dt} = \frac{e\sigma A\left(T_1^{\,4} - T_2^{\,4}\right)}{ms}$

$= \frac{1 \times 6 \times 10^{-8} \times 6 \times 25 \times 10^{-4} \times [(500)^4 - (300)^4]}{1 \times 400}$

$= \frac{36 \times 25 \times 544}{400} \times 10^{-4} = 1224 \times 10^{-4} = 0.1224°\text{C/s} \approx 0.12°\text{C/s}.$

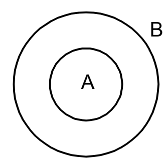
46. $Q = e\sigma A(T_2^{\,4} - T_1^{\,4})$

For any body, $210 = eA\sigma[(500)^4 - (300)^4]$

For black body, $700 = 1 \times A\sigma[(500)^4 - (300)^4]$

Dividing $\frac{210}{700} = \frac{e}{1} \Rightarrow e = 0.3$

47. $A_A = 20\text{ cm}^2,\qquad A_B = 80\text{ cm}^2$



$(mS)_A = 42\text{ J/°C},\qquad (mS)_B = 82\text{ J/°C},$

$T_A = 100°C,\qquad T_B = 20°C$

K_B is low thus it is a poor conducter and K_A is high.

Thus A will absorb no heat and conduct all

$\left(\frac{E}{t}\right)_A = \sigma A_A\,[(373)^4 - (293)^4] \qquad \Rightarrow (mS)_A\left(\frac{d\theta}{dt}\right)_A = \sigma A_A\,[(373)^4 - (293)^4]$

$\Rightarrow \left(\frac{d\theta}{dt}\right)_A = \frac{\sigma A_a\left[(373)^4 - (293)^4\right]}{(mS)_A} = \frac{6 \times 10^{-8}\left[(373)^4 - (293)^4\right]}{42} = 0.03\text{ °C/S}$

Similarly $\left(\frac{d\theta}{dt}\right)_B = 0.043\text{ °C/S}$

48. $\frac{Q}{t} = eAe(T_2^{\,4} - T_1^{\,4})$

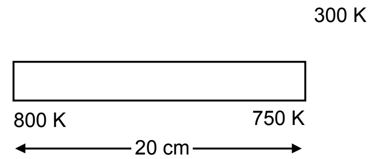
$\Rightarrow \frac{Q}{At} = 1 \times 6 \times 10^{-8}\,[(300)^4 - (290)^4] \qquad = 6 \times 10^{-8}\,(81 \times 10^8 - 70.7 \times 10^8) = 6 \times 10.3$

$\frac{Q}{t} = \frac{KA(\theta_1 - \theta_2)}{l}$

$\Rightarrow \frac{Q}{tA} = \frac{K(\theta_1 - \theta_2)}{l} = \frac{K \times 17}{0.5} = 6 \times 10.3 = \frac{K \times 17}{0.5} \quad \Rightarrow K = \frac{6 \times 10.3 \times 0.5}{17} = 1.8$

49. $\sigma = 6 \times 10^{-8}$ w/m^2-k^4

L = 20 cm = 0.2 m, K = ?



$\Rightarrow E = \frac{KA(\theta_1 - \theta_2)}{d} = A\sigma(T_1^4 - T_2^4)$

$\Rightarrow K = \frac{s(T_1 - T_2) \times d}{\theta_1 - \theta_2} = \frac{6 \times 10^{-8} \times (750^4 - 300^4) \times 2 \times 10^{-1}}{50}$

$\Rightarrow K = 73.993 \approx 74.$

50. v = 100 cc

$\Delta\theta = 5°C$

t = 5 min

For water

$\frac{mS\Delta\theta}{dt} = \frac{KA}{l}\Delta\theta$

$\Rightarrow \frac{100 \times 10^{-3} \times 1000 \times 4200}{5} = \frac{KA}{l}$

For Kerosene

$\frac{ms}{at} = \frac{KA}{l}$

$\Rightarrow \frac{100 \times 10^{-3} \times 800 \times 2100}{t} = \frac{KA}{l}$

$\Rightarrow \frac{100 \times 10^{-3} \times 800 \times 2100}{t} = \frac{100 \times 10^{-3} \times 1000 \times 4200}{5}$

$\Rightarrow T = \frac{5 \times 800 \times 2100}{1000 \times 4200} = 2$ min

51. 50°C 45°C 40°C

Let the surrounding temperature be 'T'°C

Avg. t = $\frac{50+45}{2} = 47.5$

Avg. temp. diff. from surrounding

T = 47.5 – T

Rate of fall of temp = $\frac{50-45}{5} = 1$ °C/mm

From Newton's Law

1°C/mm = bA × t

$\Rightarrow bA = \frac{1}{t} = \frac{1}{47.5 - T}$ …(1)

In second case,

Avg, temp = $\frac{40+45}{2} = 42.5$

Avg. temp. diff. from surrounding

t′ = 42.5 – t

Rate of fall of temp = $\frac{45-40}{8} = \frac{5}{8}$ °C/mm

From Newton's Law

$\frac{5}{B} = bAt'$

$\Rightarrow \frac{5}{8} = \frac{1}{(47.5 - T)} \times (42.5 - T)$

By C & D [Componendo & Dividendo method]

We find, T = 34.1°C

52. Let the water eq. of calorimeter = m

$\frac{(m + 50 \times 10^{-3}) \times 4200 \times 5}{10}$ = Rate of heat flow

$\frac{(m + 100 \times 10^{-3}) \times 4200 \times 5}{18}$ = Rate of flow

$\Rightarrow \frac{(m + 50 \times 10^{-3}) \times 4200 \times 5}{10} = \frac{(m + 100 \times 10^{-3}) \times 4200 \times 5}{18}$

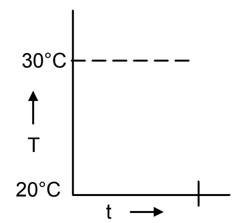
$\Rightarrow (m + 50 \times 10^{-3})18 = 10m + 1000 \times 10^{-3}$

$\Rightarrow 18m + 18 \times 50 \times 10^{-3} = 10m + 1000 \times 10^{-3}$

$\Rightarrow 8m = 100 \times 10^{-3}$ kg

$\Rightarrow m = 12.5 \times 10^{-3}$ kg = 12.5 g

53. In steady state condition as no heat is absorbed, the rate of loss of heat by conduction is equal to that of the supplied.



i.e. H = P

m = 1Kg, Power of Heater = 20 W, Room Temp. = 20°C

(a) H = $\frac{d\theta}{dt}$ = P = 20 watt

(b) by Newton's law of cooling

$\frac{-d\theta}{dt} = K(\theta - \theta_0)$

$-20 = K(50 - 20) \Rightarrow K = 2/3$

Again, $\frac{-d\theta}{dt} = K(\theta - \theta_0) = \frac{2}{3} \times (30 - 20) = \frac{20}{3}$ w

(c) $\left(\frac{dQ}{dt}\right)_{20} = 0$, $\quad \left(\frac{dQ}{dt}\right)_{30} = \frac{20}{3}$ $\quad \left(\frac{dQ}{dt}\right)_{avg} = \frac{10}{3}$

T = 5 min = 300 ′

Heat liberated = $\frac{10}{3} \times 300$ = 1000 J

Net Heat absorbed = Heat supplied – Heat Radiated = 6000 – 1000 = 5000 J

Now, $m\Delta\theta'$ = 5000

$\Rightarrow S = \frac{5000}{m\Delta\theta} = \frac{5000}{1 \times 10}$ = 500 J $Kg^{-1}°C^{-1}$

54. Given:

Heat capacity = m × s = 80 J/°C

$\left(\frac{d\theta}{dt}\right)_{increase}$ = 2 °C/s

$\left(\frac{d\theta}{dt}\right)_{decrease}$ = 0.2 °C/s

(a) Power of heater = $mS\left(\frac{d\theta}{dt}\right)_{increasing}$ = 80 × 2 = 160 W

(b) Power radiated = $mS\left(\frac{d\theta}{dt}\right)_{decreasing}$ = 80 × 0.2 = 16 W

(c) Now $mS\left(\frac{d\theta}{dt}\right)_{decreasing} = K(T - T_0)$

$\Rightarrow 16 = K(30 - 20)$ $\qquad \Rightarrow K = \frac{16}{10} = 1.6$

Now, $\frac{d\theta}{dt} = K(T - T_0) = 1.6 \times (30 - 25) = 1.6 \times 5 = 8$ W

(d) P.t = H $\Rightarrow$ 8 × t

55. $\frac{d\theta}{dt} = -K(T - T_0)$

Temp. at t = 0 is θ_1

(a) Max. Heat that the body can loose = $\Delta Q_m = ms(\theta_1 - \theta_0)$

($\therefore$ as, $\Delta t = \theta_1 - \theta_0$)

(b) if the body loses 90% of the max heat the decrease in its temp. will be

$\frac{\Delta Q_m \times 9}{10ms} = \frac{(\theta_1 - \theta_0) \times 9}{10}$

If it takes time t_1, for this process, the temp. at t_1

$= \theta_1 - (\theta_1 - \theta_0)\frac{9}{10} = \frac{10\theta_1 - 9\theta_1 - 9\theta_0}{10} = \frac{\theta_1 - 9\theta_0}{10} \times 1$

Now, $\frac{d\theta}{dt} = -K(\theta - \theta_1)$

Let $\theta = \theta_1$ at t = 0; & θ be temp. at time t

$$\int_{\theta}^{\theta} \frac{d\theta}{\theta - \theta_o} = -K\int_0^t dt$$

or, $\ln\frac{\theta - \theta_0}{\theta_1 - \theta_0} = -Kt$

or, $\theta - \theta_0 = (\theta_1 - \theta_0)\, e^{-kt}$ ...(2)

Putting value in the Eq (1) and Eq (2)

$\frac{\theta_1 - 9\theta_0}{10} - \theta_0 = (\theta_1 - \theta_0)\, e^{-kt}$

$\Rightarrow t_1 = \frac{\ln 10}{k}$

♣ ♣ ♣ ♣ ♣

CHAPTER – 29
ELECTRIC FIELD AND POTENTIAL
EXERCISES

1. $\varepsilon_0 = \dfrac{\text{Coulomb}^2}{\text{Newton m}^2} = I^1M^{-1}L^{-3}T^4$

$\therefore F = \dfrac{kq_1q_2}{r^2}$

2. $q_1 = q_2 = q = 1.0$ C distance between = 2 km = 1 × 10^3 m

so, force = $\dfrac{kq_1q_2}{r^2}$ $F = \dfrac{(9\times10^9)\times1\times1}{(2\times10^3)^2} = \dfrac{9\times10^9}{2^2\times10^6}$ = 2,25 × 10^3 N

The weight of body = mg = 40 × 10 N = 400 N

So, $\dfrac{\text{wt of body}}{\text{force between charges}} = \left(\dfrac{2.25\times10^3}{4\times10^2}\right)^{-1} = (5.6)^{-1} = \dfrac{1}{5.6}$

So, force between charges = 5.6 weight of body.

3. q = 1 C, Let the distance be χ

F = 50 × 9.8 = 490

$F = \dfrac{Kq^2}{\chi^2} \Rightarrow 490 = \dfrac{9\times10^9\times1^2}{\chi^2}$ or $\chi^2 = \dfrac{9\times10^9}{490} = 18.36\times10^6$

$\Rightarrow \chi = 4.29\times10^3$ m

4. charges 'q' each, AB = 1 m

wt, of 50 kg person = 50 × g = 50 × 9.8 = 490 N

$F_C = \dfrac{kq_1q_2}{r^2}$ $\therefore \dfrac{kq^2}{r^2} = 490$ N

$\Rightarrow q^2 = \dfrac{490\times r^2}{9\times10^9} = \dfrac{490\times1\times1}{9\times10^9}$

$\Rightarrow q = \sqrt{54.4\times10^{-9}} = 23.323\times10^{-5}$ coulomb ≈ 2.3×10^{-4} coulomb

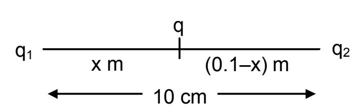
5. Charge on each proton = a= 1.6×10^{-19} coulomb

Distance between charges = 10×10^{-15} metre = r

Force = $\dfrac{kq^2}{r^2} = \dfrac{9\times10^9\times1.6\times1.6\times10^{-38}}{10^{-30}}$ = 9 × 2.56 × 10 = 230.4 Newton

6. $q_1 = 2.0\times10^{-6}$ $q_2 = 1.0\times10^{-6}$ r = 10 cm = 0.1 m

Let the charge be at a distance x from q_1



$F_1 = \dfrac{Kq_1q}{\chi^2}$ $F_2 = \dfrac{kqq_2}{(0.1-\chi)^2}$

$= \dfrac{9.9\times2\times10^{-6}\times10^9\times q}{\chi^2}$

Now since the net force is zero on the charge q. $\Rightarrow f_1 = f_2$

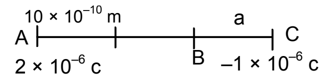
$\Rightarrow \dfrac{kq_1q}{\chi^2} = \dfrac{kqq_2}{(0.1-\chi)^2}$

$\Rightarrow 2(0.1-\chi)^2 = \chi^2 \Rightarrow \sqrt{2}\,(0.1-\chi) = \chi$

$\Rightarrow \chi = \dfrac{0.1\sqrt{2}}{1+\sqrt{2}}$ = 0.0586 m = 5.86 cm ≈ 5.9 cm From larger charge

7. $q_1 = 2 \times 10^{-6}$ c $\quad q_2 = -1 \times 10^{-6}$ c $\quad r = 10$ cm $= 10 \times 10^{-2}$ m

Let the third charge be a $\quad$ so, $F_{-AC} = -F_{-BC}$



$\Rightarrow \dfrac{kQq_1}{r_1^2} = \dfrac{-KQq_2}{r_2^2} \qquad \Rightarrow \dfrac{2\times10^{-6}}{(10+\chi)^2} = \dfrac{1\times10^{-6}}{\chi^2}$

$\Rightarrow 2\chi^2 = (10+\chi)^2 \Rightarrow \sqrt{2}\,\chi = 10 + \chi \Rightarrow \chi(\sqrt{2} - 1) = 10 \Rightarrow \chi = \dfrac{-10}{1.414-1} = 24.14$ cm χ

So, distance = 24.14 + 10 = 34.14 cm from larger charge

8. Minimum charge of a body is the charge of an electron

Wo, $q = 1.6 \times 10^{-19}$ c $\qquad \chi = 1$ cm $= 1 \times 10^{-2}$ cm

So, $F = \dfrac{kq_1q_2}{r^2} = \dfrac{9\times10^9 \times 1.6\times1.6\times10^{-19}\times10^{-19}}{10^{-2}\times10^{-2}} = 23.04 \times 10^{-38+9+2+2} = 23.04 \times 10^{-25} = 2.3 \times 10^{-24}$

9. No. of electrons of 100 g water $= \dfrac{10\times100}{18} = 55.5$ Nos $\qquad$ Total charge = 55.5

No. of electrons in 18 g of $H_2O = 6.023 \times 10^{23} \times 10 = 6.023 \times 10^{24}$

No. of electrons in 100 g of $H_2O = \dfrac{6.023\times10^{24}\times100}{18} = 0.334 \times 10^{26} = 3.334 \times 10^{25}$

Total charge $= 3.34 \times 10^{25} \times 1.6 \times 10^{-19} = 5.34 \times 10^{6}$ c

10. Molecular weight of $H_2O = 2 \times 1 \times 16 = 16$

No. of electrons present in one molecule of H_2O = 10

18 gm of H_2O has 6.023×10^{23} molecule

18 gm of H_2O has $6.023 \times 10^{23} \times 10$ electrons

100 gm of H_2O has $\dfrac{6.023\times10^{24}}{18}\times100$ electrons

So number of protons $= \dfrac{6.023\times10^{26}}{18}$ protons (since atom is electrically neutral)

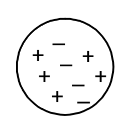
Charge of protons $= \dfrac{1.6\times10^{-19}\times6.023\times10^{26}}{18}$ coulomb $= \dfrac{1.6\times6.023\times10^{7}}{18}$ coulomb

Charge of electrons = $= \dfrac{1.6\times6.023\times10^{7}}{18}$ coulomb

Hence Electrical force $= \dfrac{9\times10^9\left(\dfrac{1.6\times6.023\times10^7}{18}\right)\times\left(\dfrac{1.6\times6.023\times10^7}{18}\right)}{(10\times10^{-2})^2}$

$= \dfrac{8\times6.023}{18}\times1.6\times6.023\times10^{25} = 2.56 \times 10^{25}$ Newton

11. Let two protons be at a distance be 13.8 femi



$F = \dfrac{9\times10^9\times1.6\times10^{-38}}{(14.8)^2\times10^{-30}} = 1.2$ N

12. F = 0.1 N

$r = 1$ cm $= 10^{-2}$ (As they rubbed with each other. So the charge on each sphere are equal)

So, $F = \dfrac{kq_1q_2}{r^2} \Rightarrow 0.1 = \dfrac{kq^2}{(10^{-2})^2} \Rightarrow q^2 = \dfrac{0.1\times10^{-4}}{9\times10^9} \Rightarrow q^2 = \dfrac{1}{9}\times10^{-14} \Rightarrow q = \dfrac{1}{3}\times10^{-7}$

1.6×10^{-19} c $\qquad$ Carries by 1 electron $\quad$ 1 c carried by $\dfrac{1}{1.6\times10^{-19}}$

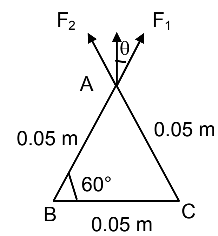
0.33×10^{-7} c carries by $\dfrac{1}{1.6\times10^{-19}}\times0.33\times10^{-7} = 0.208 \times 10^{12} = 2.08 \times 10^{11}$

13. $F = \frac{kq_1q_2}{r^2} = \frac{9\times10^9\times1.6\times1.6\times10^{-19}\times10^{-19}}{(2.75\times10^{-10})^2} = \frac{23.04\times10^{-29}}{7.56\times10^{-20}}$ = 3.04 × 10^{-9}

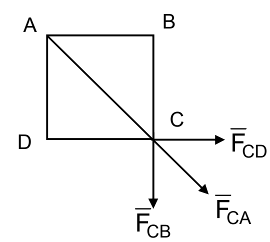
14. Given: mass of proton = 1.67 × 10^{-27} kg = M_p
k = 9 × 10^9 Charge of proton = 1.6 × 10^{-19} c = C_p
G = 6.67 × 10^{-11} Let the separation be 'r'
$Fe = \frac{k(C_p)^2}{r^2}$, $fg= \frac{G(M_p)^2}{r^2}$
Now, $Fe : Fg = \frac{K(C_p)^2}{r^2}\times\frac{r^2}{G(M_p)^2} = \frac{9\times10^9\times(1.6\times10^{-19})^2}{6.67\times10^{-11}\times(1.67\times10^{-27})^2}$ = 9 × 2.56 × 10^{38} ≈ 1,24 ×10^{38}

15. Expression of electrical force $F = C\times e^{\frac{-kr}{r^2}}$
Since e^{-kr} is a pure number. So, dimensional formulae of $F = \frac{\text{dimensional formulae of C}}{\text{dimensional formulae of } r^2}$
Or, $[MLT^{-2}][L^2]$ = dimensional formulae of C = $[ML^3T^{-2}]$
Unit of C = unit of force × unit of r^2 = Newton × m^2 = Newton–m^2
Since –kr is a number hence dimensional formulae of
$k = \frac{1}{\text{dimentional formulae of r}} = [L^{-1}]$ Unit of k = m^{-1}

16. Three charges are held at three corners of a equilateral trangle.
Let the charges be A, B and C. It is of length 5 cm or 0.05 m
Force exerted by B on A = F_1 force exerted by C on A = F_2
So, force exerted on A = resultant $F_1 = F_2$
$\Rightarrow F = \frac{kq_2}{r^2} = \frac{9\times10^9\times2\times2\times2\times10^{-12}}{5\times5\times10^{-4}} = \frac{36}{25}\times10 = 14.4$
Now, force on A = 2 × F cos 30° since it is equilateral Δ.
$\Rightarrow$ Force on A = 2 × 1.44 × $\sqrt{\frac{3}{2}}$ = 24.94 N.



17. $q_1 = q_2 = q_3 = q_4 = 2\times10^{-6}$ C
v = 5 cm = 5 × 10^{-2} m
so force on $\bar{c} = \bar{F}_{CA} + \bar{F}_{CB} + \bar{F}_{CD}$
so Force along × Component = $\bar{F}_{CD} + \bar{F}_{CA}\cos 45° + 0$
$= \frac{k(2\times10^{-6})^2}{(5\times10^{-2})^2} + \frac{k(2\times10^{-6})^2}{(5\times10^{-2})^2}\frac{1}{2\sqrt{2}} = kq^2\left(\frac{1}{25\times10^{-4}} + \frac{1}{50\sqrt{2}\times10^{-4}}\right)$
$= \frac{9\times10^9\times4\times10^{-12}}{24\times10^{-4}}\left(1+\frac{1}{2\sqrt{2}}\right)$ = 1.44 (1.35) = 19.49 Force along % component = 19.49
So, Resultant $R = \sqrt{Fx^2+Fy^2} = 19.49\sqrt{2}$ = 27.56



18. R = 0.53 A° = 0.53 × 10^{-10} m
$F = \frac{Kq_1q_2}{r^2} = \frac{9\times10^9\times1.6\times1.6\times10^{-38}}{0.53\times0.53\times10^{-10}\times10^{-10}}$ = 82.02 × 10^{-9} N

19. Fe from previous problem No. 18 = 8.2 × 10^{-8} N Ve = ?
Now, M_e = 9.12 × 10^{-31} kg r = 0.53 × 10^{-10} m
Now, $Fe = \frac{M_ev^2}{r} \Rightarrow v^2 = \frac{Fe\times r}{m_e} = \frac{8.2\times10^{-8}\times0.53\times10^{-10}}{9.1\times10^{-31}}$ = 0.4775 × 10^{13} = 4.775 × 10^{12} m^2/s^2
$\Rightarrow$ v = 2.18 × 10^6 m/s

20. Electric force feeled by 1 c due to 1×10^{-8} c.

$F_1 = \dfrac{k \times 1 \times 10^{-8} \times 1}{(10 \times 10^{-2})^2} = k \times 10^{-6}$ N. electric force feeled by 1 c due to 8×10^{-8} c.

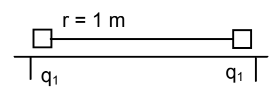
$F_2 = \dfrac{k \times 8 \times 10^{-8} \times 1}{(23 \times 10^{-2})^2} = \dfrac{k \times 8 \times \times 10^{-8} \times 10^2}{9} = \dfrac{28k \times 10^{-6}}{4} = 2k \times 10^{-6}$ N.

Similarly $F_3 = \dfrac{k \times 27 \times 10^{-8} \times 1}{(30 \times 10^{-2})^2} = 3k \times 10^{-6}$ N

So, $F = F_1 + F_2 + F_3 + \ldots\ldots + F_{10} = k \times 10^{-6}\,(1 + 2 + 3 + \ldots\ldots + 10)$ N

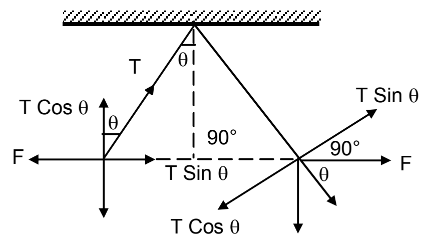
$= k \times 10^{-6} \times \dfrac{10 \times 11}{2} = 55k \times 10^{-6} = 55 \times 9 \times 10^9 \times 10^{-6}\text{ N} = 4.95 \times 10^3$ N

21. Force exerted $= \dfrac{kq_1^{\,2}}{r^2}$



$= \dfrac{9 \times 10^9 \times 2 \times 2 \times 10^{-16}}{1^2} = 3.6 \times 10^{-6}$ is the force exerted on the string

22. $q_1 = q_2 = 2 \times 10^{-7}$ c m = 100 g
$l = 50\text{ cm} = 5 \times 10^{-2}$ m $d = 5 \times 10^{-2}$ m



(a) Now Electric force

$F = K\dfrac{q^2}{r^2} = \dfrac{9 \times 10^9 \times 4 \times 10^{-14}}{25 \times 10^{-4}}\text{ N} = 14.4 \times 10^{-2}\text{ N} = 0.144$ N

(b) The components of Resultant force along it is zero,
because mg balances T cos θ and so also.
F = mg = T sin θ

(c) Tension on the string
T sin θ = F T cos θ = mg

$\text{Tan}\,\theta = \dfrac{F}{mg} = \dfrac{0.144}{100 \times 10^{-3} \times 9.8} = 0.14693$

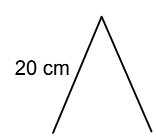
But $T\cos\theta = 10^2 \times 10^{-3} \times 10 = 1$ N

$\Rightarrow T = \dfrac{1}{\cos\theta} = \sec\theta$

$\Rightarrow T = \dfrac{F}{\sin\theta}$,

Sin θ = 0.145369 ; Cos θ = 0.989378;

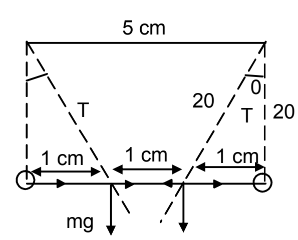
23. $q = 2.0 \times 10^{-8}$ c n= ? T = ? $\text{Sin}\,\theta = \dfrac{1}{20}$



Force between the charges

$F = \dfrac{Kq_1q_2}{r^2} = \dfrac{9 \times 10^9 \times 2 \times 10^{-8} \times 2 \times 10^{-8}}{(3 \times 10^{-2})^2} = 4 \times 10^{-3}$ N

$mg\sin\theta = F \Rightarrow m = \dfrac{F}{g\sin\theta} = \dfrac{4 \times 10^{-3}}{10 \times (1/20)} = 8 \times 10^{-3} = 8$ gm

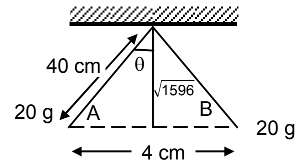


$\text{Cos}\,\theta = \sqrt{1 - \text{Sin}^2\theta} = \sqrt{1 - \dfrac{1}{400}} = \sqrt{\dfrac{400 - 1}{400}} = 0.99 \approx 1$

So, T = mg cos θ
Or $T = 8 \times 10^{-3}\ 10 \times 0.99 = 8 \times 10^{-2}$ M

24. $T \cos\theta = mg$ …(1)

$T \sin\theta = Fe$ …(2)

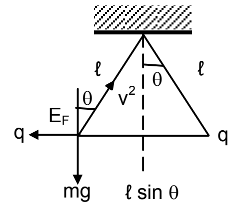


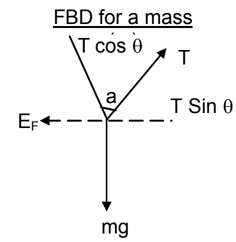
Solving, (2)/(1) we get, $\tan\theta = \dfrac{Fe}{mg} = \dfrac{kq^2}{r} \times \dfrac{1}{mg}$

$\Rightarrow \dfrac{2}{\sqrt{1596}} = \dfrac{9 \times 10^9 \times q^2}{(0.04)^2 \times 0.02 \times 9.8}$

$\Rightarrow q^2 = \dfrac{(0.04)^2 \times 0.02 \times 9.8 \times 2}{9 \times 10^9 \times \sqrt{1596}} = \dfrac{6.27 \times 10^{-4}}{9 \times 10^9 \times 39.95} = 17 \times 10^{-16} c^2$

$\Rightarrow q = \sqrt{17 \times 10^{-16}} = 4.123 \times 10^{-8}$ c





25. Electric force $= \dfrac{kq^2}{(\ell \sin Q + \ell \sin Q)^2} = \dfrac{kq^2}{4\ell^2 \sin^2}$

So, $T \cos\theta = ms$ (For equilibrium) $T \sin\theta = Ef$

Or $\tan\theta = \dfrac{Ef}{mg}$

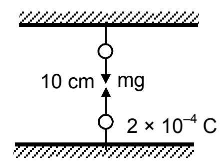
$\Rightarrow mg = Ef \cot\theta = \dfrac{kq^2}{4\ell^2 \sin^2\theta} \cot\theta = \dfrac{q^2 \cot\theta}{\ell^2 \sin^2\theta 16\pi E_0}$

or $m = \dfrac{q^2 \cot\theta}{16\pi E_0 \ell^2 \text{Sin}^2\theta g}$ unit.

26. Mass of the bob = 100 g = 0.1 kg

So Tension in the string = 0.1 × 9.8 = 0.98 N.

For the Tension to be 0, the charge below should repel the first bob.

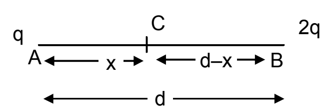


$\Rightarrow F = \dfrac{kq_1q_2}{r^2}$ $\quad T - mg + F = 0 \Rightarrow T = mg - f \quad T = mg$

$\Rightarrow 0.98 = \dfrac{9 \times 10^9 \times 2 \times 10^{-4} \times q_2}{(0.01)^2} \Rightarrow q_2 = \dfrac{0.98 \times 1 \times 10^{-2}}{9 \times 2 \times 10^5} = 0.054 \times 10^{-9}$ N

27. Let the charge on C = q

So, net force on c is equal to zero



So $F_{\overline{AC}} + F_{\overline{BA}} = 0$, But $F_{AC} = F_{BC} \Rightarrow \dfrac{kqQ}{x^2} = \dfrac{k2qQ}{(d-x)^2}$

$\Rightarrow 2x^2 = (d - x)^2 \Rightarrow \sqrt{2}\, x = d - x$

$\Rightarrow x = \dfrac{d}{\sqrt{2}+1} = \dfrac{d}{(\sqrt{2}+1)} \times \dfrac{(\sqrt{2}-1)}{(\sqrt{2}-1)} = d(\sqrt{2}-1)$

For the charge on rest, $F_{AC} + F_{AB} = 0$

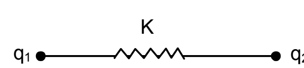
$(2.414)^2 \dfrac{kqQ}{d^2} + \dfrac{kq(2q)}{d^2} = 0 \Rightarrow \dfrac{kq}{d^2}[(2.414)^2 Q + 2q] = 0$

$\Rightarrow 2q = -(2.414)^2 Q$

$\Rightarrow Q = \dfrac{2}{-(\sqrt{2}+1)^2} q = -\left(\dfrac{2}{3+2\sqrt{2}}\right) q = -(0.343)\, q = -(6 - 4\sqrt{2})$

28. K = 100 N/m $\quad \ell = 10$ cm $= 10^{-1}$ m $\quad q = 2.0 \times 10^{-8}$ c Find ℓ = ?

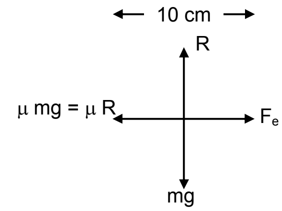
Force between them $F = \dfrac{kq_1q_2}{r^2} = \dfrac{9 \times 10^9 2 \times 10^{-8} \times 2 \times 10^{-8}}{10^{-2}} = 36 \times 10^{-5}$ N



So, $F = -kx$ or $x = \dfrac{F}{-K} = \dfrac{36 \times 10^{-5}}{100} = 36 \times 10^{-7}$ cm $= 3.6 \times 10^{-6}$ m

29. $q_A = 2 \times 10^{-6}$ C $M_b = 80$ g $\mu = 0.2$

Since B is at equilibrium, So, $Fe = \mu R$



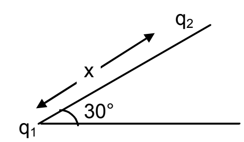
$\Rightarrow \frac{Kq_Aq_B}{r^2} = \mu R = \mu m \times g$

$\Rightarrow \frac{9\times10^9\times2\times10^{-6}\times q_B}{0.01} = 0.2 \times 0.08 \times 9.8$

$\Rightarrow q_B = \frac{0.2\times0.08\times9.8\times0.01}{9\times10^9\times2\times10^{-6}} = 8.7 \times 10^{-8}$ C Range = $\pm 8.7 \times 10^{-8}$ C

30. $q_1 = 2 \times 10^{-6}$ c Let the distance be r unit

$\therefore F_{repulsion} = \frac{kq_1q_2}{r^2}$



For equilibrium $\frac{kq_1q_2}{r^2} = mg \sin\theta$

$\Rightarrow \frac{9\times10^9\times4\times10^{-12}}{r^2} = m \times 9.8 \times \frac{1}{2}$

$\Rightarrow r^2 = \frac{18\times4\times10^{-3}}{m\times9.8} = \frac{72\times10^{-3}}{9.8\times10^{-1}} = 7.34 \times 10^{-2}$ metre

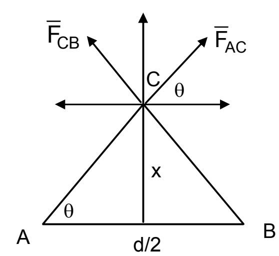
$\Rightarrow r = 2.70924 \times 10^{-1}$ metre from the bottom.

31. Force on the charge particle 'q' at 'c' is only the x component of 2 forces

So, $F_{on\ c} = F_{CB} \sin\theta + F_{AC} \sin\theta$ But $|\bar{F}_{CB}| = |\bar{F}_{AC}|$

$= 2F_{CB}\sin\theta = 2\frac{KQq}{x^2+(d/2)^2}\times\frac{x}{\left[x^2+d^2/4\right]^{1/2}} = \frac{2k\theta qx}{(x^2+d^2/4)^{3/2}} = \frac{16kQq}{(4x^2+d^2)^{3/2}}x$

For maximum force $\frac{dF}{dx} = 0$



$\frac{d}{dx}\left(\frac{16kQqx}{(4x^2+d^2)^{3/2}}\right) = 0 \Rightarrow K\left[\frac{(4x^2+d^2)-x\left[3/2\left[4x^2+d^2\right]^{1/2}8x\right]}{[4x^2+d^2]^3}\right] = 0$

$\Rightarrow \frac{K(4x^2+d^2)^{1/2}\left[(4x^2+d^2)^3-12x^2\right]}{(4x^2+d^2)^3} = 0 \Rightarrow (4x^2+d^2)^3 = 12x^2$

$\Rightarrow 16x^4 + d^4 + 8x^2d^2 = 12x^2$ $d^4 + 8x^2d^2 = 0$

$\Rightarrow d^2 = 0$ $d^2 + 8x^2 = 0$ $\Rightarrow d^2 = 8x^2 \Rightarrow d = \frac{d}{2\sqrt{2}}$

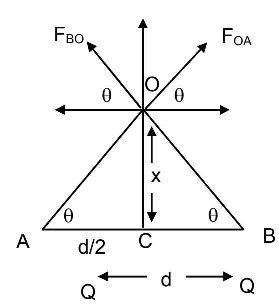
32. (a) Let Q = charge on A & B Separated by distance d

q = charge on c displaced $\perp$ to –AB

So, force on 0 = $\bar{F}_{AB} + \bar{F}_{BO}$

But $F_{AO}\cos\theta = F_{BO}\cos\theta$

So, force on '0' in due to vertical component.



$\bar{F} = F_{AO}\sin\theta + F_{BO}\sin\theta$ $|\bar{F}_{AO}| = |\bar{F}_{BO}|$

$= 2\frac{KQq}{(d/2^2+x^2)}\sin\theta$ $F = \frac{2KQq}{(d/2)^2+x^2}\sin\theta$

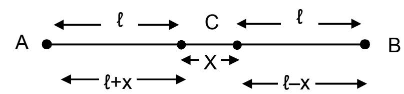
$= \frac{4\times2\times kQq}{(d^2+4x^2)}\times\frac{x}{[(d/2)^2+x^2]^{1/2}} = \frac{2kQq}{[(d/2)^2+x^2]^{3/2}}x$ = Electric force $\Rightarrow F \propto x$

(b) When x << d $F = \frac{2kQq}{[(d/2)^2 + x^2]^{3/2}} x$ x<<d

$\Rightarrow F = \frac{2kQq}{(d^2/4)^{3/2}} x \Rightarrow F \propto x$ $a = \frac{F}{m} = \frac{1}{m}\left[\frac{2kQqx}{[(d^2/4) + \ell^2}\right]$

So time period $T = 2\pi\sqrt{\frac{\ell}{g}} = 2\pi\sqrt{\frac{\ell}{a}}$

33. $F_{AC} = \frac{KQq}{(\ell + x)^2}$ $F_{CA} = \frac{KQq}{(\ell - x)^2}$



Net force = $KQq\left[\frac{1}{(\ell - x)^2} - \frac{1}{(\ell + x)^2}\right]$

$= KQq\left[\frac{(\ell + x)^2 - (\ell - x)^2}{(\ell + x)^2(\ell - x)^2}\right] = KQq\left[\frac{4\ell x}{(\ell^2 - x^2)^2}\right]$

x<<< l = d/2 neglecting x w.r.t. ℓ We get

net $F = \frac{KQq4\ell x}{\ell^4} = \frac{KQq4x}{\ell^3}$ acceleration $= \frac{4KQqx}{m\ell^3}$

Time period $= 2\pi\sqrt{\frac{\text{displacement}}{\text{acceleration}}} = 2\pi\sqrt{\frac{xm\ell^3}{4KQqx}} = 2\pi\sqrt{\frac{m\ell^3}{4KQq}}$

$= \sqrt{\frac{4\pi^2 m\ell^3 4\pi\varepsilon_0}{4Qq}} = \sqrt{\frac{4\pi^3 m\ell^3\varepsilon_0}{Qq}} = \sqrt{4\pi^3 md^3\varepsilon_0 8Qq} = \left[\frac{\pi^3 md^3\varepsilon_0}{2Qq}\right]^{1/2}$

34. $F_e = 1.5 \times 10^{-3}$ N, $q = 1 \times 10^{-6}$ C, $F_e = q \times E$

$\Rightarrow E = \frac{F_e}{q} = \frac{1.5 \times 10^{-3}}{1 \times 10^{-6}} = 1.5 \times 10^3$ N/C

35. $q_2 = 2 \times 10^{-6}$ C, $q_1^2 = -4 \times 10^{-6}$ C, r = 20 cm = 0.2 m

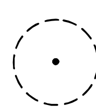
(E_1 = electric field due to q_1, E_2 = electric field due to q_2)

$\Rightarrow \frac{(r - x)^2}{x^2} = \frac{-q_2}{q_1} \Rightarrow \frac{(r - 1)^2}{x} = \frac{-q_2}{q_1} = \frac{4 \times 10^{-6}}{2 \times 10^{-6}} = \frac{1}{2}$

$\Rightarrow \left(\frac{r}{x} - 1\right) = \frac{1}{\sqrt{2}} = \frac{1}{1.414} \Rightarrow \frac{r}{x} = 1.414 + 1 = 2.414$

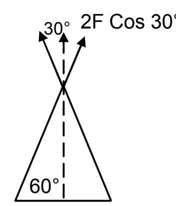
$\Rightarrow x = \frac{r}{2.414} = \frac{20}{2.414} = 8.285$ cm

36. $EF = \frac{KQ}{r^2}$



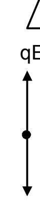
$5 \text{ N/C} = \frac{9 \times 10^9 \times Q}{4^2}$

$\Rightarrow \frac{4 \times 20 \times 10^{-2}}{9 \times 10^9} = Q \Rightarrow Q = 8.88 \times 10^{-11}$



37. m = 10, $mg = 10 \times 10^{-3} g \times 10^{-3}$ kg, $q = 1.5 \times 10^{-6}$ C

But $qE = mg \Rightarrow (1.5 \times 10^{-6}) E = 10 \times 10^{-6} \times 10$

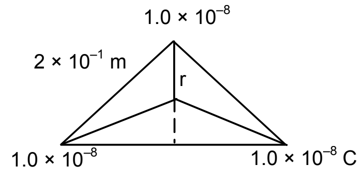


$\Rightarrow E = \frac{10 \times 10^{-4} \times 10}{1.5 \times 10^{-6}} = \frac{100}{1.5} = 66.6$ N/C

$= \frac{100 \times 10^3}{1.5} = \frac{10^{5+1}}{15} = 6.6 \times 10^3$

38. $q = 1.0 \times 10^{-8}$ C, $\ell = 20$ cm

$E = ?$ $V = ?$



Since it forms an equipotential surface.

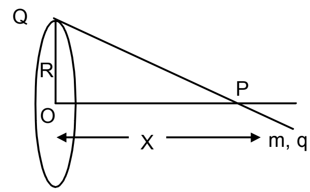
So the electric field at the centre is <u>Zero</u>.

$$r = \frac{2}{3}\sqrt{(2\times10^{-1})^2 - (10^{-1})^2} = \frac{2}{3}\sqrt{4\times10^{-2} - 10^{-2}}$$

$$= \frac{2}{3}\sqrt{10^{-2}(4-1)} = \frac{2}{3}\times10^{-2}\times1.732 = 1.15\times10^{-1}$$

$$V = \frac{3\times9\times10^{9}1\times10^{-8}}{1\times10^{-1}} = 23\times10^{2} = 2.3\times10^{3}\text{ V}$$

39. We know : Electric field 'E' at 'P' due to the charged ring



$$= \frac{KQx}{(R^2+x^2)^{3/2}} = \frac{KQx}{R^3}$$

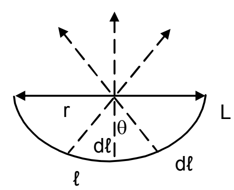
Force experienced 'F' = Q × E = $\frac{q\times K\times Qx}{R^3}$

Now, amplitude = x

$$\text{So, } T = 2\pi\sqrt{\frac{x}{KQqx/mR^3}} = 2\pi\sqrt{\frac{mR^3x}{KQqx}} = 2\pi\sqrt{\frac{4\pi\varepsilon_0 mR^3}{Qq}} = \sqrt{\frac{4\pi^2\times4\pi\varepsilon_0 mR^3}{qQ}}$$

$$\Rightarrow T = \left[\frac{16\pi^3\varepsilon_0 mR^3}{qQ}\right]^{1/2}$$

40. λ = Charge per unit length = $\frac{Q}{L}$



dq_1 for a length dl = λ × dl

Electric field at the centre due to charge = $k\times\frac{dq}{r^2}$

The horizontal Components of the Electric field balances each other. Only the vertical components remain.

∴ Net Electric field along vertical

$$d_E = 2\,E\cos\theta = \frac{Kdq\times\cos\theta}{r^2} = \frac{2kCos\theta}{r^2}\times\lambda\times dl \qquad \left[\text{but } d\theta = \frac{d\ell}{r} = d\ell = rd\theta\right]$$

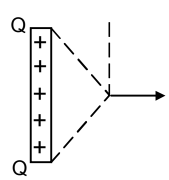
$$\Rightarrow \frac{2k\lambda}{r^2}Cos\theta\times rd\theta = \frac{2k\lambda}{r}Cos\theta\times d\theta$$

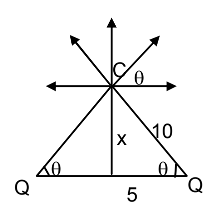
$$\text{or } E = \int_0^{\pi/2}\frac{2k\lambda}{r}Cos\theta\times d\theta = \int_0^{\pi/2}\frac{2k\lambda}{r}Sin\theta = \frac{2k\lambda l}{r} = \frac{2K\theta}{Lr}$$

but $L = \pi R \Rightarrow r = \frac{L}{\pi}$

$$\text{So } E = \frac{2k\theta}{L\times(L/\pi)} = \frac{2k\pi\theta}{L^2} = \frac{2}{4\pi\varepsilon_0}\times\frac{\pi\theta}{L^2} = \frac{\theta}{2\varepsilon_0 L^2}$$

41. $G = 50\ \mu C = 50\times10^{-6}$ C



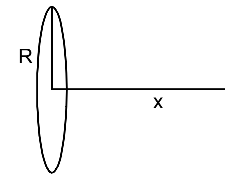


We have, E = $\frac{2KQ}{r}$ for a charged cylinder.

$$\Rightarrow E = \frac{2\times9\times10^{9}\times50\times10^{-6}}{5\sqrt{3}} = \frac{9\times10^{-5}}{5\sqrt{3}} = 1.03\times10^{-5}$$

42. Electric field at any point on the axis at a distance x from the center of the ring is

$$E = \frac{xQ}{4\pi\varepsilon_0(R^2+x^2)^{3/2}} = \frac{KxQ}{(R^2+x^2)^{3/2}}$$



Differentiating with respect to x

$$\frac{dE}{dx} = \frac{KQ(R^2+x^2)^{3/2} - KxQ(3/2)(R^2+x^2)^{11/2}2x}{(r^2+x^2)^3}$$

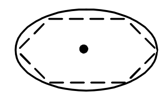
Since at a distance x, Electric field is maximum.

$$\frac{dE}{dx} = 0 \Rightarrow KQ\,(R^2+x^2)^{3/2} - Kx^2\,Q3(R^2+x^2)^{1/2} = 0$$

$$\Rightarrow KQ\,(R^2+x^2)^{3/2} = Kx^2\,Q3(R^2+x^2)^{1/2} \Rightarrow R^2 + x^2 = 3x^2$$

$$\Rightarrow 2x^2 = R^2 \Rightarrow x^2 = \frac{R^2}{2} \Rightarrow x = \frac{R}{\sqrt{2}}$$

43. Since it is a regular hexagon. So, it forms an equipotential surface. Hence the charge at each point is equal. Hence the net entire field at the centre is Zero.



44. Charge/Unit length = $\frac{Q}{2\pi a} = \lambda$; Charge of $d\ell = \frac{Qd\ell}{2\pi a}C$

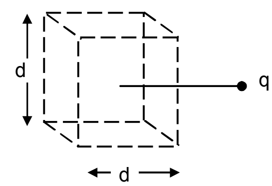
Initially the electric field was '0' at the centre. Since the element '$d\ell$' is removed so, net electric field must $\frac{K\times q}{a^2}$ Where q = charge of element $d\ell$

$$E = \frac{Kq}{a^2} = \frac{1}{4\pi\varepsilon_0}\times\frac{Qd\ell}{2\pi a}\times\frac{1}{a^2} = \frac{Qd\ell}{8\pi^2\varepsilon_0 a^3}$$

45. We know,

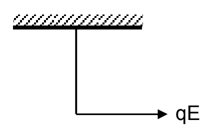
Electric field at a point due to a given charge

'E' = $\frac{Kq}{r^2}$ Where q = charge, r = Distance between the point and the charge



So, 'E' = $\frac{1}{4\pi\varepsilon_0}\times\frac{q}{d^2}$ [$\therefore$ r = 'd' here]

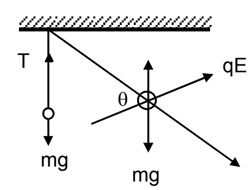
46. E = 20 kv/m = 20 × 10^3 v/m, m = 80 × 10^{-5} kg, c = 20 × 10^{-5} C



$$\tan\theta = \left(\frac{qE}{mg}\right)^{-1} \quad [\,T\sin\theta = mg,\ \ T\cos\theta = qe]$$

$$\tan\theta = \left(\frac{2\times10^{-8}\times20\times10^3}{80\times10^{-6}\times10}\right)^{-1} = \left(\frac{1}{2}\right)^{-1}$$

$$1+\tan^2\theta = \frac{1}{4}+1 = \frac{5}{4} \qquad [\cos\theta = \frac{1}{\sqrt{5}},\ \sin\theta = \frac{2}{\sqrt{5}}]$$



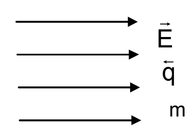
$$T\sin\theta = mg \Rightarrow T\times\frac{2}{\sqrt{5}} = 80\times10^{-6}\times10$$

$$\Rightarrow T = \frac{8\times10^{-4}\times\sqrt{5}}{2} = 4\times\sqrt{5}\times10^{-4} = 8.9\times10^{-4}$$

47. Given

u = Velocity of projection, $\vec{E}$ = Electric field intensity

q = Charge; m = mass of particle



We know, Force experienced by a particle with charge 'q' in an electric field $\vec{E}$ = qE

$\therefore$ acceleration produced = $\frac{qE}{m}$

As the particle is projected against the electric field, hence deceleration = $\frac{qE}{m}$

So, let the distance covered be 's'

Then, $v^2 = u^2 + 2as$ [where a = acceleration, v = final velocity]

Here $0 = u^2 - 2 \times \frac{qE}{m} \times S \Rightarrow S = \frac{u^2 m}{2qE}$ units

48. $m = 1\ g = 10^{-3}$ kg, $u = 0$, $q = 2.5 \times 10^{-4}$ C ; $E = 1.2 \times 10^4$ N/c ; $S = 40$ cm $= 4 \times 10^{-1}$ m

a) $F = qE = 2.5 \times 10^{-4} \times 1.2 \times 10^4 = 3$ N

So, $a = \frac{F}{m} = \frac{3}{10^{-3}} = 3 \times 10^3$

$E_q = mg = 10^{-3} \times 9.8 = 9.8 \times 10^{-3}$ N

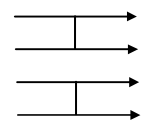
b) $S = \frac{1}{2}at^2$ or $t = \sqrt{\frac{2a}{g}} = \sqrt{\frac{2 \times 4 \times 10^{-1}}{3 \times 10^3}} = 1.63 \times 10^{-2}$ sec

$v^2 = u^2 + 2as = 0 + 2 \times 3 \times 10^3 \times 4 \times 10^{-1} = 24 \times 10^2 \Rightarrow v = \sqrt{24 \times 10^2} = 4.9 \times 10 = 49$ m/sec

work done by the electric force $w = F \rightarrow td = 3 \times 4 \times 10^{-1} = 12 \times 10^{-1} = 1.2$ J

49. $m = 100$ g, $q = 4.9 \times 10^{-5}$, $F_g = mg$, $F_e = qE$

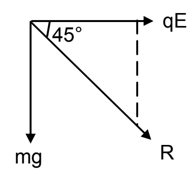
$\vec{E} = 2 \times 10^4$ N/C



So, the particle moves due to the et resultant R

$R = \sqrt{F_g^2 + F_e^2} = \sqrt{(0.1 \times 9.8)^2 + (4.9 \times 10^{-5} \times 2 \times 10^4)^2}$

$= \sqrt{0.9604 + 96.04 \times 10^{-2}} = \sqrt{1.9208} = 1.3859$ N



$\tan\theta = \frac{F_g}{F_e} = \frac{mg}{qE} = 1$ So, $\theta = 45°$

$\therefore$ Hence path is straight along resultant force at an angle 45° with horizontal

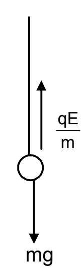
Disp. Vertical = $(1/2) \times 9.8 \times 2 \times 2 = 19.6$ m

Disp. Horizontal = $S = (1/2)\ at^2 = \frac{1}{2} \times \frac{qE}{m} \times t^2 = \frac{1}{2} \times \frac{0.98}{0.1} \times 2 \times 2 = 19.6$ m

Net Dispt. = $\sqrt{(19.6)^2 + (19.6)^2} = \sqrt{768.32} = 27.7$ m

50. $m = 40$ g, $q = 4 \times 10^{-6}$ C

Time for 20 oscillations = 45 sec. Time for 1 oscillation = $\frac{45}{20}$ sec



When no electric field is applied, $T = 2\pi\sqrt{\frac{\ell}{g}} \Rightarrow \frac{45}{20} = 2\pi\sqrt{\frac{\ell}{10}}$

$\Rightarrow \frac{\ell}{10} = \left(\frac{45}{20}\right)^2 \times \frac{1}{4\pi^2} \Rightarrow \ell = \frac{(45)^2 \times 10}{(20)^2 \times 4\pi^2} = 1.2836$

When electric field is not applied,

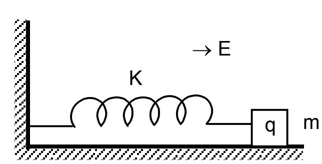
$T = 2\pi\sqrt{\frac{\ell}{g-a}}$ $[a = \frac{qE}{m} = 2.5] = 2\pi\sqrt{\frac{1.2836}{10-2.5}} = 2.598$

Time for 1 oscillation = 2.598

Time for 20 oscillation = $2.598 \times 20 = 51.96$ sec ≈ 52 sec.

51. $F = qE$, $F = -Kx$

Where x = amplitude

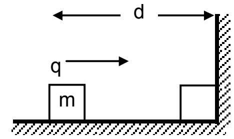


$qE = -Kx$ or $x = \frac{-qE}{K}$

52. The block does not undergo. SHM since here the acceleration is not proportional to displacement and not always opposite to displacement. When the block is going towards the wall the acceleration is along displacement and when going away from it the displacement is opposite to acceleration.

Time taken to go towards the wall is the time taken to goes away from it till velocity is

d = ut + (1/2) at^2



$\Rightarrow d = \frac{1}{2} \times \frac{qE}{m} \times t^2$

$\Rightarrow t^2 = \frac{2dm}{qE} \Rightarrow t = \sqrt{\frac{2md}{qE}}$

∴ Total time taken for to reach the wall and com back (Time period)

$= 2t = 2\sqrt{\frac{2md}{qE}} = \sqrt{\frac{8md}{qE}}$

53. E = 10 n/c, S = 50 cm = 0.1 m

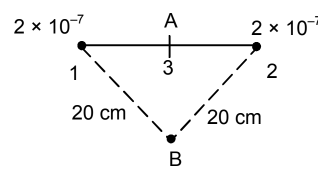
$E = \frac{dV}{dr}$ or, V = E × r = 10 × 0.5 = 5 cm

54. Now, $V_B - V_A$ = Potential diff = ? Charge = 0.01 C

Work done = 12 J Now, Work done = Pot. Diff × Charge

$\Rightarrow$ Pot. Diff = $\frac{12}{0.01}$ = 1200 Volt

55. When the charge is placed at A,



$E_1 = \frac{Kq_1q_2}{r} + \frac{Kq_3q_4}{r}$

$= \frac{9\times10^9(2\times10^{-7})^2}{0.1} + \frac{9\times10^9(2\times10^{-7})^2}{0.1}$

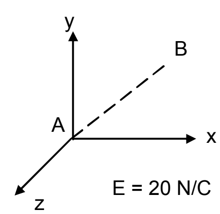
$= \frac{2\times9\times10^9\times4\times10^{-14}}{0.1} = 72 \times 10^{-4}$ J

When charge is placed at B,

$E_2 = \frac{Kq_1q_2}{r} + \frac{Kq_3q_4}{r} = \frac{2\times9\times10^9\times4\times10^{-14}}{0.2} = 36 \times 10^{-4}$ J

Work done = $E_1 - E_2 = (72 - 36) \times 10^{-4} = 36 \times 10^{-4}$ J = 3.6×10^{-3} J

56. (a) A = (0, 0) B = (4, 2)



$V_B - V_A = E \times d = 20 \times \sqrt{16} = 80$ V

(b) A(4m, 2m), B = (6m, 5m)

$\Rightarrow V_B - V_A = E \times d = 20 \times \sqrt{(6-4)^2} = 20 \times 2 = 40$ V

(c) A(0, 0) B = (6m, 5m)

$\Rightarrow V_B - V_A = E \times d = 20 \times \sqrt{(6-0)^2} = 20 \times 6 = 120$ V.

57. (a) The Electric field is along x-direction

Thus potential difference between (0, 0) and (4, 2) is,

$\delta V = -E \times \delta x = -20 \times (40) = -80$ V

Potential energy $(U_B - U_A)$ between the points = $\delta V \times q$

$= -80 \times (-2) \times 10^{-4} = 160 \times 10^{-4} = 0.016$ J.

(b) A = (4m, 2m) B = (6m, 5m)

$\delta V = -E \times \delta x = -20 \times 2 = -40$ V

Potential energy $(U_B - U_A)$ between the points = $\delta V \times q$

$= -40 \times (-2 \times 10^{-4}) = 80 \times 10^{-4} = 0.008$ J

(c) A = (0, 0) B = (6m, 5m)

$\delta V = -E \times \delta x = -20 \times 6 = -120$ V

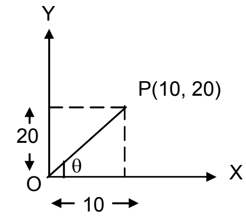
Potential energy $(U_B - U_A)$ between the points A and B

$= \delta V \times q = -120 \times (-2 \times 10^{-4}) = 240 \times 10^{-4} = 0.024$ J

58. E = $\left(\hat{i}20 + \hat{j}30\right)$ N/CV = at (2m, 2m) r = (2i + 2j)

So, V= $-\vec{E}\times\vec{r}$ = –(i20 + 30J) (2 $\hat{i}$ + 2j) = –(2 × 20 + 2× 30) = – 100 V

59. E = $\vec{i}$ × Ax = 100 $\vec{i}$



$\int_v^0 dv = -\int E \times d\ell$ V = $-\int_0^{10} 10x \times dx = -\int_0^{10} \frac{1}{2} \times 10 \times x^2$

0 – V = $-\left[\frac{1}{2}\times 1000\right]$ = – 500 ⇒ V = 500 Volts

60. V(x, y, z) = A(xy + yz + zx)

(a) A = $\frac{\text{Volt}}{m^2} = \frac{ML^2T^{-2}}{ITL^2}$ = $[MT^{-3}I^{-1}]$

(b) E = $-\frac{\delta V\hat{i}}{\delta x} - \frac{\delta V\hat{j}}{\delta y} - \frac{\delta V\hat{k}}{\delta z} = -\left[\frac{\delta}{\delta x}[A(xy+yz+zx) + \frac{\delta}{\delta y}[A(xy+yz+zx) + \frac{\delta}{\delta z}[A(xy+yz+zx)\right]$

= $-\left[(Ay+Az)\hat{i} + (Ax+Az)\hat{j} + (Ay+Ax)\hat{k}\right] = -A(y+z)\hat{i} + A(x+z)\hat{j} + A(y+x)\hat{k}$

(c) A = 10 SI unit, r = (1m, 1m, 1m)

E = –10(2) $\hat{i}$ – 10(2) $\hat{j}$ – 10(2) $\hat{k}$ = – 20 $\hat{i}$ – 20 $\hat{j}$ – 20 $\hat{k}$ = $\sqrt{2o^2 + 20^2 + 20^2}$ = $\sqrt{1200}$ = 34.64 ≈ 35 N/C

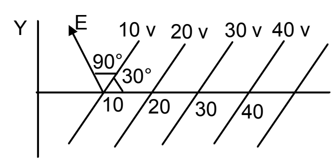
61. $q_1 = q_2 = 2 \times 10^{-5}$ C

Each are brought from infinity to 10 cm a part d = 10×10^{-2} m

So work done = negative of work done. (Potential E)

P.E = $\int_{\infty}^{10} F \times ds$ P.E. = $K \times \frac{q_1 q_2}{r} = \frac{9\times 10^9 \times 4 \times 10^{-10}}{10 \times 10^{-2}}$ = 36 J

62. (a) The angle between potential E dℓ = dv



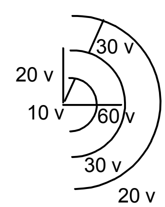
Change in potential = 10 V = dV

As E = ⊥r dV (As potential surface)

So, E dℓ = dV ⇒ E dℓ Cos(90° + 30°) = – dv

⇒ E(10× 10^{-2}) cos 120° = – dV

⇒ E = $\frac{-dV}{10\times 10^{-2}\text{Cos}120°} = -\frac{10}{10^{-1}\times(-1/2)}$ = 200 V/m making an angle 120° with y-axis

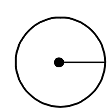


(b) As Electric field intensity is ⊥r to Potential surface

So, E = $\frac{kq}{r^2}r = \frac{kq}{r} \Rightarrow \frac{kq}{r}$ = 60 v q = $\frac{6}{K}$

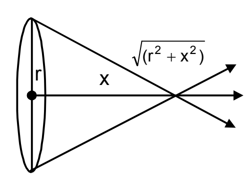
So, E = $\frac{kq}{r^2} = \frac{6\times k}{k \times r^2}$ v.m = $\frac{6}{r^2}$ v.m

63. Radius = r So, 2πr = Circumference



Charge density = λ Total charge = 2πr × λ

Electric potential = $\frac{Kq}{r} = \frac{1}{4\pi\varepsilon_0} \times \frac{2\pi r\lambda}{(x^2+r^2)^{1/2}} = \frac{r\lambda}{2\varepsilon_0(x^2+r^2)^{1/2}}$



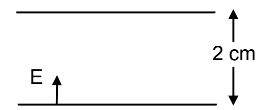
So, Electric field = $\frac{V}{r}\text{Cos}\theta$

= $\frac{r\lambda}{2\varepsilon_0(x^2+r^2)^{1/2}} \times \frac{1}{(x^2+r^2)^{1/2}}$

= $\frac{r\lambda}{2\varepsilon_0(x^2+r^2)^{1/2}} \times \frac{x}{(x^2+r^2)^{1/2}} = \frac{r\lambda x}{2\varepsilon_0(x^2+r^2)^{3/2}}$

64. $\vec{E}$ = 1000 N/C

(a) V = E × dℓ = $1000 \times \frac{2}{100}$ = 20 V



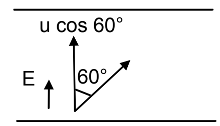
(b) u = ? $\vec{E}$ = 1000, = 2/100 m

$a = \frac{F}{m} = \frac{q \times E}{m} = \frac{1.6 \times 10^{-19} \times 1000}{9.1 \times 10^{-31}}$ = 1.75 × 10^{14} m/s^2

0 = u^2 –2 × 1.75 × 10^{14} × 0.02 ⇒ u^2 = 0.04 × 1.75 × 10^{14} ⇒ u = 2.64 × 10^6 m/s.

(c) Now, U = u Cos 60° V = 0, s = ?

a = 1.75 × 10^{14} m/s^2 $V^2 = u^2 - 2as$



$\Rightarrow s = \frac{(u\text{Cos}60°)^2}{2 \times a} = \frac{\left(2.64 \times 10^6 \times \frac{1}{2}\right)^2}{2 \times 1.75 \times 10^{14}} = \frac{1.75 \times 10^{12}}{3.5 \times 10^{14}}$ = 0.497 × 10^{-2} ≈ 0.005 m ≈ 0.50 cm

65. E = 2 N/C in x-direction

(a) Potential aat the origin is O. dV = – E_x dx – E_y dy – E_z dz

⇒ V – 0 = – 2x ⇒ V = – 2x

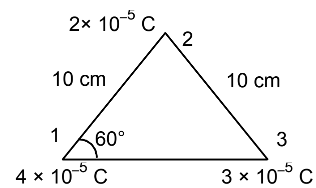
(b) (25 – 0) = – 2x ⇒ x = – 12.5 m

(c) If potential at origin is 100 v, v – 100 = – 2x ⇒ V = – 2x + 100 = 100 – 2x

(d) Potential at ∞ IS 0, V – V′ = – 2x ⇒ V′ = V + 2x = 0 + 2∞ ⇒ V′ = ∞

Potential at origin is ∞. No, it is not practical to take potential at ∞ to be zero.

66. Amount of work done is assembling the charges is equal to the net potential energy



So, P.E. = $U_{12} + U_{13} + U_{23}$

$= \frac{Kq_1q_2}{r_{12}} + \frac{Kq_1q_3}{r_{13}} + \frac{Kq_2q_3}{r_{23}} = \frac{K \times 10^{-10}}{r}[4 \times 2 + 4 \times 3 + 3 \times 2]$

$= \frac{9 \times 10^9 \times 10^{-10}}{10^{-1}}(8 + 12 + 6)$ = 9 × 26 = 234 J

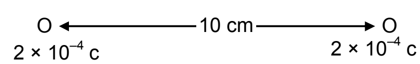
67. K.C. decreases by 10 J. Potential = 100 v to 200 v.

So, change in K.E = amount of work done

⇒ 10J = (200 – 100) v × q_0 ⇒ 100 q_0 = 10 v

$\Rightarrow q_0 = \frac{10}{100}$ = 0.1 C

68. m = 10 g; $F = \frac{KQ}{r} = \frac{9 \times 10^9 \times 2 \times 10^{-4}}{10 \times 10^{-2}}$ F = 1.8 × 10^{-7}



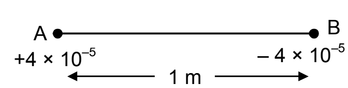
F = m × a ⇒ $a = \frac{1.8 \times 10^{-7}}{10 \times 10^{-3}}$ = 1.8 × 10^{-3} m/s^2

$V^2 - u^2 = 2as \Rightarrow V^2 = u^2 + 2as$

$V = \sqrt{0 + 2 \times 1.8 \times 10^{-3} \times 10 \times 10^{-2}} = \sqrt{3.6 \times 10^{-4}}$ = 0.6 × 10^{-2} = 6 × 10^{-3} m/s.

69. $q_1 = q_2$ = 4 × 10^{-5}; s = 1m, m = 5 g = 0.005 kg

$F = K\frac{q^2}{r^2} = \frac{9 \times 10^9 \times (4 \times 10^{-5})^2}{1^2}$ = 14.4 N



Acceleration ‘a’ = $\frac{F}{m} = \frac{14.4}{0.005}$ = 2880 m/s^2

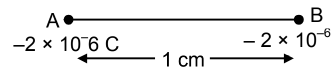
Now u = 0, s = 50 cm = 0.5 m, a = 2880 m/s^2, V =?

$V^2 = u^2 + 2as \Rightarrow V^2$ = = 2 × 2880 × 0.5

$\Rightarrow V = \sqrt{2880}$ = 53.66 m/s ≈ 54 m/s for each particle

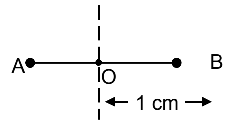
70. E = 2.5 × 104 P = 3.4×10^{-30} τ = PE sin θ
= P × E × 1 = $3.4 \times 10^{-30} \times 2.5 \times 10^{4} = 8.5 \times 10^{-26}$

71. (a) Dipolemoment = q × ℓ



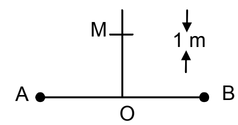
(Where q = magnitude of charge ℓ = Separation between the charges)
= $2 \times 10^{-6} \times 10^{-2}$ cm = 2×10^{-8} cm

(b) We know, Electric field at an axial point of the dipole



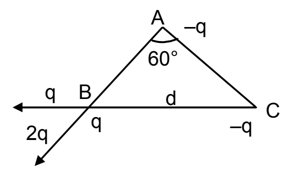
$$= \frac{2KP}{r^3} = \frac{2 \times 9 \times 10^9 2 \times 10^{-8}}{(1 \times 10^{-2})^3} = 36 \times 10^7 \text{ N/C}$$

(c) We know, Electric field at a point on the perpendicular bisector about 1m away from centre of dipole.



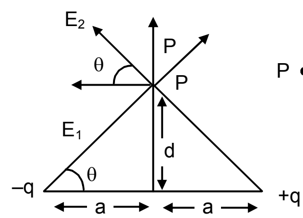
$$= \frac{KP}{r^3} = \frac{9 \times 10^9 2 \times 10^{-8}}{1^3} = 180 \text{ N/C}$$

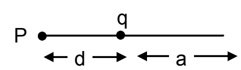
72. Let –q & –q are placed at A & C
Where 2q on B So length of A = d
So the dipole moment = (q × d) = P
So, Resultant dipole moment



P= $[(qd)^2 + (qd)^2 + 2qd \times qd \text{ Cos } 60°]^{1/2} = [3\,q^2d^2]^{1/2} = \sqrt{3}\,qd = \sqrt{3}\,P$

73. (a) P = 2qa
(b) E_1 Sin θ = E_2 sin θ Electric field intensity
= E_1 Cos θ + E_2 Cos θ = 2 E_1 Cos θ

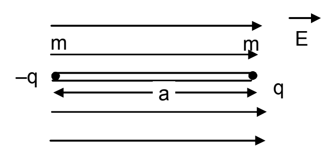




$$E_1 = \frac{Kqp}{a^2 + d^2} \text{ so } E = \frac{2KPQ}{a^2 + d^2}\frac{a}{(a^2 + d^2)^{1/2}} = \frac{2Kq \times a}{(a^2 + d^2)^{3/2}}$$

When a << d $$= \frac{2Kqa}{(d^2)^{3/2}} = \frac{PK}{d^3} = \frac{1}{4\pi\varepsilon_0}\frac{P}{d^3}$$

74. Consider the rod to be a simple pendulum.



For simple pendulum T = $2\pi\sqrt{\ell / g}$ (ℓ = length, q = acceleration)
Now, force experienced by the charges

F = Eq Now, acceleration = $\frac{F}{m} = \frac{Eq}{m}$

Hence length = a so, Time period = $2\pi\sqrt{\frac{a}{(Eq/m)}} = 2\pi\sqrt{\frac{ma}{Eq}}$

75. 64 grams of copper have 1 mole 6.4 grams of copper have 0.1 mole
1 mole = No atoms 0.1 mole = (no × 0.1) atoms
= $6 \times 10^{23} \times 0.1$ atoms = 6×10^{22} atoms
1 atom contributes 1 electron 6×10^{22} atoms contributes 6×10^{22} electrons.

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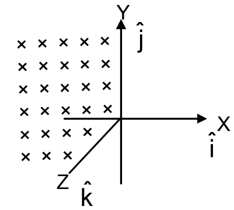
CHAPTER – 30
GAUSS'S LAW

1. Given : $\vec{E}$ = 3/5 E_0 $\hat{i}$ + 4/5 E_0 $\hat{j}$

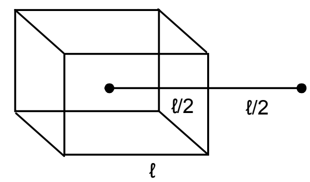


E_0 = 2.0 × 10^3 N/C The plane is parallel to yz-plane.

Hence only 3/5 E_0 $\hat{i}$ passes perpendicular to the plane whereas 4/5 E_0 $\hat{j}$ goes parallel. Area = 0.2m^2 (given)

∴ Flux = $\vec{E} + \vec{A}$ = 3/5 × 2 × 10^3 × 0.2 = 2.4 × 10^2 Nm^2/c = 240 Nm^2/c

2. Given length of rod = edge of cube = ℓ



Portion of rod inside the cube = ℓ/2

Total charge = Q.

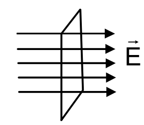
Linear charge density = λ = Q/ℓ of rod.

We know: Flux α charge enclosed.

Charge enclosed in the rod inside the cube.

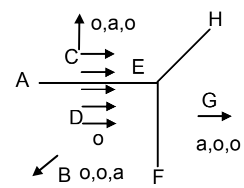
= ℓ/2 ε_0 × Q/ℓ = Q/2 ε_0

3. As the electric field is uniform.



Considering a perpendicular plane to it, we find that it is an equipotential surface. Hence there is no net current flow on that surface. Thus, net charge in that region is zero.

4. Given: E = $\frac{E_0\chi}{\ell}\hat{i}$ ℓ= 2 cm, a = 1cm.



E_0 = 5 × 10^3 N/C. From fig. We see that flux passes mainly through surface areas. ABDC & EFGH. As the AEFB & CHGD are paralled to the Flux. Again in ABDC a = 0; hence the Flux only passes through the surface are EFGH.

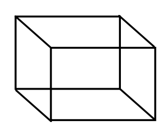
E = $\frac{E_c x}{\ell}\hat{i}$

Flux = $\frac{E_0\chi}{L}$ × Area = $\frac{5\times10^3\times a}{\ell}\times a^2$ = $\frac{5\times10^3\times a^3}{\ell}$ = $\frac{5\times10^3\times(0.01)^{-3}}{2\times10^{-2}}$ = 2.5 × 10^{-1}

Flux = $\frac{q}{\varepsilon_0}$ so, q = ε_0 × Flux

= 8.85 × 10^{-12} × 2.5 × 10^{-1} = 2.2125 × 10^{-12} c

5. According to Gauss's Law Flux = $\frac{q}{\varepsilon_0}$



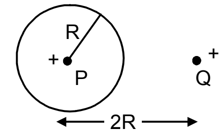
Since the charge is placed at the centre of the cube. Hence the flux passing through the six surfaces = $\frac{Q}{6\varepsilon_0}$ × 6 = $\frac{Q}{\varepsilon_0}$

6. Given – A charge is placed o a plain surface with area = a^2, about a/2 from its centre.

Assumption : let us assume that the given plain forms a surface of an imaginary cube. Then the charge is found to be at the centre of the cube.

Hence flux through the surface = $\frac{Q}{\varepsilon_0}\times\frac{1}{6}$ = $\frac{Q}{6\varepsilon_0}$

7. Given: Magnitude of the two charges placed = 10^{-7}c.

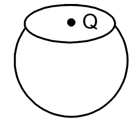


We know: from Gauss's law that the flux experienced by the sphere is only due to the internal charge and not by the external one.

Now $\oint \vec{E}.\vec{ds} = \frac{Q}{\varepsilon_0}$ = $\frac{10^{-7}}{8.85\times10^{-12}}$ = 1.1 × 10^4 N-m^2/C.

8. We know: For a spherical surface

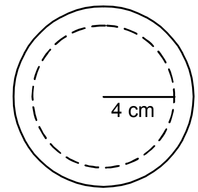
Flux = $\oint \vec{E}.ds = \frac{q}{\varepsilon_0}$ [by Gauss law]



Hence for a hemisphere = total surface area = $\frac{q}{\varepsilon_0} \times \frac{1}{2}$ = $\frac{q}{2\varepsilon_0}$

9. Given: Volume charge density = 2.0 × 10^{-4} c/m^3

In order to find the electric field at a point 4cm = 4 × 10^{-2} m from the centre let us assume a concentric spherical surface inside the sphere.



Now, $\oint E.ds = \frac{q}{\varepsilon_0}$

But σ = $\frac{q}{4/3\pi R^3}$ so, q = σ × 4/3 π R^3

Hence = $\frac{\sigma \times 4/3 \times 22/7 \times (4 \times 10^{-2})^3}{\varepsilon_0} \times \frac{1}{4 \times 22/7 \times (4 \times 10^{-2})^2}$

= 2.0 × 10^{-4} 1/3 × 4 × 10^{-2} × $\frac{1}{8.85 \times 10^{-12}}$ = 3.0 × 10^5 N/C

10. Charge present in a gold nucleus = 79 × 1.6 × 10^{-19} C

Since the surface encloses all the charges we have:

(a) $\oint \vec{E}.\overrightarrow{ds} = \frac{q}{\varepsilon_0} = \frac{79 \times 1.6 \times 10^{-19}}{8.85 \times 10^{-12}}$

E = $\frac{q}{\varepsilon_0 ds}$ = $\frac{79 \times 1.6 \times 10^{-19}}{8.85 \times 10^{-12}} \times \frac{1}{4 \times 3.14 \times (7 \times 10^{-15})^2}$ [$\therefore$area = $4\pi r^2$]

= 2.3195131 × 10^{21} N/C

(b) For the middle part of the radius. Now here r = 7/2 × 10^{-15}m

Volume = 4/3 π r^3 = $\frac{48}{3} \times \frac{22}{7} \times \frac{343}{8} \times 10^{-45}$

Charge enclosed = ζ × volume [ζ : volume charge density]

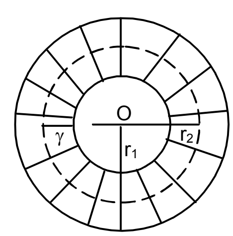
But ζ= $\frac{\text{Net charge}}{\text{Net volume}}$ = $\frac{7.9 \times 1.6 \times 10^{-19} c}{\left(\frac{4}{3}\right) \times \pi \times 343 \times 10^{-45}}$

Net charged enclosed = $\frac{7.9 \times 1.6 \times 10^{-19}}{\left(\frac{4}{3}\right) \times \pi \times 343 \times 10^{-45}} \times \frac{4}{3}\pi \times \frac{343}{8} \times 10^{-45}$ = $\frac{7.9 \times 1.6 \times 10^{-19}}{8}$

$\oint \vec{E}\overrightarrow{ds}$ = $\frac{q \text{ enclosed}}{\varepsilon_0}$

$\Rightarrow$ E = $\frac{7.9 \times 1.6 \times 10^{-19}}{8 \times \varepsilon_0 \times S}$ = $\frac{7.9 \times 1.6 \times 10^{-19}}{8 \times 8.85 \times 10^{-12} \times 4\pi \times \frac{49}{4} \times 10^{-30}}$ = 1.159 × 10^{21}N/C

11. Now, Volume charge density = $\frac{Q}{\frac{4}{3} \times \pi \times (r_2^{\,3} - r_1^{\,3})}$



$\therefore$ ζ = $\frac{3Q}{4\pi(r_2^{\,3} - r_1^{\,3})}$

Again volume of sphere having radius x = $\frac{4}{3}\pi x^3$

Now charge enclosed by the sphere having radius

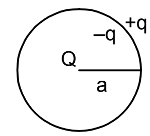
$$\chi = \left(\frac{4}{3}\pi\chi^3 - \frac{4}{3}\pi r_1^{\,3}\right)\times\frac{Q}{\frac{4}{3}\pi r_2^{\,3} - \frac{4}{3}\pi r_1^{\,3}} = Q\left(\frac{\chi^3 - r_1^{\,3}}{r_2^{\,3} - r_1^{\,3}}\right)$$

Applying Gauss's law – $E \times 4\pi\chi^2 = \frac{q\ \text{enclosed}}{\varepsilon_0}$

$$\Rightarrow E = \frac{Q}{\varepsilon_0}\left(\frac{\chi^3 - r_1^{\,3}}{r_2^{\,3} - r_1^{\,3}}\right)\times\frac{1}{4\pi\chi^2} = \frac{Q}{4\pi\varepsilon_0\chi^2}\left(\frac{\chi^3 - r_1^{\,3}}{r_2^{\,3} - r_1^{\,3}}\right)$$

12. Given: The sphere is uncharged metallic sphere.

Due to induction the charge induced at the inner surface = –Q, and that outer surface = +Q.



(a) Hence the surface charge density at inner and outer surfaces = $\frac{\text{charge}}{\text{total surface area}}$

$= -\frac{Q}{4\pi a^2}$ and $\frac{Q}{4\pi a^2}$ respectively.

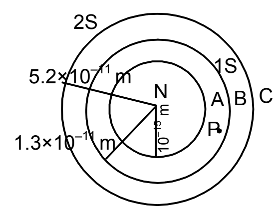
(b) Again if another charge 'q' is added to the surface. We have inner surface charge density = $-\frac{Q}{4\pi a^2}$, because the added charge does not affect it.

On the other hand the external surface charge density = $Q + \frac{q}{4\pi a^2}$ as the 'q' gets added up.

(c) For electric field let us assume an imaginary surface area inside the sphere at a distance 'x' from centre. This is same in both the cases as the 'q' in ineffective.

Now, $\oint E.ds = \frac{Q}{\varepsilon_0}$ So, $E = \frac{Q}{\varepsilon_0}\times\frac{1}{4\pi x^2} = \frac{Q}{4\pi\varepsilon_0 x^2}$

13. (a) Let the three orbits be considered as three concentric spheres A, B & C.



Now, Charge of 'A' = $4 \times 1.6 \times 10^{-16}$ c

Charge of 'B' = $2 \times 1.6 \times 10^{-16}$ c

Charge of 'C' = $2 \times 1.6 \times 10^{-16}$ c

As the point 'P' is just inside 1s, so its distance from centre = 1.3×10^{-11} m

Electric field = $\frac{Q}{4\pi\varepsilon_0 x^2} = \frac{4\times1.6\times10^{-19}}{4\times3.14\times8.85\times10^{-12}\times(1.3\times10^{-11})^2} = 3.4 \times 10^{13}$ N/C

(b) For a point just inside the 2 s cloud

Total charge enclosed = $4 \times 1.6 \times 10^{-19} - 2 \times 1.6 \times 10^{-19} = 2 \times 1.6 \times 10^{-19}$

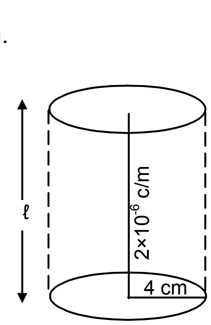
Hence, Electric filed,

$$\vec{E} = \frac{2\times1.6\times10^{-19}}{4\times3.14\times8.85\times10^{-12}\times(5.2\times10^{-11})^2} = 1.065 \times 10^{12}\ \text{N/C} \approx 1.1 \times 10^{12}\ \text{N/C}$$

14. Drawing an electric field around the line charge we find a cylinder of radius 4×10^{-2} m.

Given: λ = linear charge density

Let the length be $\ell = 2 \times 10^{-6}$ c/m



We know $\oint E.dl = \frac{Q}{\varepsilon_0} = \frac{\lambda\ell}{\varepsilon_0}$

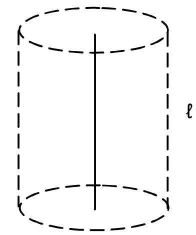
$\Rightarrow E \times 2\pi r \ell = \frac{\lambda\ell}{\varepsilon_0} \Rightarrow E = \frac{\lambda}{\varepsilon_0 \times 2\pi r}$

For, $r = 2 \times 10^{-2}$ m & $\lambda = 2 \times 10^{-6}$ c/m

$$\Rightarrow E = \frac{2\times10^{-6}}{8.85\times10^{-12}\times2\times3.14\times2\times10^{-2}} = 8.99 \times 10^5\ \text{N/C} \approx 9 \times 10^5\ \text{N/C}$$

15. Given :
$\lambda = 2 \times 10^{-6}$ c/m
For the previous problem.
$E = \dfrac{\lambda}{\epsilon_0\ 2\pi r}$ for a cylindrical electricfield.

Now, For experienced by the electron due to the electric filed in wire = centripetal force.

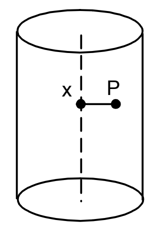


$Eq = mv^2$ $\left[\begin{array}{l}\text{we know}, m_e = 9.1\times10^{-31}\text{kg},\\ v_e = ?,\ r = \text{assumed radius}\end{array}\right]$

$\Rightarrow \dfrac{1}{2}\ Eq = \dfrac{1}{2}\dfrac{mv^2}{r}$

$\Rightarrow$ KE = 1/2 × E × q × r = $\dfrac{1}{2} \times \dfrac{\lambda}{\varepsilon_0 2\pi r} \times 1.6 \times 10^{-19} = 2.88 \times 10^{-17}$ J.

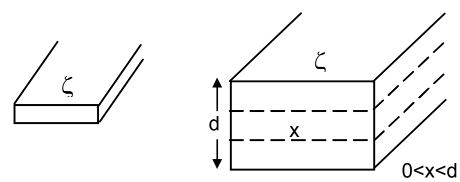
16. Given: Volume charge density = ζ
Let the height of cylinder be h.
$\therefore$ Charge Q at P = $\zeta \times 4\pi\chi^2 \times h$



For electric field $\oint E.ds = \dfrac{Q}{\varepsilon_0}$

$E = \dfrac{Q}{\varepsilon_0 \times ds} = \dfrac{\zeta \times 4\pi\chi^2 \times h}{\varepsilon_0 \times 2 \times \pi \times \chi \times h} = \dfrac{2\zeta\chi}{\varepsilon_0}$

17. $\oint E.dA = \dfrac{Q}{\varepsilon_0}$



Let the area be A.
Uniform change distribution density is ζ
$Q = \zeta A$
$E = \dfrac{Q}{\varepsilon_0} \times dA = \dfrac{\zeta \times a \times \chi}{\varepsilon_0 \times A} = \dfrac{\zeta\chi}{\varepsilon_0}$

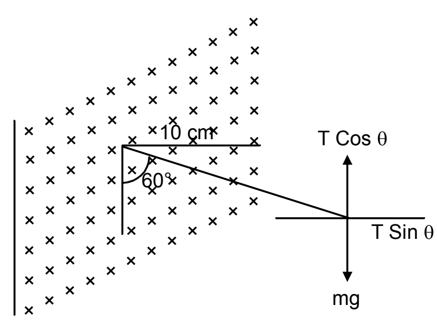
18. $Q = -2.0 \times 10^{-6}$C Surface charge density = 4×10^{-6} C/m^2

We know $\vec{E}$ due to a charge conducting sheet = $\dfrac{\sigma}{2\varepsilon_0}$

Again Force of attraction between particle & plate

$= Eq = \dfrac{\sigma}{2\varepsilon_0} \times q = \dfrac{4\times10^{-6}\times2\times10^{-6}}{2\times8\times10^{-12}} = 0.452$N

19. Ball mass = 10g
Charge = 4×10^{-6} c
Thread length = 10 cm
Now from the fig, T cosθ = mg
T sinθ = electric force



Electric force = $\dfrac{\sigma q}{2\varepsilon_0}$ (σ surface charge density)

T sinθ = $\dfrac{\sigma q}{2\varepsilon_0}$, T cosθ = mg

Tan θ = $\dfrac{\sigma q}{2mg\varepsilon_0}$

$\sigma = \dfrac{2mg\varepsilon_0 \tan\theta}{q} = \dfrac{2\times8.85\times10^{-12}\times10\times10^{-3}\times9.8\times1.732}{4\times10^{-6}} = 7.5 \times 10^{-7}$ C/m^2

20. (a) Tension in the string in Equilibrium

$T \cos 60° = mg$

$\Rightarrow T = \dfrac{mg}{\cos 60°} = \dfrac{10 \times 10^{-3} \times 10}{1/2} = 10^{-1} \times 2 = 0.20\ N$

(b) Straingtening the same figure.

Now the resultant for 'R'

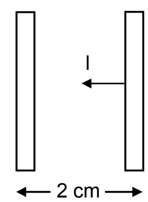
Induces the acceleration in the pendulum.

$T = 2 \times \pi \sqrt{\dfrac{\ell}{g}} = 2\pi \sqrt{\dfrac{\ell}{\left[g^2 + \left(\dfrac{\sigma q}{2\varepsilon_0 m}\right)^2\right]^{1/2}}} = 2\pi \sqrt{\dfrac{\ell}{\left[100 + \left(0.2 \times \dfrac{\sqrt{3}}{2 \times 10^{-2}}\right)^2\right]^{1/2}}}$

$= 2\pi \sqrt{\dfrac{\ell}{(100+300)^{1/2}}} = 2\pi \sqrt{\dfrac{\ell}{20}} = 2 \times 3.1416 \times \sqrt{\dfrac{10 \times 10^{-2}}{20}} = 0.45$ sec.

21. $s = 2cm = 2 \times 10^{-2}m$, $u = 0$, $a = ?$ $t = 2\mu s = 2 \times 10^{-6}s$

Acceleration of the electron, $s = (1/2)\, at^2$



$2 \times 10^{-2} = (1/2) \times a \times (2 \times 10^{-6})^2 \Rightarrow a = \dfrac{2 \times 2 \times 10^{-2}}{4 \times 10^{-12}} \Rightarrow a = 10^{10}\ m/s^2$

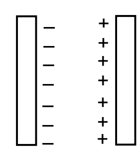
The electric field due to charge plate $= \dfrac{\sigma}{\varepsilon_0}$

Now, electric force $= \dfrac{\sigma}{\varepsilon_0} \times q$ = acceleration $= \dfrac{\sigma}{\varepsilon_0} \times \dfrac{q}{m_e}$

Now $\dfrac{\sigma}{\varepsilon_0} \times \dfrac{q}{m_e} = 10^{10}$

$\Rightarrow \sigma = \dfrac{10^{10} \times \varepsilon_0 \times m_e}{q} = \dfrac{10^{10} \times 8.85 \times 10^{-12} \times 9.1 \times 10^{-31}}{1.6 \times 10^{-19}}$

$= 50.334 \times 10^{-14} = 0.50334 \times 10^{-12}\ c/m^2$

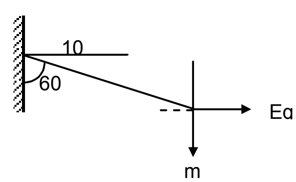


22. Given: Surface density = σ

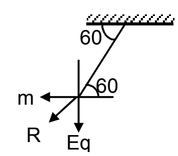
(a) & (c) For any point to the left & right of the dual plater, the electric field is zero.

As there are no electric flux outside the system.

(b) For a test charge put in the middle.



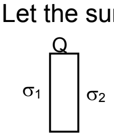
It experiences a fore $\dfrac{\sigma q}{2\varepsilon_0}$ towards the (-ve) plate.



Hence net electric field $\dfrac{1}{q}\left(\dfrac{\sigma q}{2\varepsilon_0} + \dfrac{\sigma q}{2\varepsilon_0}\right) = \dfrac{\sigma}{\varepsilon_0}$

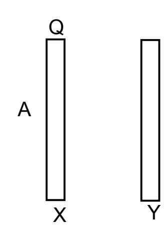
23. (a) For the surface charge density of a single plate.

Let the surface charge density at both sides be σ_1 & σ_2



= Now, electric field at both ends.

$= \dfrac{\sigma_1}{2\varepsilon_0}$ & $\dfrac{\sigma_2}{2\varepsilon_0}$

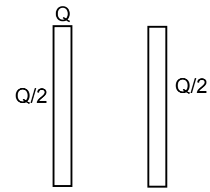


Due to a net balanced electric field on the plate $\dfrac{\sigma_1}{2\varepsilon_0}$ & $\dfrac{\sigma_2}{2\varepsilon_0}$

$\therefore \sigma_1 = \sigma_2$ So, $q_1 = q_2 = Q/2$

$\therefore$ Net surface charge density = Q/2A

(b) Electric field to the left of the plates = $\frac{\sigma}{\varepsilon_0}$



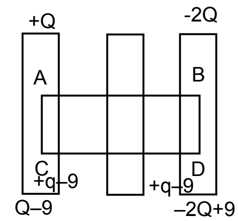
Since σ = Q/2A Hence Electricfield = Q/2Aε_0

This must be directed toward left as 'X' is the charged plate.

(c) & (d) Here in both the cases the charged plate 'X' acts as the only source of electric field, with (+ve) in the inner side and 'Y' attracts towards it with (-ve) he in its inner side. So for the middle portion E = $\frac{Q}{2A\varepsilon_0}$ towards right.

(d) Similarly for extreme right the outerside of the 'Y' plate acts as positive and hence it repels to the right with E = $\frac{Q}{2A\varepsilon_0}$

24. Consider the Gaussian surface the induced charge be as shown in figure.



The net field at P due to all the charges is Zero.

$\therefore$ –2Q +9/2A ε_0 (left) +9/2A ε_0 (left) + 9/2A ε_0 (right) + Q – 9/2A ε_0 (right) = 0

$\Rightarrow$ –2Q + 9 – Q + 9 = 0 $\Rightarrow$ 9 = 3/2 Q

$\therefore$ charge on the right side of right most plate

= –2Q + 9 = – 2Q + 3/2 Q = – Q/2

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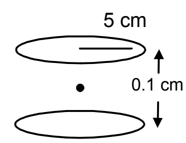
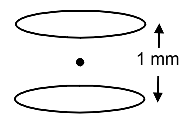
CHAPTER – 31
CAPACITOR

1. Given that
   Number of electron = 1×10^{12}
   Net charge Q = $1 \times 10^{12} \times 1.6 \times 10^{-19} = 1.6 \times 10^{-7}$ C
   $\therefore$ The net potential difference = 10 L.
   $\therefore$ Capacitance – C = $\frac{q}{v} = \frac{1.6 \times 10^{-7}}{10} = 1.6 \times 10^{-8}$ F.

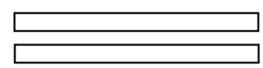
2. $A = \pi r^2 = 25\ \pi cm^2$
   d = 0.1 cm
   $c = \frac{\varepsilon_0 A}{d} = \frac{8.854 \times 10^{-12} \times 25 \times 3.14}{0.1} = 6.95 \times 10^{-5}\ \mu F$.



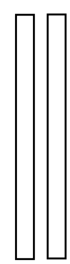
3. Let the radius of the disc = R
   $\therefore$ Area = πR^2
   C = 1f
   D = 1 mm = 10^{-3} m
   $\therefore C = \frac{\varepsilon_0 A}{d}$
   $\Rightarrow 1 = \frac{8.85 \times 10^{-12} \times \pi r^2}{10^{-3}} \Rightarrow r^2 = \frac{10^{-3} \times 10^{12}}{8.85 \times \pi} = \frac{10^9}{27.784}$ = 5998.5 m = 6 Km



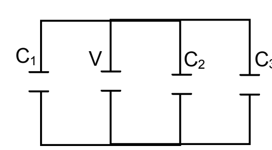
4. A = 25 cm^2 = 2.5×10^{-3} cm^2
   d = 1 mm = 0.01 m
   V = 6V Q = ?
   $C = \frac{\varepsilon_0 A}{d} = \frac{8.854 \times 10^{-12} \times 2.5 \times 10^{-3}}{0.01}$
   $Q = CV = \frac{8.854 \times 10^{-12} \times 2.5 \times 10^{-3}}{0.01} \times 6 = 1.32810 \times 10^{-10}$ C
   W = Q × V = $1.32810 \times 10^{-10} \times 6 = 8 \times 10^{-10}$ J.

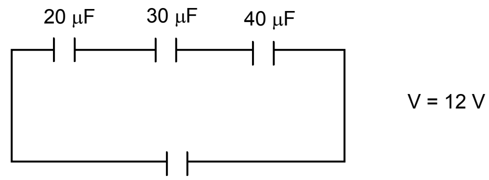


5. Plate area A = 25 cm^2 = 2.5×10^{-3} m
   Separation d = 2 mm = 2×10^{-3} m
   Potential v = 12 v
   (a) We know $C = \frac{\varepsilon_0 A}{d} = \frac{8.85 \times 10^{-12} \times 2.5 \times 10^{-3}}{2 \times 10^{-3}} = 11.06 \times 10^{-12}$ F
   $C = \frac{q}{v} \Rightarrow 11.06 \times 10^{-12} = \frac{q}{12}$
   $\Rightarrow q_1 = 1.32 \times 10^{-10}$ C.
   (b) Then d = decreased to 1 mm
   $\therefore$ d = 1 mm = 1×10^{-3} m
   $C = \frac{\varepsilon_0 A}{d} = \frac{q}{v} = \frac{8.85 \times 10^{-12} \times 2.5 \times 10^{-3}}{1 \times 10^{-3}} = \frac{2}{12}$
   $\Rightarrow q_2 = 8.85 \times 2.5 \times 12 \times 10^{-12} = 2.65 \times 10^{-10}$ C.
   $\therefore$ The extra charge given to plate = $(2.65 - 1.32) \times 10^{-10} = 1.33 \times 10^{-10}$ C.



6. $C_1 = 2\ \mu F$, $C_2 = 4\ \mu F$,
   $C_3 = 6\ \mu F$ V = 12 V
   $cq = C_1 + C_2 + C_3 = 2 + 4 + 6 = 12\ \mu F = 12 \times 10^{-6}$ F
   $q_1 = 12 \times 2 = 24\ \mu C$, $q_2 = 12 \times 4 = 48\ \mu C$, $q_3 = 12 \times 6 = 72\ \mu C$



7. 

∴ The equivalent capacity.

$C = \dfrac{C_1C_2C_3}{C_2C_3 + C_1C_3 + C_1C_2} = \dfrac{20 \times 30 \times 40}{30 \times 40 + 20 \times 40 + 20 \times 30} = \dfrac{24000}{2600} = 9.23\ \mu F$

(a) Let Equivalent charge at the capacitor = q

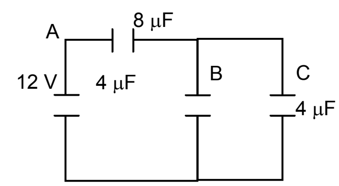
$C = \dfrac{q}{V} \Rightarrow q = C \times V = 9.23 \times 12 = 110\ \mu C$ on each.

As this is a series combination, the charge on each capacitor is same as the equivalent charge which is 110 μC.

(b) Let the work done by the battery = W

$\therefore V = \dfrac{W}{q} \Rightarrow W = Vq = 110 \times 12 \times 10^{-6} = 1.33 \times 10^{-3}$ J.

8. $C_1 = 8\ \mu F$, $C_2 = 4\ \mu F$, $C_3 = 4\ \mu F$



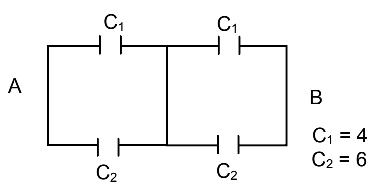
$Ceq = \dfrac{(C_2 + C_3) \times C_1}{C_1 + C_2 + C_3}$

$= \dfrac{8 \times 8}{16} = 4\ \mu F$

Since B & C are parallel & are in series with A

So, $q_1 = 8 \times 6 = 48\ \mu C$ $q_2 = 4 \times 6 = 24\ \mu C$ $q3 = 4 \times 6 = 24\ \mu C$

9. (a)



∴ C_1, C_1 are series & C_2, C_2 are series as the V is same at p & q. So no current pass through p & q.

$\dfrac{1}{C} = \dfrac{1}{C_1} = \dfrac{1}{C_2} \Rightarrow \dfrac{1}{C} = \dfrac{1+1}{C_1C_2}$

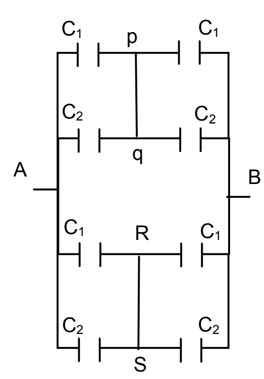
$C_p = \dfrac{C_1}{2} = \dfrac{4}{2} = 2\ \mu F$

And $C_q = \dfrac{C_2}{2} = \dfrac{6}{2} = 3\ \mu F$

$\therefore C = C_p + C_q = 2 + 3 = 5\ \mu F$

(b) $C_1 = 4\ \mu F$, $C_2 = 6\ \mu F$,

In case of p & q, q = 0



$\therefore C_p = \dfrac{C_1}{2} = \dfrac{4}{2} = 2\ \mu F$

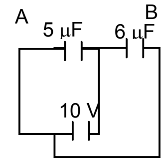
$C_q = \dfrac{C_2}{2} = \dfrac{6}{2} = 3\ \mu F$

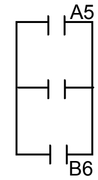
& $C' = 2 + 3 = 5\ \mu F$

C & $C' = 5\ \mu F$

∴ The equation of capacitor $C = C' + C'' = 5 + 5 = 10\ \mu F$

10. V = 10 v



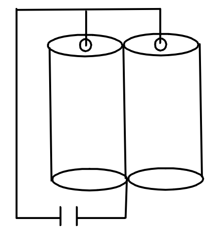


Ceq = $C_1 + C_2$ [∴ They are parallel]

= 5 + 6 = 11 μF

q = CV = 11 × 10 110 μC

11. The capacitance of the outer sphere = 2.2 μF



C = 2.2 μF

Potential, V = 10 v

Let the charge given to individual cylinder = q.

$C = \frac{q}{V}$

⇒ q = CV = 2.2 × 10 = 22 μF

∴ The total charge given to the inner cylinder = 22 + 22 = 44 μF

12. $C = \frac{q}{V}$, Now $V = \frac{Kq}{R}$

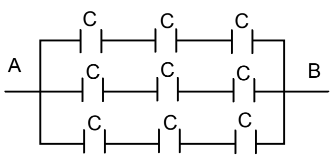
So, $C_1 = \frac{q}{(Kq/R_1)} = \frac{R_1}{K} = 4\pi\varepsilon_0 R_1$

Similarly $c_2 = 4\pi\varepsilon_0 R_2$

The combination is necessarily parallel.

Hence Ceq = $4\pi\varepsilon_0 R_1 + 4\pi\varepsilon_0 R_2 = 4\pi\varepsilon_0(R_1 + R_2)$

13.



∴C = 2 μF

∴ In this system the capacitance are arranged in series. Then the capacitance is parallel to each other.

(a) ∴ The equation of capacitance in one row

$C = \frac{C}{3}$

(b) and three capacitance of capacity $\frac{C}{3}$ are connected in parallel

∴ The equation of capacitance

$C = \frac{C}{3} + \frac{C}{3} + \frac{C}{3} = C = 2\ \mu F$

As the volt capacitance on each row are same and the individual is

$= \frac{\text{Total}}{\text{No. of capacitance}} = \frac{60}{3} = 20\ V$

14. Let there are 'x' no of capacitors in series ie in a row

So, x × 50 = 200

⇒ x = 4 capacitors.

Effective capacitance in a row = $\frac{10}{4}$

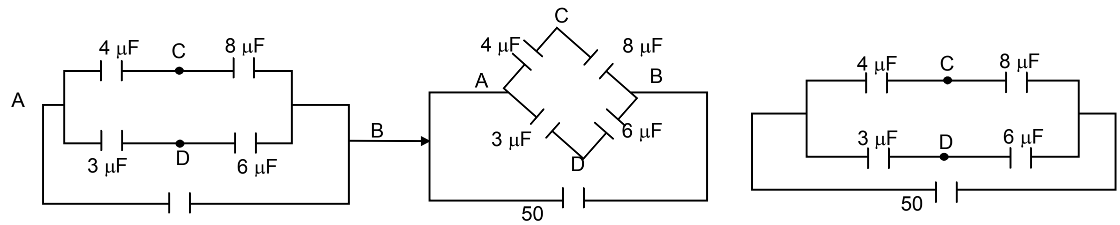
Now, let there are 'y' such rows,

So, $\frac{10}{4} \times y = 10$

⇒ y = 4 capacitor.

So, the combinations of four rows each of 4 capacitors.

15.



(a) Capacitor = $\frac{4 \times 8}{4+8}$ = $\frac{8}{3}$ μ

and $\frac{6 \times 3}{6+3}$ = 2 μF

(i) The charge on the capacitance $\frac{8}{3}$ μF

∴ Q = $\frac{8}{3}$ × 50 = $\frac{400}{3}$

∴ The potential at 4 μF = $\frac{400}{3 \times 4}$ = $\frac{100}{3}$

at 8 μF = $\frac{400}{3 \times 8}$ = $\frac{100}{6}$

The Potential difference = $\frac{100}{3} - \frac{100}{6}$ = $\frac{50}{3}$ μV

(ii) Hence the effective charge at 2 μF = 50 × 2 = 100 μF

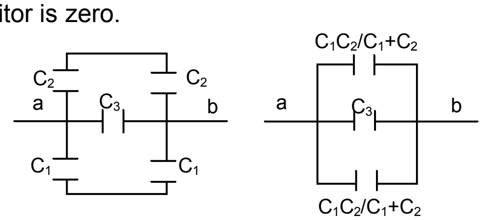
∴ Potential at 3 μF = $\frac{100}{3}$; Potential at 6 μF = $\frac{100}{6}$

∴Difference = $\frac{100}{3} - \frac{100}{6}$ = $\frac{50}{3}$ μV

∴ The potential at C & D is $\frac{50}{3}$ μV

(b) ∴ $\frac{P}{q} = \frac{R}{S}$ = $\frac{1}{2}$ = $\frac{1}{2}$ = It is balanced. So from it is cleared that the wheat star bridge balanced. So the potential at the point C & D are same. So no current flow through the point C & D. So if we connect another capacitor at the point C & D the charge on the capacitor is zero.

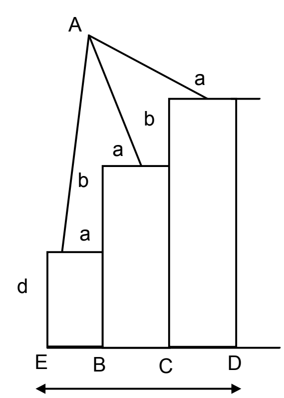
16. Ceq between a & b



$= \frac{C_1C_2}{C_1+C_2} + C_3 + \frac{C_1C_2}{C_1+C_2}$

$= C_3 + \frac{2C_1C_2}{C_1+C_2}$ (∴The three are parallel)

17. In the figure the three capacitors are arranged in parallel.



All have same surface area = a = $\frac{A}{3}$

First capacitance C_1 = $\frac{\varepsilon_0 A}{3d}$

2nd capacitance C_2 = $\frac{\varepsilon_0 A}{3(b+d)}$

3rd capacitance C_3 = $\frac{\varepsilon_0 A}{3(2b+d)}$

Ceq = $C_1 + C_2 + C_3$

$= \frac{\varepsilon_0 A}{3d} + \frac{\varepsilon_0 A}{3(b+d)} + \frac{\varepsilon_0 A}{3(2b+d)} = \frac{\varepsilon_0 A}{3}\left(\frac{1}{d} + \frac{1}{b+d} + \frac{1}{2b+d}\right)$

$= \frac{\varepsilon_0 A}{3}\left(\frac{(b+d)(2b+d)+(2b+d)d+(b+d)d}{d(b+d)(2b+d)}\right)$

$= \frac{\varepsilon_0 A\left(3d^2+6bd+2b^2\right)}{3d(b+d)(2b+d)}$

18. (a) $C = \frac{2\varepsilon_0 L}{\ln(R_2/R_1)} = \frac{e \times 3.14 \times 8.85 \times 10^{-2} \times 10^{-1}}{\ln 2}$ [ln2 = 0.6932]

$= 80.17 \times 10^{-13} \Rightarrow 8$ PF

(b) Same as R_2/R_1 will be same.

19. Given that

$C = 100$ PF $= 100 \times 10^{-12}$ F　　$C_{cq} = 20$ PF $= 20 \times 10^{-12}$ F

$V = 24$ V　　$q = 24 \times 100 \times 10^{-12} = 24 \times 10^{-10}$

$q_2 = ?$

Let q_1 = The new charge 100 PF　　V_1 = The Voltage.

Let the new potential is V_1

After the flow of charge, potential is same in the two capacitor

$V_1 = \frac{q_2}{C_2} = \frac{q_1}{C_1}$

$= \frac{q - q_1}{C_2} = \frac{q_1}{C_1}$

$= \frac{24 \times 10^{-10} - q_1}{24 \times 10^{-12}} = \frac{q_1}{100 \times 10^{-12}}$

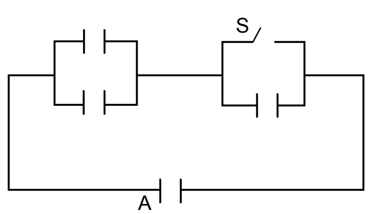
$= 24 \times 10^{-10} - q_1 = \frac{q_1}{5}$

$= 6q_1 = 120 \times 10^{-10}$

$= q_1 = \frac{120}{6} \times 10^{-10} = 20 \times 10^{-10}$

$\therefore V_1 = \frac{q_1}{C_1} = \frac{20 \times 10^{-10}}{100 \times 10^{-12}} = 20$ V

20.



Initially when 's' is not connected,

$C_{eff} = \frac{2C}{3} q = \frac{2C}{3} \times 50 = \frac{5}{2} \times 10^{-4} = 1.66 \times 10^{-4}$ C

After the switch is made on,

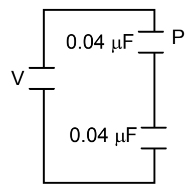
Then $C_{eff} = 2C = 10^{-5}$

$Q = 10^{-5} \times 50 = 5 \times 10^{-4}$

Now, the initial charge will remain stored in the stored in the short capacitor

Hence net charge flowing

$= 5 \times 10^{-4} - 1.66 \times 10^{-4} = 3.3 \times 10^{-4}$ C.

21. 

Given that mass of particle m = 10 mg

Charge 1 = – 0.01 μC

A = 100 cm^2 Let potential = V

The Equation capacitance C = $\frac{0.04}{2}$ = 0.02 μF

The particle may be in equilibrium, so that the wt. of the particle acting down ward, must be balanced by the electric force acting up ward.

∴ qE = Mg

Electric force = qE = $q\frac{V}{d}$ where V – Potential, d – separation of both the plates.

$= q\frac{VC}{\varepsilon_0 A}$ $C = \frac{\varepsilon_0 A}{q}$ $d = \frac{\varepsilon_0 A}{C}$

qE = mg

$= \frac{QVC}{\varepsilon_0 A}$ = mg

$= \frac{0.01 \times 0.02 \times V}{8.85 \times 10^{-12} \times 100}$ = 0.1 × 980

$\Rightarrow V = \frac{0.1 \times 980 \times 8.85 \times 10^{-10}}{0.0002}$ = 0.00043 = 43 MV

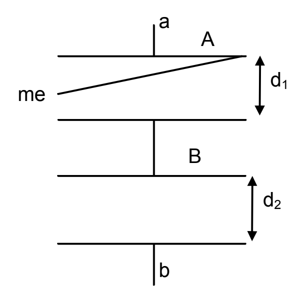
22. Let mass of electron = μ

Charge electron = e

We know, 'q'

For a charged particle to be projected in side to plates of a parallel plate capacitor with electric field E,

$y = \frac{1qE}{2m}\left(\frac{x}{\mu}\right)^2$



where y – Vertical distance covered or

x – Horizontal distance covered

μ – Initial velocity

From the given data,

$y = \frac{d_1}{2}$, $E = \frac{V}{R} = \frac{qd_1}{\varepsilon_0 a^2 \times d_1} = \frac{q}{\varepsilon_0 a^2}$, x = a, μ = ?

For capacitor A –

$V_1 = \frac{q}{C_1} = \frac{qd_1}{\varepsilon_0 a^2}$ as $C_1 = \frac{\varepsilon_0 a^2}{d_1}$

Here q = chare on capacitor.

q = C × V where C = Equivalent capacitance of the total arrangement = $\frac{\varepsilon_0 a^2}{d_1 + d_2}$

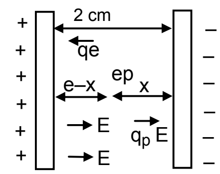
So, $q = \frac{\varepsilon_0 a^2}{d_1 + d_2} \times V$

Hence $E = \frac{q}{\varepsilon_0 a^2} = \frac{\varepsilon_0 a^2 \times V}{(d_1 + d_2)\varepsilon_0 a^2} = \frac{V}{(d_1 + d_2)}$

Substituting the data in the known equation, we get, $\frac{d_1}{2} = \frac{1}{2} \times \frac{e \times V}{(d_1 + d_2)m} \times \frac{a^2}{u^2}$

$\Rightarrow u^2 = \frac{Vea^2}{d_1 m(d_1 + d_2)} \Rightarrow u = \left(\frac{Vea^2}{d_1 m(d_1 + d_2)}\right)^{1/2}$

23. The acceleration of electron $a_e = \frac{qeme}{Me}$



The acceleration of proton = $\frac{qpe}{Mp}$ = ap

The distance travelled by proton $X = \frac{1}{2} apt^2$ …(1)

The distance travelled by electron …(2)

From (1) and (2) $\Rightarrow 2 - X = \frac{1}{2} a_c t^2$ $x = \frac{1}{2} a_c t^2$

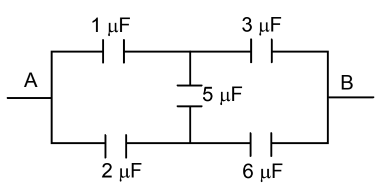
$\Rightarrow \frac{x}{2-x} = \frac{a_p}{a_c} = \frac{\left(\frac{q_p E}{M_p}\right)}{\left(\frac{q_c F}{M_c}\right)}$

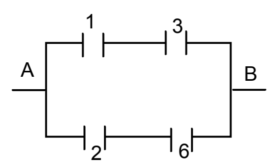
$= \frac{x}{2-x} = \frac{M_c}{M_p} = \frac{9.1 \times 10^{-31}}{1.67 \times 10^{-27}} = \frac{9.1}{1.67} \times 10^{-4} = 5.449 \times 10^{-4}$

$\Rightarrow x = 10.898 \times 10^{-4} - 5.449 \times 10^{-4} x$

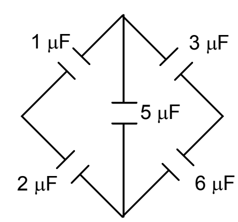
$\Rightarrow x = \frac{10.898 \times 10^{-4}}{1.0005449} = 0.001089226$

24. (a)





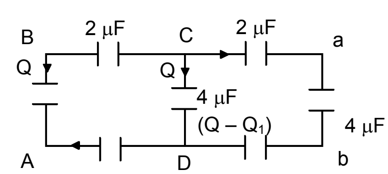
As the bridge in balanced there is no current through the 5 μF capacitor
So, it reduces to
similar in the case of (b) & (c)
as 'b' can also be written as.



$Ceq = \frac{1 \times 3}{1+3} + \frac{2 \times 6}{2+6} = \frac{3}{48} + \frac{12}{8} = \frac{6+12}{8} = 2.25\ \mu F$

25. (a) By loop method application in the closed circuit ABCabDA

$-12 + \frac{2Q}{2\mu F} + \frac{Q_1}{2\mu F} + \frac{Q_1}{4\mu F} = 0$ …(1)



In the close circuit ABCDA

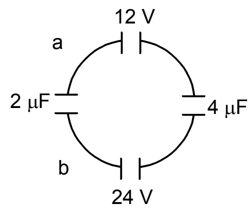
$-12 + \frac{Q}{2\mu F} + \frac{Q + Q_1}{4\mu F} = 0$ …(2)

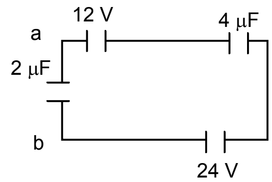
From (1) and (2) $2Q + 3Q_1 = 48$ …(3)
And $3Q - q_1 = 48$ and subtracting $Q = 4Q_1$, and substitution in equation

$2Q + 3Q_1 = 48 \Rightarrow 8\,Q_1 + 3Q_1 = 48 \Rightarrow 11Q_1 = 48,\ q_1 = \frac{48}{11}$

$Vab = \frac{Q_1}{4\mu F} = \frac{48}{11 \times 4} = \frac{12}{11}$ V

(b)



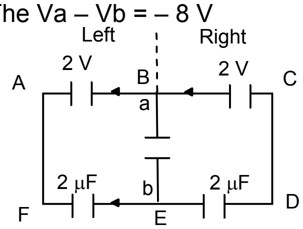


The potential = 24 – 12 = 12

Potential difference $V = \frac{(2 \times 0 + 12 \times 4)}{2+4} = \frac{48}{6} = 8$ V

$\therefore$ The Va – Vb = – 8 V

(c)

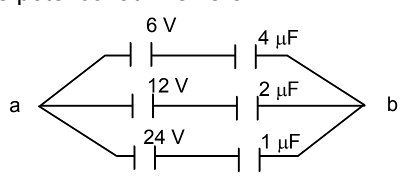


From the figure it is cleared that the left and right branch are symmetry and reversed, so the current go towards BE from BAFEB same as the current from EDCBE.

$\therefore$The net charge Q = 0 $\quad\quad \therefore V = \frac{Q}{C} = \frac{0}{C} = 0 \quad\quad \therefore$Vab = 0

$\therefore$ The potential at K is zero.

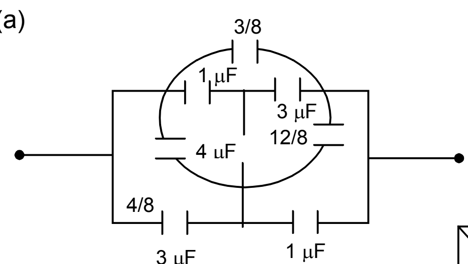
(d)

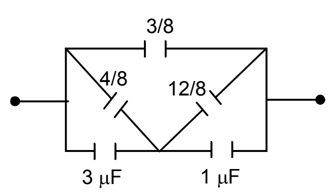


The net potential = $\frac{\text{Net charge}}{\text{Net capacitance}} = \frac{24+24+24}{7} = \frac{72}{7}$ 10.3 V

$\therefore$Va – Vb = – 10.3 V

26. (a)





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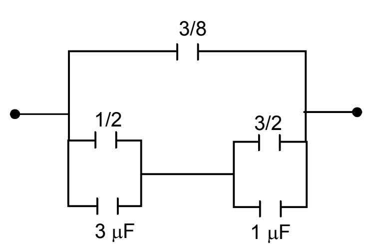
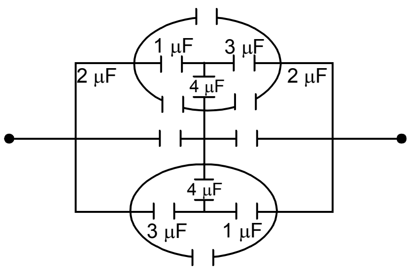


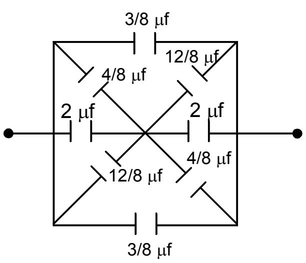
By star Delta conversion

$$C_{eff} = \frac{3}{8} + \left[\frac{\left(3+\frac{1}{2}\right) \times \left(\frac{3}{2}+1\right)}{\left(3+\frac{1}{2}\right) + \left(\frac{3}{2}+1\right)} \right] = \frac{3}{8} + \frac{35}{24} = \frac{9+35}{24} = \frac{11}{6}\ \mu F$$

(b)



by star Delta convensor



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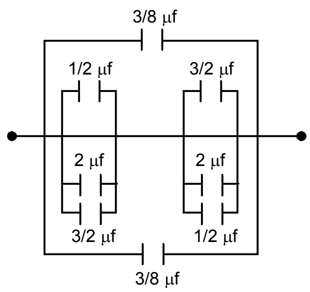

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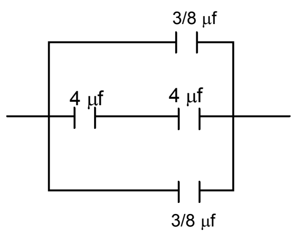
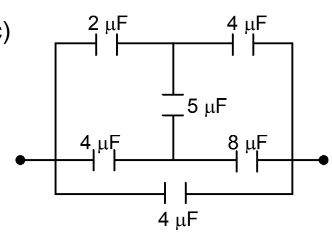


$$= \frac{3}{8} + \frac{16}{8} + \frac{3}{8} = \frac{11}{4}\ \mu f$$

(c)



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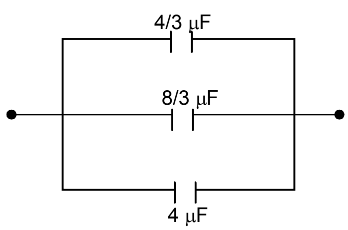
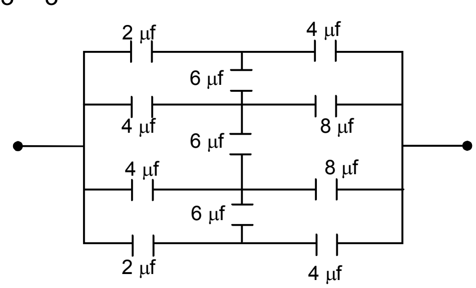


$$\text{Cef} = \frac{4}{3} + \frac{8}{3} + 4 = 8\ \mu F$$

(d)



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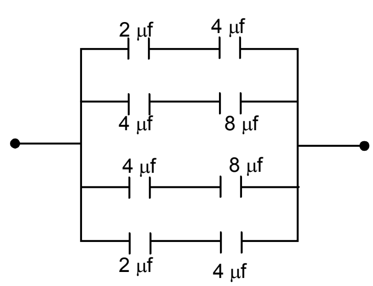

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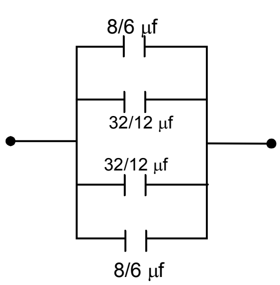
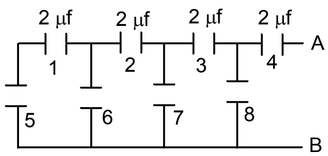


$$\text{Cef} = \frac{3}{8} + \frac{32}{12} + \frac{32}{12} + \frac{8}{6} = \frac{16+32}{6} = 8\ \mu f$$

27.



= C_5 and C_1 are in series

$C_{eq} = \frac{2 \times 2}{2+2} = 1$

This is parallel to $C_6 = 1 + 1 = 2$

Which is series to $C_2 = \frac{2 \times 2}{2+2} = 1$

Which is parallel to $C_7 = 1 + 1 = 2$

Which is series to $C_3 = \frac{2 \times 2}{2+2} = 1$

Which is parallel to $C_8 = 1 + 1 = 2$

This is series to $C_4 = \frac{2 \times 2}{2+2} = 1$

28.

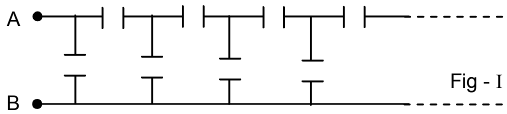


Fig - I

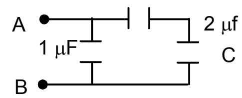


Fig – II

Let the equivalent capacitance be C. Since it is an infinite series. So, there will be negligible change if the arrangement is done an in Fig – II

$C_{eq} = \frac{2 \times C}{2+C} + 1 \Rightarrow C = \frac{2C+2+C}{2+C}$

$\Rightarrow (2 + C) \times C = 3C + 2$

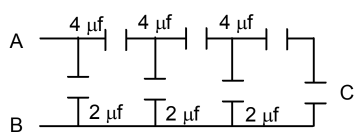
$\Rightarrow C^2 - C - 2 = 0$

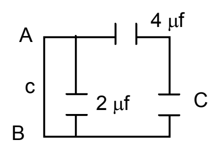
$\Rightarrow (C - 2)(C + 1) = 0$

$C = -1$ (Impossible)

So, $C = 2\ \mu F$

29.





= C and 4 μf are in series

So, $C_1 = \frac{4 \times C}{4+C}$

Then C_1 and 2 μf are parallel

$C = C_1 + 2\ \mu f$

$\Rightarrow \frac{4 \times C}{4+C} + 2 \Rightarrow \frac{4C+8+2C}{4+C} = C$

$\Rightarrow 4C + 8 + 2C = 4C + C^2 = C^2 - 2C - 8 = 0$

$C = \frac{2 \pm \sqrt{4+4 \times 1 \times 8}}{2} = \frac{2 \pm \sqrt{36}}{2} = \frac{2 \pm 6}{2}$

$C = \frac{2+6}{2} = 4\ \mu f$

$\therefore$ The value of C is 4 μf

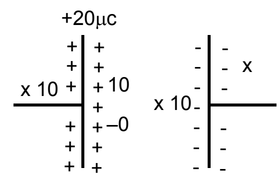
30. $q_1 = +2.0 \times 10^{-8}$ c $\qquad$ $q_2 = -1.0 \times 10^{-8}$ c

$C = 1.2 \times 10^{-3}\ \mu F = 1.2 \times 10^{-9}$ F

net q = $\frac{q_1 - q_2}{2} = \frac{3.0 \times 10^{-8}}{2}$

$V = \frac{q}{c} = \frac{3 \times 10^{-8}}{2} \times \frac{1}{1.2 \times 10^{-9}} = 12.5$ V

31. $\therefore$ Given that



Capacitance = 10 μF

Charge = 20 μc

$\therefore$ The effective charge = $\frac{20-0}{2} = 10\ \mu F$

$\therefore C = \frac{q}{V} \Rightarrow V = \frac{q}{C} = \frac{10}{10} = 1$ V

32. $q_1 = 1\ \mu C = 1 \times 10^{-6}$ C $\qquad$ $C = 0.1\ \mu F = 1 \times 10^{-7}$F

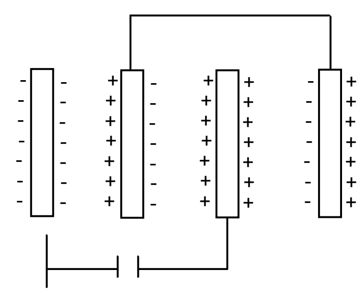
$q_2 = 2\ \mu C = 2 \times 10^{-6}$ C

net q = $\frac{q_1 - q_2}{2} = \frac{(1-2)\times 10^{-6}}{2} = -0.5 \times 10^{-6}$ C

Potential 'V' = $\frac{q}{c} = \frac{1 \times 10^{-7}}{-5 \times 10^{-7}} = -5$ V

But potential can never be (–)ve. So, V = 5 V

33. Here three capacitors are formed



And each of

$A = \frac{96}{\varepsilon_0} \times 10^{-12}$ f.m.

$d = 4$ mm $= 4 \times 10^{-3}$ m

$\therefore$Capacitance of a capacitor

$$C = \frac{\varepsilon_0 A}{d} = \frac{\varepsilon_0 \frac{96 \times 10^{-12}}{\varepsilon_0}}{4 \times 10^{-3}} = 24 \times 10^{-9} \text{ F.}$$

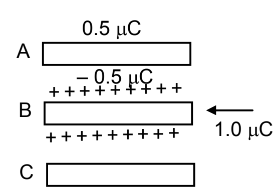
$\therefore$ As three capacitor are arranged is series

So, $C_{eq} = \frac{C}{q} = \frac{24 \times 10^{-9}}{3} = 8 \times 10^{-9}$

$\therefore$ The total charge to a capacitor = $8 \times 10^{-9} \times 10 = 8 \times 10^{-8}$ c

$\therefore$ The charge of a single Plate = $2 \times 8 \times 10^{-8} = 16 \times 10^{-8} = 0.16 \times 10^{-6} = 0.16$ μc.

34. (a) When charge of 1 μc is introduced to the B plate, we also get 0.5 μc charge on the upper surface of Plate 'A'.

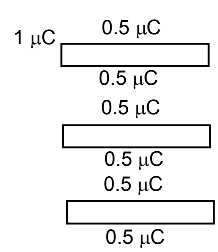


(b) Given $C = 50\ \mu F = 50 \times 10^{-9}$ F $= 5 \times 10^{-8}$ F

Now charge = 0.5×10^{-6} C

$V = \frac{q}{C} = \frac{5 \times 10^{-7} C}{5 \times 10^{-8} F} = 10$ V

35. Here given,



Capacitance of each capacitor, C = 50 μf = 0.05 μf

Charge Q = 1 μF which is given to upper plate = 0.5 μc charge appear on outer and inner side of upper plate and 0.5 μc of charge also see on the middle.

(a) Charge of each plate = 0.5 μc

Capacitance = 0.5 μf

$\therefore C = \frac{q}{V} \;\therefore V = \frac{q}{C} = \frac{0.5}{0.05} = 10$ v

(b) The charge on lower plate also = 0.5 μc

Capacitance = 0.5 μF

$\therefore C = \frac{q}{V} \Rightarrow V = \frac{q}{C} = \frac{0.5}{0.05} = 10$ V

$\therefore$ The potential in 10 V

36. $C_1 = 20$ PF $= 20 \times 10^{-12}$ F, $C_2 = 50$ PF $= 50 \times 10^{-12}$ F

Effective $C = \frac{C_1C_2}{C_1 + C_2} = \frac{2\times10^{-11} \times 5\times10^{-11}}{2\times10^{-11} + 5\times10^{-11}} = 1.428 \times 10^{-11}$ F

Charge 'q' $= 1.428 \times 10^{-11} \times 6 = 8.568 \times 10^{-11}$ C

$V_1 = \frac{q}{C_1} = \frac{8.568\times10^{-11}}{2\times10^{-11}} = 4.284$ V

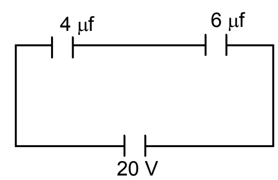
$V_2 = \frac{q}{C_2} = \frac{8.568\times10^{-11}}{5\times10^{-11}} = 1.71$ V

Energy stored in each capacitor

$E_1 = (1/2)\, C_1V_1^2 = (1/2) \times 2 \times 10^{-11} \times (4.284)^2 = 18.35 \times 10^{-11} \approx 184$ PJ

$E_2 = (1/2)\, C_2V_2^2 = (1/2) \times 5 \times 10^{-11} \times (1.71)^2 = 7.35 \times 10^{-11} \approx 73.5$ PJ

37. $\therefore C_1 = 4\ \mu$F, $C_2 = 6\ \mu$F, V = 20 V



Eq. capacitor $C_{eq} = \frac{C_1C_2}{C_1 + C_2} = \frac{4\times6}{4+6} = 2.4$

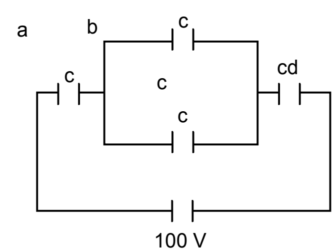
$\therefore$ The Eq Capacitance $C_{eq} = 2.5\ \mu$F

$\therefore$ The energy supplied by the battery to each plate

$E = (1/2)\, CV^2 = (1/2) \times 2.4 \times 20^2 = 480\ \mu$J

$\therefore$ The energy supplies by the battery to capacitor $= 2 \times 480 = 960\ \mu$J

38. C = 10 μF $= 10 \times 10^{-6}$ F



For a & d

$q = 4 \times 10^{-4}$ C

$c = 10^{-5}$ F

$E = \frac{1}{2}\frac{q^2}{c} = \frac{1}{2}\frac{(4\times10^{-4})^2}{10^{-5}} = 8 \times 10^{-3}$ J = 8 mJ

For b & c

$q = 4 \times 10^{-4}$ c

$C_{eq} = 2c = 2 \times 10^{-5}$ F

$V = \frac{4\times10^{-4}}{2\times10^{-5}} = 20$ V

$E = (1/2)\, cv^2 = (1/2) \times 10^{-5} \times (20)^2 = 2 \times 10^{-3}$ J = 2 mJ

39. Stored energy of capacitor $C_1 = 4.0$ J

$= \frac{1}{2}\frac{q^2}{c^2} = 4.0$ J

When then connected, the charge shared

$\frac{1}{2}\frac{q_1^2}{c^2} = \frac{1}{2}\frac{q_2^2}{c^2} \Rightarrow q_1 = q_2$

So that the energy should divided.

$\therefore$ The total energy stored in the two capacitors each is 2 J.

40. Initial charge stored = C × V = 12 × 2 × 10^{-6} = 24 × 10^{-6} c

Let the charges on 2 & 4 capacitors be q_1 & q_2 respectively

There, V = $\frac{q_1}{C_1} = \frac{q_2}{C_2} \Rightarrow \frac{q_1}{2} = \frac{q_2}{4} \Rightarrow q_2 = 2q_1$.

or $q_1 + q_2 = 24 \times 10^{-6}$ C

$\Rightarrow q_1 = 8 \times 10^{-6}\ \mu c$

$q_2 = 2q_1 = 2 \times 8 \times 10^{-6} = 16 \times 10^{-6}\ \mu c$

$E_1 = (1/2) \times C_1 \times V_1^2 = (1/2) \times 2 \times \left(\frac{8}{2}\right)^2 = 16\ \mu J$

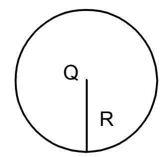
$E_2 = (1/2) \times C_2 \times V_2^2 = (1/2) \times 4 \times \left(\frac{8}{4}\right)^2 = 8\ \mu J$

41. Charge = Q

Radius of sphere = R

$\therefore$ Capacitance of the sphere = C = $4\pi\varepsilon_0 R$

Energy = $\frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}\frac{Q^2}{4\pi\varepsilon_0 R} = \frac{Q^2}{8\pi\varepsilon_0 R}$



42. Q = CV = $4\pi\varepsilon_0 R \times V$

E = $\frac{1}{2}\frac{q^2}{C}$ [$\therefore$ 'C' in a spherical shell = 4 $\pi\varepsilon_0$R]

E = $\frac{1}{2}\frac{16\pi^2\varepsilon_0{}^2 \times R^2 \times V^2}{4\pi\varepsilon_0 \times 2R}$ = 2 $\pi\varepsilon_0 RV^2$ ['C' of bigger shell = 4 $\pi\varepsilon_0$R]

43. $\sigma = 1 \times 10^{-4}$ c/m^2

a = 1 cm = 1 × 10^{-2} m $a^3 = 10^{-6}$ m

The energy stored in the plane = $\frac{1}{2}\frac{\sigma^2}{\varepsilon_0} = \frac{1}{2}\frac{(1\times10^{-4})^2}{8.85\times10^{-12}} = \frac{10^4}{17.7}$ = 564.97

The necessary electro static energy stored in a cubical volume of edge 1 cm infront of the plane

= $\frac{1}{2}\frac{\sigma^2}{\varepsilon_0}a^3$ = 265 × 10^{-6} = 5.65 × 10^{-4} J

44. area = a = 20 cm^2 = 2 × 10^{-2} m^2

d = separation = 1 mm = 10^{-3} m

Ci = $\frac{\varepsilon_0 \times 2\times10^{-3}}{10^{-3}} = 2\varepsilon_0$ Cf = $\frac{\varepsilon_0 \times 2\times10^{-3}}{2\times10^{-3}} = \varepsilon_0$

$q_i = 24\ \varepsilon_0$ $q_f = 12\ \varepsilon_0$ So, q flown out 12 ε_0. ie, $q_i - q_f$.

(a) So, q = 12 × 8.85 × 10^{-12} = 106.2 × 10^{-12} C = 1.06 × 10^{-10} C

(b) Energy absorbed by battery during the process

= q × v = 1.06 × 10^{-10} C × 12 = 12.7 × 10^{-10} J

(c) Before the process

E_i = (1/2) × Ci × v^2 = (1/2) × 2 × 8.85 × 10^{-12} × 144 = 12.7 × 10^{-10} J

After the force

E_i = (1/2) × Cf × v^2 = (1/2) × 8.85 × 10^{-12} × 144 = 6.35 × 10^{-10} J

(d) Workdone = Force × Distance

= $\frac{1}{2}\frac{q^2}{\varepsilon_0 A}$ = 1 × 10^3 = $\frac{1}{2}\times\frac{12\times12\times\varepsilon_0\times\varepsilon_0\times10^{-3}}{\varepsilon_0\times2\times10^{-3}}$

(e) From (c) and (d) we have calculated, the energy loss by the separation of plates is equal to the work done by the man on plate. Hence no heat is produced in transformer.

45. (a) Before reconnection

$C = 100\ \mu f$ $V = 24\ V$

$q = CV = 2400\ \mu c$ (Before reconnection)

After connection

When $C = 100\ \mu f$ $V = 12\ V$

$q = CV = 1200\ \mu c$ (After connection)

(b) $C = 100,$ $V = 12\ V$

$\therefore q = CV = 1200\ v$

(c) We know $V = \frac{W}{q}$

$W = vq = 12 \times 1200 = 14400\ J = 14.4\ mJ$

The work done on the battery.

(d) Initial electrostatic field energy $Ui = (1/2)\ CV_1^2$

Final Electrostatic field energy $Uf = (1/2)\ CV_2^2$

$\therefore$Decrease in Electrostatic

Field energy $= (1/2)\ CV_1^2 - (1/2)\ CV_2^2$

$= (1/2)\ C(V_1^2 - V_2^2) = (1/2) \times 100(576 - 144) = 21600J$

$\therefore$ Energy = 21600 j = 21.6 mJ

(e)After reconnection

$C = 100\ \mu c,$ $V = 12\ v$

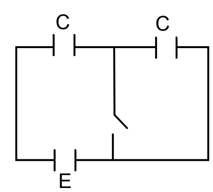
$\therefore$ The energy appeared $= (1/2)\ CV^2 = (1/2) \times 100 \times 144 = 7200\ J = 7.2\ mJ$

This amount of energy is developed as heat when the charge flow through the capacitor.

46. (a) Since the switch was open for a long time, hence the charge flown must be due to the both, when the switch is closed.

$C_{ef} = C/2$

So $q = \frac{E \times C}{2}$



(b) Workdone $= q \times v = \frac{EC}{2} \times E = \frac{E^2C}{2}$

(c) $E_i = \frac{1}{2} \times \frac{C}{2} \times E^2 = \frac{E^2C}{4}$

$E_f = (1/2) \times C \times E^2 = \frac{E^2C}{2}$

$E_i - E_f = \frac{E^2C}{4}$

(d) The net charge in the energy is wasted as heat.

47. $C_1 = 5\ \mu f$ $V_1 = 24\ V$

$q_1 = C_1V_1 = 5 \times 24 = 120\ \mu c$

and $C_2 = 6\ \mu f$ $V_2 = R$

$q_2 = C_2V_2 = 6 \times 12 = 72$

$\therefore$ Energy stored on first capacitor

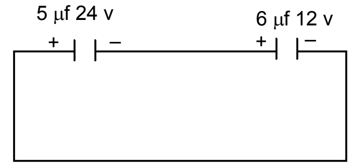
$E_i = \frac{1}{2}\frac{q_1^2}{C_1} = \frac{1}{2} \times \frac{(120)^2}{2} = 1440\ J = 1.44\ mJ$

Energy stored on 2nd capacitor

$E_2 = \frac{1}{2}\frac{q_2^2}{C_2} = \frac{1}{2} \times \frac{(72)^2}{6} = 432\ J = 4.32\ mJ$

(b) C_1V_1 C_2V_2

Let the effective potential = V



$$V = \frac{C_1V_1 - C_2V_2}{C_1 + C_2} = \frac{120 - 72}{5 + 6} = 4.36$$

The new charge $C_1V = 5 \times 4.36 = 21.8\ \mu c$

and $C_2V = 6 \times 4.36 = 26.2\ \mu c$

(c) $U_1 = (1/2)\ C_1V^2$

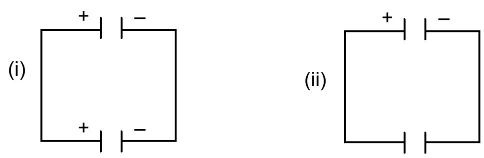
$U_2 = (1/2)\ C_2V^2$

$U_f = (1/2)\ V^2\ (C_1 + C_2) = (1/2)\ (4.36)^2\ (5 + 6) = 104.5 \times 10^{-6}$ J = 0.1045 mJ

But $U_i = 1.44 + 0.433 = 1.873$

$\therefore$ The loss in KE = 1.873 – 0.1045 = 1.7687 = 1.77 mJ

48.



When the capacitor is connected to the battery, a charge Q = CE appears on one plate and –Q on the other. When the polarity is reversed, a charge –Q appears on the first plate and +Q on the second. A charge 2Q, therefore passes through the battery from the negative to the positive terminal.

The battery does a work.

$W = Q \times E = 2QE = 2CE^2$

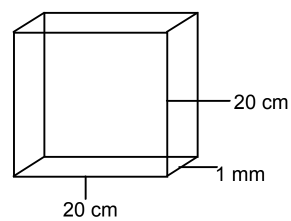
In this process. The energy stored in the capacitor is the same in the two cases. Thus the workdone by battery appears as heat in the connecting wires. The heat produced is therefore,

$2CE^2 = 2 \times 5 \times 10^{-6} \times 144 = 144 \times 10^{-5}$ J = 1.44 mJ [have C = 5 μf V = E = 12V]

49. A = 20 cm × 20 cm = 4×10^{-2} m

d = 1 m = 1×10^{-3} m

k = 4 t = d



$$C = \frac{\varepsilon_0 A}{d - t + \frac{t}{k}} = \frac{\varepsilon_0 A}{d - d + \frac{d}{k}} = \frac{\varepsilon_0 Ak}{d}$$

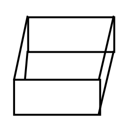
$$= \frac{8.85 \times 10^{-12} \times 4 \times 10^{-2} \times 4}{1 \times 10^{-3}} = 141.6 \times 10^{-9}\ \text{F} = 1.42\ \text{nf}$$

50. Dielectric const. = 4

F = 1.42 nf, V = 6 V

Charge supplied = q = CV = $1.42 \times 10^{-9} \times 6 = 8.52 \times 10^{-9}$ C

Charge Induced = q(1 – 1/k) = $8.52 \times 10^{-9} \times (1 - 0.25) = 6.39 \times 10^{-9}$ = 6.4 nc



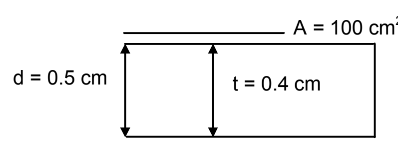
Net charge appearing on one coated surface = $\frac{8.52\mu c}{4}$ = 2.13 nc

51. Here

Plate area = 100 cm^2 = 10^{-2} m^2

Separation d = .5 cm = 5×10^{-3} m

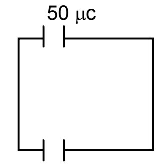
Thickness of metal t = .4 cm = 4×10^{-3} m



$$C = \frac{\varepsilon_0 A}{d - t + \frac{t}{k}} = \frac{\varepsilon_0 A}{d - t} = \frac{8.585 \times 10^{-12} \times 10^{-2}}{(5 - 4) \times 10^{-3}} = 88\ \text{pF}$$

Here the capacitance is independent of the position of metal. At any position the net separation is d – t. As d is the separation and t is the thickness.

52. Initial charge stored = 50 μc



Let the dielectric constant of the material induced be 'k'.
Now, when the extra charge flown through battery is 100.
So, net charge stored in capacitor = 150 μc

Now $C_1 = \frac{\varepsilon_0 A}{d}$ or $\frac{q_1}{V} = \frac{\varepsilon_0 A}{d}$ …(1)

$C_2 = \frac{\varepsilon_0 Ak}{d}$ or, $\frac{q_2}{V} = \frac{\varepsilon_0 Ak}{d}$ …(2)

Deviding (1) and (2) we get $\frac{q_1}{q_2} = \frac{1}{k}$

$\Rightarrow \frac{50}{150} = \frac{1}{k} \Rightarrow k = 3$

53. C = 5 μf V = 6 V d = 2 mm = 2×10^{-3} m.

(a) the charge on the +ve plate

q = CV = 5 μf × 6 V = 30 μc

(b) $E = \frac{V}{d} = \frac{6V}{2 \times 10^{-3} m} = 3 \times 10^3$ V/M

(c) d = 2×10^{-3} m

t = 1×10^{-3} m

k = 5 or $C = \frac{\varepsilon_0 A}{d}$ $\Rightarrow 5 \times 10^{-6} = \frac{8.85 \times A \times 10^{-12}}{2 \times 10^{-3}} \times 10^{-9} \Rightarrow A = \frac{10^4}{8.85}$

When the dielectric placed on it

$$C_1 = \frac{\varepsilon_0 A}{d - t + \frac{t}{k}} = \frac{8.85 \times 10^{-12} \times \frac{10^4}{8.85}}{10^{-3} + \frac{10^{-3}}{5}} = \frac{10^{-12} \times 10^4 \times 5}{6 \times 10^{-3}} = \frac{5}{6} \times 10^{-5} = 0.00000833 = 8.33\ \mu F.$$

(d) C = 5×10^{-6} f. V = 6 V

$\therefore$ Q = CV = 3×10^{-5} f = 30 μf

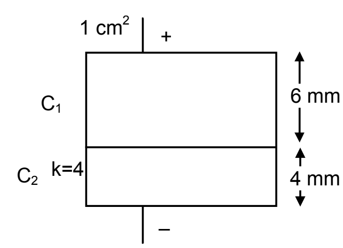
C′ = 8.3×10^{-6} f

V = 6 V

$\therefore$ Q′ = C′V = $8.3 \times 10^{-6} \times 6 \approx 50$ μF

$\therefore$ charge flown = Q′ – Q = 20 μF

54. Let the capacitances be C_1 & C_2 net capacitance 'C' = $\frac{C_1 C_2}{C_1 + C_2}$



Now $C_1 = \frac{\varepsilon_0 Ak_1}{d_1}$ $C_2 = \frac{\varepsilon_0 Ak_2}{d_2}$

$$C = \frac{\frac{\varepsilon_0 Ak_1}{d_1} \times \frac{\varepsilon_0 Ak_2}{d_2}}{\frac{\varepsilon_0 Ak_1}{d_1} + \frac{\varepsilon_0 Ak_2}{d_2}} = \frac{\varepsilon_0 A\left(\frac{k_1 k_2}{d_1 d_2}\right)}{\varepsilon_0 A\left(\frac{k_1 d_2 + k_2 d_1}{d_1 d_2}\right)} = \frac{8.85 \times 10^{-12} \times 10^{-2} \times 24}{6 \times 4 \times 10^{-3} + 4 \times 6 \times 10^{-3}}$$

= 4.425×10^{-11} C = 44.25 pc.

55. A = 400 cm² = 4×10^{-2} m²

d = 1 cm = 1×10^{-3} m

V = 160 V

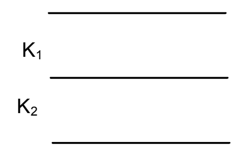
t = 0.5 = 5×10^{-4} m

k = 5

$$C = \frac{\varepsilon_0 A}{d - t + \frac{t}{k}} = \frac{8.85 \times 10^{-12} \times 4 \times 10^{-2}}{10^{-3} - 5 \times 10^{-4} + \frac{5 \times 10^{-4}}{5}} = \frac{35.4 \times 10^{-4}}{10^{-3} - 0.5}$$

56. (a) Area = A

Separation = d



$C_1 = \frac{\varepsilon_0 A k_1}{d/2}$ $\qquad$ $C_2 = \frac{\varepsilon_0 A k_2}{d/2}$

$$C = \frac{C_1 C_2}{C_1 + C_2} = \frac{\frac{2\varepsilon_0 A k_1}{d} \times \frac{2\varepsilon_0 A k_2}{d}}{\frac{2\varepsilon_0 A k_1}{d} + \frac{2\varepsilon_0 A k_2}{d}} = \frac{\frac{(2\varepsilon_0 A)^2 k_1 k_2}{d^2}}{(2\varepsilon_0 A)\frac{k_1 d + k_2 d}{d^2}} = \frac{2k_1 k_2 \varepsilon_0 A}{d(k_1 + k_2)}$$

(b) similarly

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{1}{\frac{3\varepsilon_0 A k_1}{d}} + \frac{1}{\frac{3\varepsilon_0 A k_2}{d}} + \frac{1}{\frac{3\varepsilon_0 A k_3}{d}}$$

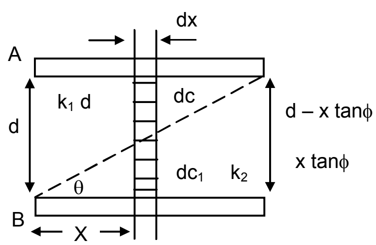
$$= \frac{d}{3\varepsilon_0 A}\left[\frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3}\right] = \frac{d}{3\varepsilon_0 A}\left[\frac{k_2 k_3 + k_1 k_3 + k_1 k_2}{k_1 k_2 k_3}\right]$$

$$\therefore C = \frac{3\varepsilon_0 A k_1 k_2 k_3}{d(k_1 k_2 + k_2 k_3 + k_1 k_3)}$$

(c) $C = C_1 + C_2$

$$= \frac{\varepsilon_0 \frac{A}{2} k_1}{d} + \frac{\varepsilon_0 \frac{A}{2} k_2}{d} = \frac{\varepsilon_0 A}{2d}(k_1 + k_2)$$

57.



Consider an elemental capacitor of with dx our at a distance 'x' from one end. It is constituted of two capacitor elements of dielectric constants k_1 and k_2 with plate separation xtanϕ and d –xtanϕ respectively in series

$$\frac{1}{dcR} = \frac{1}{dc_1} + \frac{1}{dc_2} = \frac{x\tan\phi}{\varepsilon_0 k_2 (bdx)} + \frac{d - x\tan\phi}{\varepsilon_0 k_1 (bdx)}$$

$$dcR = \frac{\varepsilon_0 bdx}{\frac{x\tan\phi}{k_2} + \frac{(d - x\tan\phi)}{k_1}}$$

$$\text{or } C_R = \varepsilon_0 b k_1 k_2 \int \frac{dx}{k_2 d + (k_1 - k_2)x\tan\phi}$$

$$= \frac{\varepsilon_0 b k_1 k_2}{\tan\phi(k_1 - k_2)} [\log_e k_2 d + (k_1 - k_2)\, x \tan\phi]a$$

$$= \frac{\varepsilon_0 b k_1 k_2}{\tan\phi(k_1 - k_2)} [\log_e k_2 d + (k_1 - k_2)\, a \tan\phi - \log_e k_2 d]$$

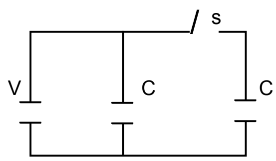
$\therefore \tan\phi = \frac{d}{a}$ and A = a × a

$$C_R = \frac{\varepsilon_0 a k_1 k_2}{\frac{d}{a}(k_1 - k_2)} \qquad \left[\log_e\left(\frac{k_1}{k_2}\right)\right]$$

$$C_R = \frac{\varepsilon_0 a^2 k_1 k_2}{d(k_1 - k_2)} \qquad \left[\log_e\left(\frac{k_1}{k_2}\right)\right]$$

$$C_R = \frac{\varepsilon_0 a^2 k_1 k_2}{d(k_1 - k_2)} \quad \ln \frac{k_1}{k_2}$$

58.



I. Initially when switch 's' is closed

Total Initial Energy = (1/2) CV^2 + (1/2) CV^2 = CV^2 …(1)

II. When switch is open the capacitance in each of capacitors varies, hence the energy also varies.

i.e. in case of 'B', the charge remains

Same i.e. cv

$C_{eff} = 3C$

$$E = \frac{1}{2} \times \frac{q^2}{c} = \frac{1}{2} \times \frac{c^2 v^2}{3c} = \frac{cv^2}{6}$$

In case of 'A'

$C_{eff} = 3c$

$$E = \frac{1}{2} \times C_{eff} v^2 = \frac{1}{2} \times 3c \times v^2 = \frac{3}{2} cv^2$$

$$\text{Total final energy} = \frac{cv^2}{6} + \frac{3cv^2}{2} = \frac{10cv^2}{6}$$

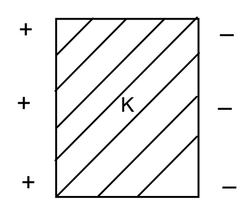
$$\text{Now, } \frac{\text{Initial Energy}}{\text{Final Energy}} = \frac{cv^2}{\frac{10cv^2}{6}} = 3$$

59. Before inserting

$$C = \frac{\varepsilon_0 A}{d} C \qquad Q = \frac{\varepsilon_0 AV}{d} C$$

After inserting

$$C = \frac{\varepsilon_0 A}{\frac{d}{k}} = \frac{\varepsilon_0 Ak}{d} \qquad Q_1 = \frac{\varepsilon_0 Ak}{d} V$$



The charge flown through the power supply

$Q = Q_1 - Q$

$$= \frac{\varepsilon_0 AkV}{d} - \frac{\varepsilon_0 AV}{d} = \frac{\varepsilon_0 AV}{d}(k-1)$$

Workdone = Charge in emf

$$= \frac{1}{2}\frac{q^2}{C} = \frac{1}{2}\frac{\frac{\varepsilon_0{}^2 A^2 V^2}{d^2}(k-1)^2}{\frac{\varepsilon_0 A}{d}(k-1)} = \frac{\varepsilon_0 AV^2}{2d}(k-1)$$

60. Capacitance = 100 μF = 10^{-4} F

P.d = 30 V

(a) q = CV = $10^{-4} \times 50 = 5 \times 10^{-3}$ c = 5 mc

Dielectric constant = 2.5

(b) New C = C′ = 2.5 × C = 2.5 × 10^{-4} F

New p.d = $\frac{q}{c^1}$ [∴'q' remains same after disconnection of battery]

$$= \frac{5 \times 10^{-3}}{2.5 \times 10^{-4}} = 20 \text{ V.}$$

(c) In the absence of the dielectric slab, the charge that must have produced

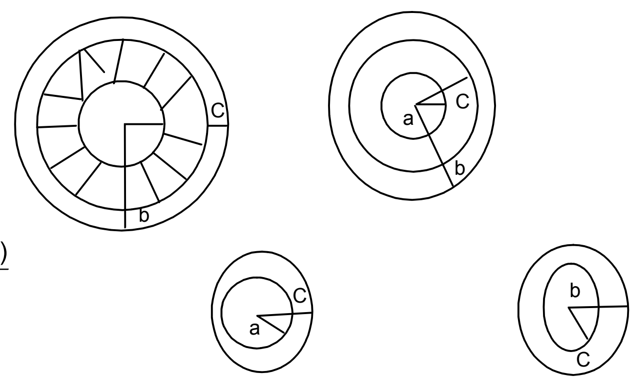
C × V = $10^{-4} \times 20 = 2 \times 10^{-3}$ c = 2 mc

(d) Charge induced at a surface of the dielectric slab

= q (1 –1/k) (where k = dielectric constant, q = charge of plate)

$$= 5 \times 10^{-3} \left(1 - \frac{1}{2.5}\right) = 5 \times 10^{-3} \times \frac{3}{5} = 3 \times 10^{-3} = 3 \text{ mc.}$$

61. Here we should consider a capacitor cac and cabc in series



$$\text{Cac} = \frac{4\pi\varepsilon_0 ack}{k(c-a)}$$

$$\text{Cbc} = \frac{4\pi\varepsilon_0 bc}{(b-c)}$$

$$\frac{1}{C} = \frac{1}{\text{Cac}} + \frac{1}{\text{Cbc}}$$

$$= \frac{(c-a)}{4\pi\varepsilon_0 ack} + \frac{(b-c)}{4\pi\varepsilon_0 bc} = \frac{b(c-a)+ka(b-c)}{k4\pi\varepsilon_0 abc}$$

$$C = \frac{4\pi\varepsilon_0 kabc}{ka(b-c)+b(c-a)}$$

62. These three metallic hollow spheres form two spherical capacitors, which are connected in series. Solving them individually, for (1) and (2)

$C_1 = \frac{4\pi\varepsilon_0 ab}{b-a}$ (∴ for a spherical capacitor formed by two spheres of radii $R_2 > R_1$)

$$C = \frac{4\pi\varepsilon_0 R_2 R_1}{R_2 - R_2}$$

Similarly for (2) and (3)

$$C_2 = \frac{4\pi\varepsilon_0 bc}{c-b}$$

$$C_{eff} = \frac{C_1 C_2}{C_1 + C_2} \frac{\frac{(4\pi\varepsilon_0)^2 ab^2 c}{(b-a)(c-a)}}{4\pi\varepsilon_0\left[\frac{ab(c-b)+bc(b-a)}{(b-a)(c-b)}\right]}$$

$$= \frac{4\pi\varepsilon_0 ab^2 c}{abc - ab^2 + b^2 c - abc} = \frac{4\pi\varepsilon_0 ab^2 c}{b^2(c-a)} = \frac{4\pi\varepsilon_0 ac}{c-a}$$

63. Here we should consider two spherical capacitor of capacitance cab and cbc in series

$$\text{Cab} = \frac{4\pi\varepsilon_0 abk}{(b-a)} \qquad \text{Cbc} = \frac{4\pi\varepsilon_0 bc}{(c-b)}$$

$$\frac{1}{C} = \frac{1}{Cab} + \frac{1}{Cbc} = \frac{(b-a)}{4\pi\varepsilon_0 abk} + \frac{(c-b)}{4\pi\varepsilon_0 bc} = \frac{c(b-a)+ka(c-b)}{k4\pi\varepsilon_0 abc}$$

$$C = \frac{4\pi\varepsilon_0 kabc}{c(b-a)+ka(c-b)}$$

64. Q = 12 μc

V = 1200 V

$\frac{v}{d} = 3 \times {}^{10-6} \frac{v}{m}$

$d = \frac{V}{(v/d)} = \frac{1200}{3\times10^{-6}} = 4 \times 10^{-4}$ m

$c = \frac{Q}{v} = \frac{12\times10^{-6}}{1200} = 10^{-8}$ f

$\therefore C = \frac{\varepsilon_0 A}{d} = 10^{-8}$ f

$\Rightarrow A = \frac{10^{-8}\times d}{\varepsilon_0} = \frac{10^{-8}\times4\times10^{-4}}{8.854\times10^{-4}}$ 0.45 m^2

65. A = 100 cm^2 = 10^{-2} m^2

d = 1 cm = 10^{-2} m

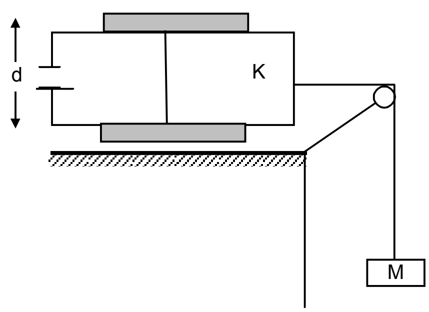
V = 24 V_0

$\therefore$ The capacitance $C = \frac{\varepsilon_0 A}{d} = \frac{8.85\times10^{-12}\times10^{-2}}{10^{-2}} = 8.85 \times 10^{-12}$

$\therefore$ The energy stored $C_1 = (1/2)\ CV^2 = (1/2) \times 10^{-12} \times (24)^2 = 2548.8 \times 10^{-12}$

$\therefore$ The forced attraction between the plates $= \frac{C_1}{d} = \frac{2548.8\times10^{-12}}{10^{-2}} = 2.54 \times 10^{-7}$ N.

66.



We knows

In this particular case the electricfield attracts the dielectric into the capacitor with a force $\frac{\varepsilon_0 bV^2(k-1)}{2d}$

Where b – Width of plates

k – Dielectric constant

d – Separation between plates

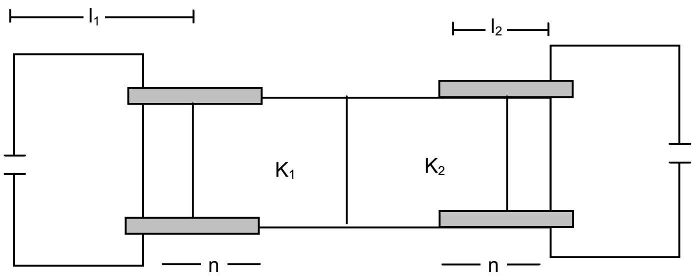
V = E = Potential difference.

Hence in this case the surfaces are frictionless, this force is counteracted by the weight.

So, $\frac{\varepsilon_0 bE^2(k-1)}{2d} = Mg$

$\Rightarrow M = \frac{\varepsilon_0 bE^2(k-1)}{2dg}$

67.



(a) Consider the left side

The plate area of the part with the dielectric is by its capacitance

$C_1 = \frac{k_1 \varepsilon_0 bx}{d}$ and with out dielectric $C_2 = \frac{\varepsilon_0 b(L_1 - x)}{d}$

These are connected in parallel

$C = C_1 + C_2 = \frac{\varepsilon_0 b}{d}[L_1 + x(k_1 - 1)]$

Let the potential V_1

$U = (1/2)\ CV_1^2 = \frac{\varepsilon_0 bv_1^2}{2d}\left[L_1 + x(k-1)\right]$ …(1)

Suppose dielectric slab is attracted by electric field and an external force F consider the part dx which makes inside further, As the potential difference remains constant at V.

The charge supply, dq = (dc) v to the capacitor

The work done by the battery is dw_b = v.dq = (dc) v^2

The external force F does a work dw_e = (–f.dx)

during a small displacement

The total work done in the capacitor is $dw_b + dw_e$ = (dc) v^2 – fdx

This should be equal to the increase dv in the stored energy.

Thus (1/2) (dk)v^2 = (dc) v^2 – fdx

$f = \frac{1}{2} v^2 \frac{dc}{dx}$

from equation (1)

$F = \frac{\varepsilon_0 bv^2}{2d}(k_1 - 1)$

$\Rightarrow V_1^2 = \frac{F \times 2d}{\varepsilon_0 b(k_1 - 1)} \Rightarrow V_1 = \sqrt{\frac{F \times 2d}{\varepsilon_0 b(k_1 - 1)}}$

For the right side, $V_2 = \sqrt{\frac{F \times 2d}{\varepsilon_0 b(k_2 - 1)}}$

$$\frac{V_1}{V_2} = \frac{\sqrt{\frac{F \times 2d}{\varepsilon_0 b(k_1 - 1)}}}{\sqrt{\frac{F \times 2d}{\varepsilon_0 b(k_2 - 1)}}}$$

$$\Rightarrow \frac{V_1}{V_2} = \frac{\sqrt{k_2 - 1}}{\sqrt{k_1 - 1}}$$

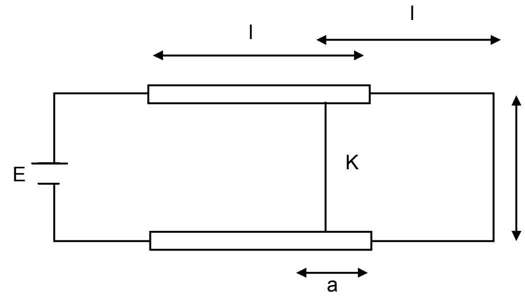
$\therefore$ The ratio of the emf of the left battery to the right battery = $\frac{\sqrt{k_2 - 1}}{\sqrt{k_1 - 1}}$

68. Capacitance of the portion with dielectrics,

$C_1 = \frac{k\varepsilon_0 A}{\ell d}$

Capacitance of the portion without dielectrics,

$C_2 = \frac{\varepsilon_0(\ell - a)A}{\ell d}$



$\therefore$ Net capacitance $C = C_1 + C_2 = \frac{\varepsilon_0 A}{\ell d}[ka + (\ell - a)]$

$C = \frac{\varepsilon_0 A}{\ell d}[\ell + a(k-1)]$

Consider the motion of dielectric in the capacitor.

Let it further move a distance dx, which causes an increase of capacitance by dc

$\therefore dQ = (dc)\ E$

The work done by the battery dw = Vdg = E (dc) E = E^2 dc

Let force acting on it be f

$\therefore$ Work done by the force during the displacement, dx = fdx

$\therefore$ Increase in energy stored in the capacitor

$\Rightarrow (1/2)\ (dc)\ E^2 = (dc)\ E^2 - fdx$

$\Rightarrow fdx = (1/2)\ (dc)\ E^2 \Rightarrow f = \frac{1}{2}\frac{E^2 dc}{dx}$

$C = \frac{\varepsilon_0 A}{\ell d}[\ell + a(k-1)]$ (here x = a)

$\Rightarrow \frac{dc}{da} = \frac{-d}{da}\left[\frac{\varepsilon_0 A}{\ell d}\{\ell + a(k-1)\}\right]$

$\Rightarrow \frac{\varepsilon_0 A}{\ell d}(k-1) = \frac{dc}{dx}$

$\Rightarrow f = \frac{1}{2}E^2\frac{dc}{dx} = \frac{1}{2}E^2\left\{\frac{\varepsilon_0 A}{\ell d}(k-1)\right\}$

$\therefore a_d = \frac{f}{m} = \frac{E^2\varepsilon_0 A(k-1)}{2\ell dm}$ $\therefore (\ell - a) = \frac{1}{2}a_d t^2$

$\Rightarrow t = \sqrt{\frac{2(\ell - a)}{a_d}} = \sqrt{\frac{2(\ell - a)2\ell dm}{E^2\varepsilon_0 A(k-1)}} = \sqrt{\frac{4m\ell d(\ell - a)}{\varepsilon_0 AE^2(k-1)}}$

$\therefore$ Time period = $2t = \sqrt{\frac{8m\ell d(\ell - a)}{\varepsilon_0 AE^2(k-1)}}$

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ELECTRIC CURRENT IN CONDUCTORS
CHAPTER - 32

1. $Q(t) = At^2 + Bt + c$

   a) $At^2 = Q$

   $\Rightarrow A = \frac{Q}{t^2} = \frac{A'T'}{T^{-2}} = A^1T^{-1}$

   b) $Bt = Q$

   $\Rightarrow B = \frac{Q}{t} = \frac{A'T'}{T} = A$

   c) $C = [Q]$

   $\Rightarrow C = A'T'$

   d) Current t = $\frac{dQ}{dt} = \frac{d}{dt}\left(At^2 + Bt + C\right)$

   $= 2At + B = 2 \times 5 \times 5 + 3 = 53$ A.

2. No. of electrons per second = 2×10^{16} electrons / sec.

   Charge passing per second = $2 \times 10^{16} \times 1.6 \times 10^{-9}$ $\frac{\text{coulomb}}{\text{sec}}$

   $= 3.2 \times 10^{-9}$ Coulomb/sec

   Current = 3.2×10^{-3} A.

3. $i' = 2\ \mu A$, t = 5 min = 5×60 sec.

   $q = i\,t = 2 \times 10^{-6} \times 5 \times 60$

   $= 10 \times 60 \times 10^{-6}\ c = 6 \times 10^{-4}\ c$

4. $i = i_0 + \alpha t$, t = 10 sec, $i_0 = 10$ A, $\alpha = 4$ A/sec.

   $q = \int_0^t idt = \int_0^t (i_0 + \alpha t)dt = \int_0^t i_0 dt + \int_0^t \alpha t dt$

   $= i_0 t + \alpha\frac{t^2}{2} = 10 \times 10 + 4 \times \frac{10 \times 10}{2}$

   $= 100 + 200 = 300$ C.

5. $i = 1$ A, A = 1 mm^2 = 1×10^{-6} m^2

   f' cu = 9000 kg/m^3

   Molecular mass has N_0 atoms

   = m Kg has (N_0/M $\times$ m) atoms = $\frac{N_0 Al9000}{63.5 \times 10^{-3}}$

   No.of atoms = No.of electrons

   $n = \frac{\text{No.of electrons}}{\text{Unit volume}} = \frac{N_0 Af}{mAl} = \frac{N_0 f}{M}$

   $= \frac{6 \times 10^{23} \times 9000}{63.5 \times 10^{-3}}$

   $i = V_d\, n\, A\, e$.

   $\Rightarrow V_d = \frac{i}{nAe} = \frac{1}{\frac{6 \times 10^{23} \times 9000}{63.5 \times 10^{-3}} \times 10^{-6} \times 1.6 \times 10^{-19}}$

   $= \frac{63.5 \times 10^{-3}}{6 \times 10^{23} \times 9000 \times 10^{-6} \times 1.6 \times 10^{-19}} = \frac{63.5 \times 10^{-3}}{6 \times 9 \times 1.6 \times 10^{26} \times 10^{-19} \times 10^{-6}}$

$= \dfrac{63.5\times10^{-3}}{6\times9\times1.6\times10} = \dfrac{63.5\times10^{-3}}{6\times9\times16}$

$= 0.074 \times 10^{-3}$ m/s = 0.074 mm/s.

6. ℓ = 1 m, r = 0.1 mm = 0.1×10^{-3} m

R = 100 Ω, f = ?

$\Rightarrow R = f\ell / a$

$\Rightarrow f = \dfrac{Ra}{\ell} = \dfrac{100\times3.14\times0.1\times0.1\times10^{-6}}{1}$

$= 3.14 \times 10^{-6} = \pi \times 10^{-6}$ Ω-m.

7. $\ell' = 2\ell$

volume of the wire remains constant.

$A\ell = A'\ell'$

$\Rightarrow A\ell = A' \times 2\ell$

$\Rightarrow A' = A/2$

f = Specific resistance

$R = \dfrac{f\ell}{A}$; $R' = \dfrac{f\ell'}{A'}$

$100\ \Omega = \dfrac{f2\ell}{A/2} = \dfrac{4f\ell}{A} = 4R$

$\Rightarrow 4 \times 100\ \Omega = 400\ \Omega$

8. ℓ = 4 m, A = 1 mm^2 = 1×10^{-6} m^2

I = 2 A, n/V = 10^{29}, t = ?

$i = n A V_d e$

$\Rightarrow e = 10^{29} \times 1 \times 10^{-6} \times V_d \times 1.6 \times 10^{-19}$

$\Rightarrow V_d = \dfrac{2}{10^{29}\times10^{-6}\times1.6\times10^{-19}}$

$= \dfrac{1}{0.8\times10^{4}} = \dfrac{1}{8000}$

$t = \dfrac{\ell}{V_d} = \dfrac{4}{1/8000} = 4\times8000$

$= 32000 = 3.2 \times 10^4$ sec.

9. $f_{cu} = 1.7 \times 10^{-8}$ Ω-m

A = 0.01 mm^2 = 0.01×10^{-6} m^2

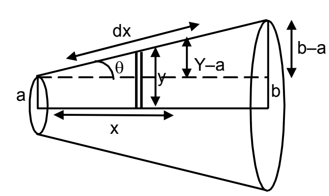
R = 1 KΩ = 10^3 Ω

$R = \dfrac{f\ell}{a}$

$\Rightarrow 10^3 = \dfrac{1.7\times10^{-8}\times\ell}{10^{-6}}$

$\Rightarrow \ell = \dfrac{10^3}{1.7} = 0.58 \times 10^3$ m = 0.6 km.

10. dR, due to the small strip dx at a distanc x d = R = $\dfrac{f\,dx}{\pi y^2}$ …(1)



$\tan\theta = \dfrac{y-a}{x} = \dfrac{b-a}{L}$

$\Rightarrow \dfrac{y-a}{x} = \dfrac{b-a}{L}$

$\Rightarrow L(y - a) = x(b - a)$

$\Rightarrow$ Ly – La = xb – xa

$\Rightarrow$ $L\frac{dy}{dx} - 0 = b - a$ (diff. w.r.t. x)

$\Rightarrow$ $L\frac{dy}{dx} = b - a$

$\Rightarrow$ dx = $\frac{Ldy}{b-a}$ …(2)

Putting the value of dx in equation (1)

dR = $\frac{fLdy}{\pi y^2(b-a)}$

$\Rightarrow$ dR = $\frac{fl}{\pi(b-a)}\frac{dy}{y^2}$

$\Rightarrow$ $\int_0^R dR = \frac{fl}{\pi(b-a)}\int_a^b \frac{dy}{y^2}$

$\Rightarrow$ R = $\frac{fl}{\pi(b-a)}\frac{(b-a)}{ab} = \frac{fl}{\pi ab}$.

11. r = 0.1 mm = 10^{-4} m
R = 1 KΩ = 10^3 Ω, V = 20 V
a) No.of electrons transferred
i = $\frac{V}{R} = \frac{20}{10^3}$ = 20 × 10^{-3} = 2 × 10^{-2} A
q = i t = 2 × 10^{-2} × 1 = 2 × 10^{-2} C.
No. of electrons transferred = $\frac{2\times10^{-2}}{1.6\times10^{-19}} = \frac{2\times10^{-17}}{1.6}$ = 1.25 × 10^{17}.
b) Current density of wire
= $\frac{i}{A} = \frac{2\times10^{-2}}{\pi\times10^{-8}} = \frac{2}{3.14}\times10^{+6}$
= 0.6369 × 10^{+6} = 6.37 × 10^5 A/m^2.

12. A = 2 × 10^{-6} m^2, I = 1 A
f = 1.7 × 10^{-8} Ω-m
E = ?
R = $\frac{f\ell}{A} = \frac{1.7\times10^{-8}\times\ell}{2\times10^{-6}}$
V = IR = $\frac{1\times1.7\times10^{-8}\times\ell}{2\times10^{-6}}$
E = $\frac{dV}{dL} = \frac{V}{l} = \frac{1.7\times10^{-8}\times\ell}{2\times10^{-6}\ell} = \frac{1.7}{2}\times10^{-2}\,V/m$
= 8.5 mV/m.

13. I = 2 m, R = 5 Ω, i = 10 A, E = ?
V = iR = 10 × 5 = 50 V
E = $\frac{V}{l} = \frac{50}{2}$ = 25 V/m.

14. $R'_{Fe} = R_{Fe}(1 + \alpha_{Fe}\,\Delta\theta)$, $R'_{Cu} = R_{Cu}(1 + \alpha_{Cu}\,\Delta\theta)$
$R'_{Fe} = R'_{Cu}$
$\Rightarrow$ $R_{Fe}(1 + \alpha_{Fe}\,\Delta\theta)$, = $R_{Cu}(1 + \alpha_{Cu}\,\Delta\theta)$

$\Rightarrow$ 3.9 [1 + 5 × 10^{-3} (20 – θ)] = 4.1 [1 + 4 x 10^{-3} (20 – θ)]

$\Rightarrow$ 3.9 + 3.9 × 5 × 10^{-3} (20 – θ) = 4.1 + 4.1 × 4 × 10^{-3} (20 – θ)

$\Rightarrow$ 4.1 × 4 × 10^{-3} (20 – θ) – 3.9 × 5 × 10^{-3} (20 – θ) = 3.9 – 4.1

$\Rightarrow$ 16.4(20 – θ) – 19.5(20 – θ) = 0.2 × 10^3

$\Rightarrow$ (20 – θ) (–3.1) = 0.2 × 10^3

$\Rightarrow$ θ – 20 = 200

$\Rightarrow$ θ = 220°C.

15. Let the voltmeter reading when, the voltage is 0 be X.

$$\frac{I_1R}{I_2R} = \frac{V_1}{V_2}$$

$$\Rightarrow \frac{1.75}{2.75} = \frac{14.4 - V}{22.4 - V} \Rightarrow \frac{0.35}{0.55} = \frac{14.4 - V}{22.4 - V}$$

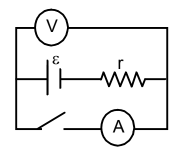
$$\Rightarrow \frac{0.07}{0.11} = \frac{14.4 - V}{22.4 - V} \Rightarrow \frac{7}{11} = \frac{14.4 - V}{22.4 - V}$$

$\Rightarrow$ 7(22.4 – V) = 11(14.4 – V) $\Rightarrow$ 156.8 – 7V = 158.4 – 11V

$\Rightarrow$ (7 – 11)V = 156.8 – 158.4 $\Rightarrow$ –4V = –1.6

$\Rightarrow$ V = 0.4 V.

16. a) When switch is open, no current passes through the ammeter. In the upper part of the circuit the Voltmenter has ∞ resistance. Thus current in it is 0.



∴ Voltmeter read the emf. (There is not Pot. Drop across the resistor).

b) When switch is closed current passes through the circuit and if its value of i.

The voltmeter reads

ε – ir = 1.45

$\Rightarrow$ 1.52 – ir = 1.45

$\Rightarrow$ ir = 0.07

$\Rightarrow$ 1 r = 0.07 $\Rightarrow$ r = 0.07 Ω.

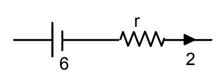
17. E = 6 V, r = 1 Ω, V = 5.8 V, R = ?

$$I = \frac{E}{R+r} = \frac{6}{R+1}, \; V = E - Ir$$

$$\Rightarrow 5.8 = 6 - \frac{6}{R+1} \times 1 \Rightarrow \frac{6}{R+1} = 0.2$$

$\Rightarrow$ R + 1 = 30 $\Rightarrow$ R = 29 Ω.

18. V = ε + ir



$\Rightarrow$ 7.2 = 6 + 2 × r

$\Rightarrow$ 1.2 = 2r $\Rightarrow$ r = 0.6 Ω.

19. a) net emf while charging

9 – 6 = 3V

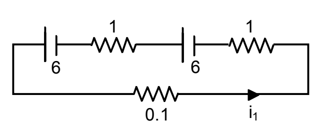
Current = 3/10 = 0.3 A

b) When completely charged.

Internal resistance ‘r’ = 1 Ω

Current = 3/1 = 3 A

20. a) $0.1i_1 + 1\, i_1 - 6 + 1i_1 - 6 = 0$



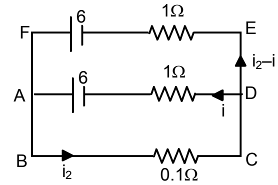
$\Rightarrow 0.1\, i_1 + 1i_1 + 1i_1 = 12$

$$\Rightarrow i_1 = \frac{12}{2.1}$$

ABCDA

$\Rightarrow 0.1i_2 + 1i - 6 = 0$

$\Rightarrow 0.1i_2 + 1i$



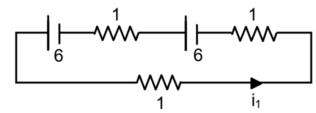
ADEFA,

$\Rightarrow$ $i - 6 + 6 - (i_2 - i)1 = 0$

$\Rightarrow$ $i - i_2 + i = 0$

$\Rightarrow$ $2i - i_2 = 0 \Rightarrow -2i \pm 0.2i = 0$

$\Rightarrow$ $i_2 = 0$.



b) $1i_1 + 1\,i_1 - 6 + 1i_1 = 0$

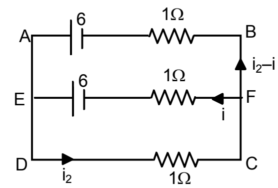
$\Rightarrow$ $3i_1 = 12 \Rightarrow i_1 = 4$

DCFED

$\Rightarrow$ $i_2 + i - 6 = 0 \Rightarrow i_2 + i = 6$

ABCDA,

$i_2 + (i_2 - i) - 6 = 0$



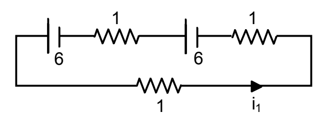
$\Rightarrow$ $i_2 + i_2 - i = 6 \Rightarrow 2i_2 - i = 6$

$\Rightarrow$ $-2i_2 \pm 2i = 6 \Rightarrow i = -2$

$i_2 + i = 6$

$\Rightarrow$ $i_2 - 2 = 6 \Rightarrow i_2 = 8$

$\frac{i_1}{i_2} = \frac{4}{8} = \frac{1}{2}$.



c) $10i_1 + 1i_1 - 6 + 1i_1 - 6 = 0$

$\Rightarrow$ $12i_1 = 12 \Rightarrow i_1 = 1$

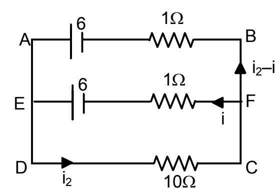
$10i_2 - i_1 - 6 = 0$

$\Rightarrow$ $10i_2 - i_1 = 6$

$\Rightarrow$ $10i_2 + (i_2 - i)1 - 6 = 0$

$\Rightarrow$ $11i_2 = 6$

$\Rightarrow$ $-i_2 = 0$

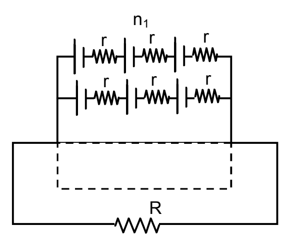


21. a) Total emf = n_1E

in 1 row

Total emf in all news = n_1E

Total resistance in one row = n_1r



Total resistance in all rows = $\frac{n_1r}{n_2}$

Net resistance = $\frac{n_1r}{n_2}$ + R

Current = $\frac{n_1E}{n_1/n_2r + R} = \frac{n_1n_2E}{n_1r + n_2R}$

b) I = $\frac{n_1n_2E}{n_1r + n_2R}$

for I = max,

$n_1r + n_2R$ = min

$\Rightarrow$ $\left(\sqrt{n_1r} - \sqrt{n_2R}\right)^2 + 2\sqrt{n_1rn_2R}$ = min

it is min, when

$\sqrt{n_1r} = \sqrt{n_2R}$

$\Rightarrow$ $n_1r = n_2R$

I is max when $n_1r = n_2R$.

22. E = 100 V, R′ = 100 kΩ = 100000 Ω

R = 1 – 100

When no other resister is added or R = 0.

$i = \frac{E}{R'} = \frac{100}{100000} = 0.001 \text{Amp}$

When R = 1

$i = \frac{100}{100000+1} = \frac{100}{100001} = 0.0009\text{A}$

When R = 100

$i = \frac{100}{100000+100} = \frac{100}{100100} = 0.000999 \text{ A}$.

Upto R = 100 the current does not upto 2 significant digits. Thus it proved.

23. A_1 = 2.4 A

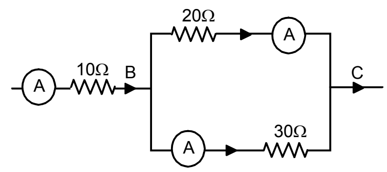
Since A_1 and A_2 are in parallel,

$\Rightarrow$ 20 × 2.4 = 30 × X

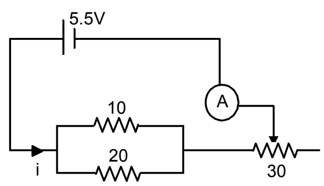
$\Rightarrow X = \frac{20 \times 2.4}{30}$ = 1.6 A.

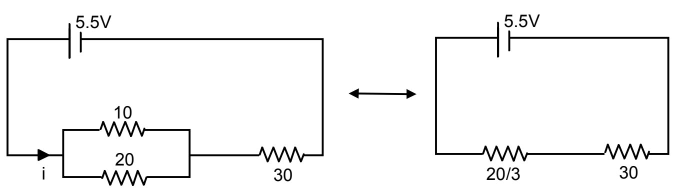
Reading in Ammeter A_2 is 1.6 A.

$A_3 = A_1 + A_2$ = 2.4 + 1.6 = 4.0 A.

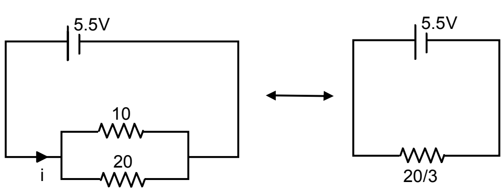


24.





$i_{min} = \frac{5.5 \times 3}{110} = 0.15$



$i_{max} = \frac{5.5 \times 3}{20} = \frac{16.5}{20} = 0.825.$

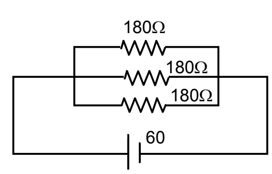
25. a) $R_{eff} = \frac{180}{3} = 60\ \Omega$

i = 60 / 60 = 1 A

b) $R_{eff} = \frac{180}{2} = 90\ \Omega$

i = 60 / 90 = 0.67 A

c) $R_{eff} = 180\ \Omega \Rightarrow$ i = 60 / 180 = 0.33 A



26. Max. R = (20 + 50 + 100) Ω = 170 Ω

Min R = $\dfrac{1}{\left(\frac{1}{20}+\frac{1}{50}+\frac{1}{100}\right)} = \dfrac{100}{8}$ = 12.5 Ω.

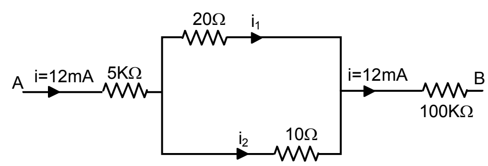
27. The various resistances of the bulbs = $\dfrac{V^2}{P}$

Resistances are $\dfrac{(15)^2}{10}, \dfrac{(15)^2}{10}, \dfrac{(15)^2}{15}$ = 45, 22.5, 15.

Since two resistances when used in parallel have resistances less than both.
The resistances are 45 and 22.5.

28. $i_1 \times 20 = i_2 \times 10$

$\Rightarrow \dfrac{i_1}{i_2} = \dfrac{10}{20} = \dfrac{1}{2}$



i_1 = 4 mA, i_2 = 8 mA

Current in 20 KΩ resistor = 4 mA
Current in 10 KΩ resistor = 8 mA
Current in 100 KΩ resistor = 12 mA

$V = V_1 + V_2 + V_3$

= 5 KΩ × 12 mA + 10 KΩ × 8 mA + 100 KΩ × 12 mA

= 60 + 80 + 1200 = 1340 volts.

29. $R_1 = R$, $i_1 = 5$ A

$R_2 = \dfrac{10R}{10+R}$, $i_2 = 6$A

Since potential constant,

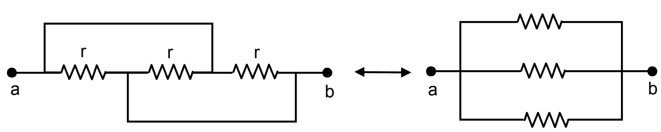
$i_1R_1 = i_2R_2$

$\Rightarrow 5 \times R = \dfrac{6 \times 10R}{10+R}$

$\Rightarrow (10 + R)5 = 60$

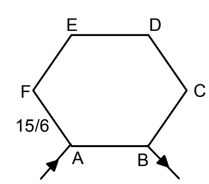
$\Rightarrow 5R = 10 \Rightarrow R = 2\ \Omega$.

30.



Eq. Resistance = r/3.

31. a) $R_{eff} = \dfrac{\frac{15\times5}{6}\times\frac{15}{6}}{\frac{15\times5}{6}+\frac{15}{6}} = \dfrac{\frac{15\times5\times15}{6\times6}}{\frac{75+15}{6}}$



$= \dfrac{15\times5\times15}{6\times90} = \dfrac{25}{12}$ = 2.08 Ω.

b) Across AC,

$R_{eff} = \dfrac{\frac{15\times4}{6}\times\frac{15\times2}{6}}{\frac{15\times4}{6}+\frac{15\times2}{6}} = \dfrac{\frac{15\times4\times15\times2}{6\times6}}{\frac{60+30}{6}}$

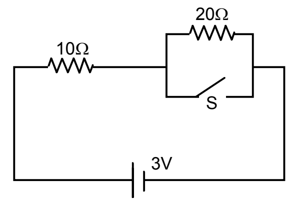
$= \dfrac{15\times4\times15\times2}{6\times90} = \dfrac{10}{3}$ = 3.33 Ω.

c) Across AD,

$$R_{eff} = \frac{\frac{15\times3}{6}\times\frac{15\times3}{6}}{\frac{15\times3}{6}+\frac{15\times3}{6}} = \frac{\frac{15\times3\times15\times3}{6\times6}}{\frac{60+30}{6}}$$

$$= \frac{15\times3\times15\times3}{6\times90} = \frac{15}{4} = 3.75\ \Omega.$$

32. a) When S is open



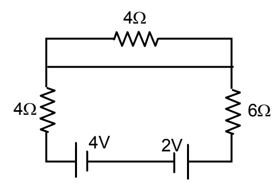
R_{eq} = (10 + 20) Ω = 30 Ω.

i = When S is closed,

R_{eq} = 10 Ω

i = (3/10) Ω = 0.3 Ω.

33. a) Current through (1) 4 Ω resistor = 0



b) Current through (2) and (3)

net E = 4V – 2V = 2V

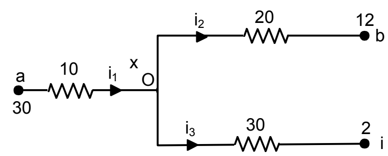
(2) and (3) are in series,

R_{eff} = 4 + 6 = 10 Ω

i = 2/10 = 0.2 A

Current through (2) and (3) are 0.2 A.

34. Let potential at the point be xV.



$(30 – x) = 10\ i_1$

$(x – 12) = 20\ i_2$

$(x – 2) = 30\ i_3$

$i_1 = i_2 + i_3$

$$\Rightarrow \frac{30-x}{10} = \frac{x-12}{20} + \frac{x-2}{30}$$

$$\Rightarrow 30 - x = \frac{x-12}{2} + \frac{x-2}{3}$$

$$\Rightarrow 30 - x = \frac{3x-36+2x-4}{6}$$

⇒ 180 – 6x = 5x – 40

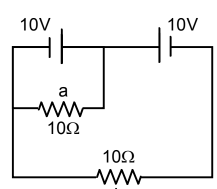
⇒ 11x = 220 ⇒ x = 220 / 11 = 20 V.

$$i_1 = \frac{30-20}{10} = 1\ A$$

$$i_2 = \frac{20-12}{20} = 0.4\ A$$

$$i_3 = \frac{20-2}{30} = \frac{6}{10} = 0.6\ A.$$

35. a) Potential difference between terminals of ‘a’ is 10 V.

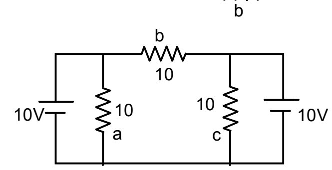


i through a = 10 / 10 = 1A

Potential different between terminals of b is 10 – 10 = 0 V

i through b = 0/10 = 0 A

b) Potential difference across ‘a’ is 10 V

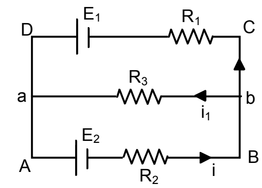


i through a = 10 / 10 = 1A

Potential different between terminals of b is 10 – 10 = 0 V

i through b = 0/10 = 0 A

36. a) In circuit, AB ba A



$E_2 + iR_2 + i_1R_3 = 0$

In circuit, $i_1R_3 + E_1 - (i - i_1)R_1 = 0$

$\Rightarrow i_1R_3 + E_1 - iR_1 + i_1R_1 = 0$

$[iR_2 + i_1R_3 \quad = -E_2]R_1$

$[iR_2 - i_1(R_1 + R_3) = E_1]\ R_2$

$iR_2R_1 + i_1R_3R_1 \quad = -E_2R_1$

$iR_2R_1 - i_1R_2(R_1 + R_3) \quad = E_1\ R_2$

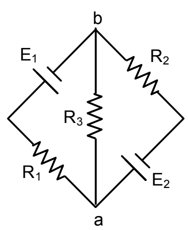
$iR_3R_1 + i_1R_2R_1 + i_1R_2R_3 = E_1R_2 - E_2R_1$

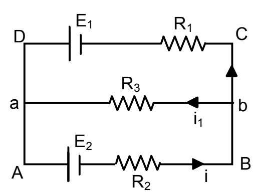
$\Rightarrow i_1(R_3R_1 + R_2R_1 + R_2R_3) = E_1R_2 - E_2R_1$

$$\Rightarrow i_1 = \frac{E_1R_2 - E_2R_1}{R_3R_1 + R_2R_1 + R_2R_3}$$

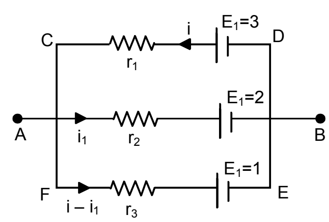
$$\Rightarrow \frac{E_1R_2R_3 - E_2R_1R_3}{R_3R_1 + R_2R_1 + R_2R_3} = \left(\frac{\frac{E_1}{R_1} - \frac{E_2}{R_2}}{\frac{1}{R_2} + \frac{1}{R_1} + \frac{1}{R_3}}\right)$$

b) $\therefore$ Same as a





37. In circuit ABDCA,



$i_1 + 2 - 3 + i = 0$

$\Rightarrow\ i + i_1 - 1 = 0$ …(1)

In circuit CFEDC,

$(i - i_1) + 1 - 3 + i = 0$

$\Rightarrow\ 2i - i_1 - 2 = 0$ …(2)

From (1) and (2)

$3i = 3 \Rightarrow i = 1$ A

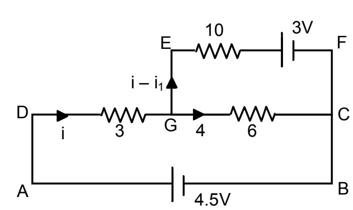
$i_1 = 1 - i = 0$ A

$i - i_1 = 1 - 0 = 1$ A

Potential difference between A and B

$= E - ir = 3 - 1.1 = 2$ V.

38. In the circuit ADCBA,



$3i + 6i_1 - 4.5 = 0$

In the circuit GEFCG,

$3i + 6i_1 = 4.5 \quad = \quad 10i - 10i_1 - 6i_1 = -3$

$\Rightarrow\ [10i - 16i_1 = -3]3$ …(1)

$[3i + 6i_1 = 4.5]\ 10$ …(2)

From (1) and (2)

$-108\ i_1 = -54$

$$\Rightarrow\ i_1 = \frac{54}{108} = \frac{1}{2} = 0.5$$

$3i + 6 \times ½ - 4.5 = 0$

$3i - 1.5 = 0 \Rightarrow i = 0.5.$

Current through 10 Ω resistor = 0 A.

39. In AHGBA,

$2 + (i - i_1) - 2 = 0$

$\Rightarrow\ i - i_1 = 0$

In circuit CFEDC,

$-(i_1 - i_2) + 2 + i_2 - 2 = 0$

$\Rightarrow\ i_2 - i_1 + i_2 = 0 \Rightarrow 2i_2 - i_1 = 0.$

In circuit BGFCB,

$-(i_1 - i_2) + 2 + (i_1 - i_2) - 2 = 0$

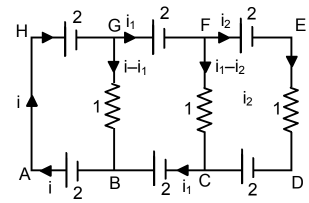
$\Rightarrow\ i_1 - i + i_1 - i_2 = 0 \qquad \Rightarrow\ 2i_1 - i - i_2 = 0 \qquad \ldots(1)$

$\Rightarrow\ i_1 - (i - i_1) - i_2 = 0 \qquad \Rightarrow\ i_1 - i_2 = 0 \qquad \ldots(2)$

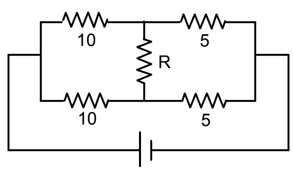
$\therefore\ i_1 - i_2 = 0$

From (1) and (2)

Current in the three resistors is 0.



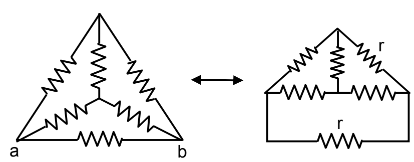
40.



For an value of R, the current in the branch is 0.

41. a) $R_{eff} = \dfrac{(2r/2)\times r}{(2r/2)+r}$

$= \dfrac{r^2}{2r} = \dfrac{r}{2}$

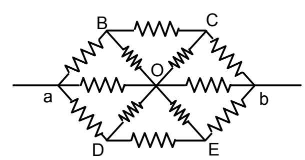


b) At 0 current coming to the junction is current going from BO = Current going along OE.

Current on CO = Current on OD

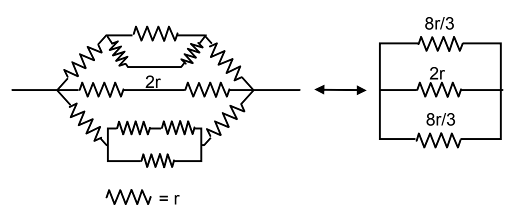
Thus it can be assumed that current coming in OC goes in OB.

Thus the figure becomes

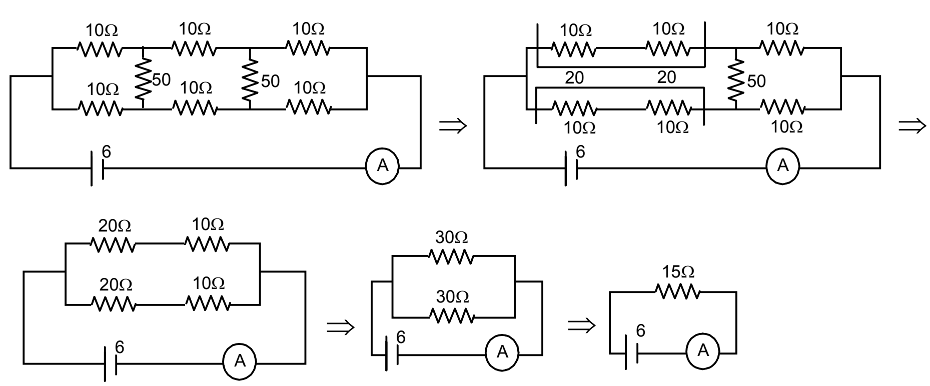


$\left[r + \left(\dfrac{2r.r}{3r}\right) + r\right] = 2r + \dfrac{2r}{3} = \dfrac{8r}{3}$

$R_{eff} = \dfrac{(8r/6)\times 2r}{(8r/6)+2r} = \dfrac{8r^2/3}{20r/6} = \dfrac{8r^2}{3}\times\dfrac{6}{20} = \dfrac{8r}{10} = 4r.$



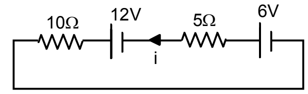
42.



$I = \dfrac{6}{15} = \dfrac{2}{5} = 0.4\ A\ .$

43. a) Applying Kirchoff's law,

$10i - 6 + 5i - 12 = 0$



$\Rightarrow 10i + 5i = 18$

$\Rightarrow 15i = 18$

$\Rightarrow i = \frac{18}{15} = \frac{6}{5} = 1.2$ A.

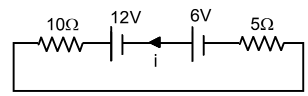
b) Potential drop across 5 Ω resistor,

$i\,5 = 1.2 \times 5$ V = 6 V

c) Potential drop across 10 Ω resistor

$i\,10 = 1.2 \times 10$ V = 12 V

d) $10i - 6 + 5i - 12 = 0$



$\Rightarrow 10i + 5i = 18$

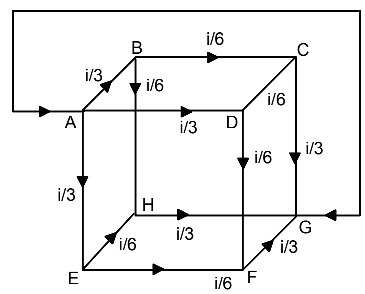
$\Rightarrow 15i = 18$

$\Rightarrow i = \frac{18}{15} = \frac{6}{5} = 1.2$ A.

Potential drop across 5 Ω resistor = 6 V

Potential drop across 10 Ω resistor = 12 V

44. Taking circuit ABHGA,



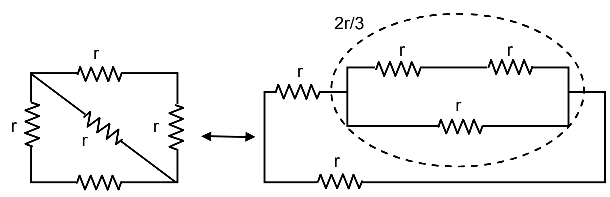
$$\frac{i}{3r} + \frac{i}{6r} + \frac{i}{3r} = V$$

$$\Rightarrow \left(\frac{2i}{3} + \frac{i}{6}\right) r = V$$

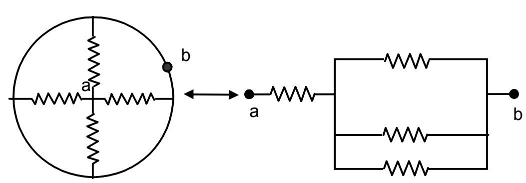
$$\Rightarrow V = \frac{5i}{6} r$$

$$\Rightarrow R_{eff} = \frac{V}{i} = \frac{5}{6r}$$

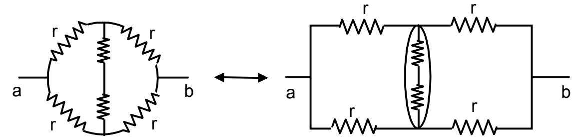
45. $R_{eff} = \dfrac{\left(\frac{2r}{3} + r\right) r}{\left(\frac{2r}{3} + r + r\right)} = \dfrac{5r}{8}$



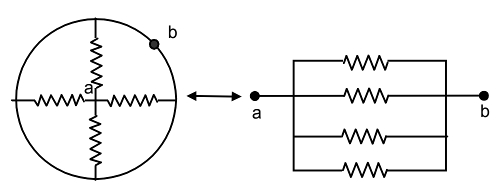
$R_{eff} = \frac{r}{3} + r = \frac{4r}{3}$



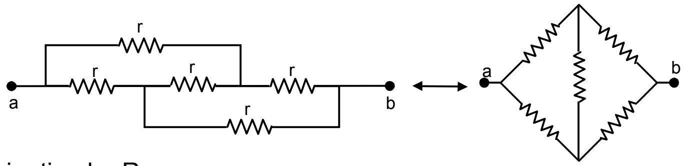
$R_{eff} = \frac{2r}{2} = r$



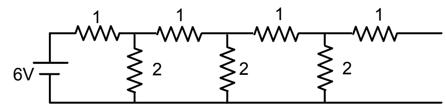
$R_{eff} = \frac{r}{4}$



$R_{eff} = r$



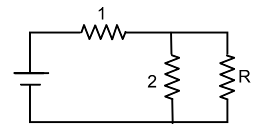
46. a) Let the equation resistance of the combination be R.



$\left(\dfrac{2R}{R+2}\right) + 1 = R$

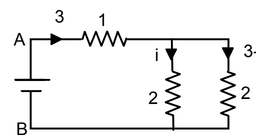
$\Rightarrow \dfrac{2R + R + 2}{R+2} = R \Rightarrow 3R + 2 = R^2 + 2R$

$\Rightarrow R^2 - R - 2 = 0$



$\Rightarrow R = \dfrac{+1 \pm \sqrt{1 + 4.1.2}}{2.1} = \dfrac{1 \pm \sqrt{9}}{2} = \dfrac{1 \pm 3}{2} = 2\ \Omega.$

b) Total current sent by battery = $\dfrac{6}{R_{eff}} = \dfrac{6}{2} = 3$



Potential between A and B

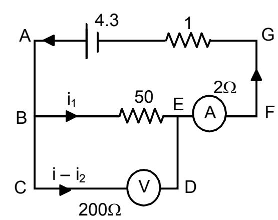
3.1 + 2.i = 6

$\Rightarrow$ 3 + 2i = 6 $\Rightarrow$ 2i = 3

$\Rightarrow$ i = 1.5 a

47. a) In circuit ABFGA,

$i_1\ 50 + 2i + i - 4.3 = 0$



$\Rightarrow 50i_1 + 3i = 4.3$ …(1)

In circuit BEDCB,

$50i_1 - (i - i_1)200 = 0$

$\Rightarrow 50i_1 - 200i + 200i_1 = 0$

$\Rightarrow 250\ i_1 - 200i = 0$

$\Rightarrow 50i_1 - 40i = 0$ …(2)

From (1) and (2)

43i = 4.3 $\Rightarrow$ i = 0.1

$5i_1 = 4 \times i = 4 \times 0.1$ $\Rightarrow i_1 = \dfrac{4 \times 0.1}{5} = 0.08$ A.

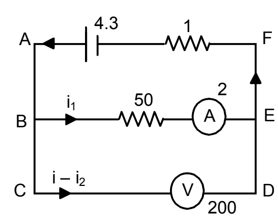
Ammeter reads a current = i = 0.1 A.

Voltmeter reads a potential difference equal to $i_1 \times 50 = 0.08 \times 50 = 4$ V.

b) In circuit ABEFA,

$50i_1 + 2i_1 + 1i - 4.3 = 0$

$\Rightarrow 52i_1 + i = 4.3$



$\Rightarrow 200 \times 52i_1 + 200\ i = 4.3 \times 200$ …(1)

In circuit BCDEB,

$(i - i_1)200 - i_1 2 - i_1 50 = 0$

$\Rightarrow 200i - 200i_1 - 2i_1 - 50i_1 = 0$

$\Rightarrow 200i - 252i_1 = 0$ …(2)

From (1) and (2)

$i_1(10652) = 4.3 \times 2 \times 100$

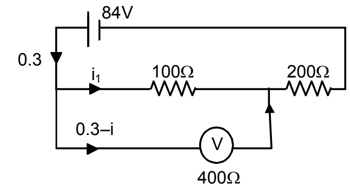
$\Rightarrow i_1 = \dfrac{4.3 \times 2 \times 100}{10652} = 0.08$

i = 4.3 – 52 × 0.08 = 0.14

Reading of the ammeter = 0.08 a

Reading of the voltmeter = $(i - i_1)200 = (0.14 - 0.08) \times 200 = 12$ V.

48. a) $R_{eff} = \frac{100 \times 400}{500} + 200 = 280$



$i = \frac{84}{280} = 0.3$

100i = (0.3 – i) 400

⇒ i = 1.2 – 4i

⇒ 5i = 1.2 ⇒ i = 0.24.

Voltage measured by the voltmeter = $\frac{0.24 \times 100}{24V}$

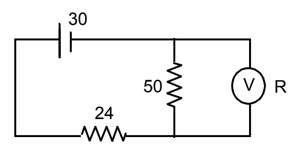
b) If voltmeter is not connected

R_{eff} = (200 + 100) = 300 Ω

$i = \frac{84}{300}$ = 0.28 A

Voltage across 100 Ω = (0.28 × 100) = 28 V.

49. Let resistance of the voltmeter be R Ω.



$R_1 = \frac{50R}{50+R}$, R_2 = 24

Both are in series.

$30 = V_1 + V_2$

$\Rightarrow 30 = iR_1 + iR_2$

$\Rightarrow 30 - iR_2 = iR_1$

$\Rightarrow iR_1 = 30 - \frac{30}{R_1 + R_2} R_2$

$\Rightarrow V_1 = 30\left(1 - \frac{R_2}{R_1 + R_2}\right)$

$\Rightarrow V_1 = 30\left(\frac{R_1}{R_1 + R_2}\right)$

$$\Rightarrow 18 = 30\left(\frac{50R}{50 + R\left(\frac{50R}{50+R} + 24\right)}\right)$$

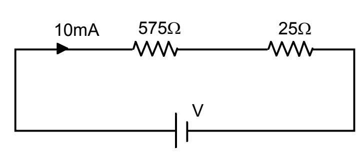
$$\Rightarrow 18 = 30\left(\frac{50R \times (50+R)}{(50+R) + (50R+24)(50+R)}\right) = \frac{30(50R)}{50R + 1200 + 24R}$$

$\Rightarrow 18 = \frac{30 \times 50 \times R}{74R + 1200}$ = 18(74R + 1200) = 1500 R

⇒ 1332R + 21600 = 1500 R ⇒ 21600 = 1.68 R

⇒ R = 21600 / 168 = 128.57.

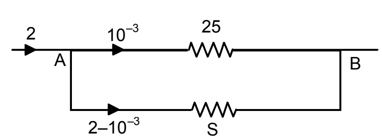
50. Full deflection current = 10 mA = (10×10^{-3})A



R_{eff} = (575 + 25)Ω = 600 Ω

$V = R_{eff} \times i = 600 \times 10 \times 10^{-3}$ = 6 V.

51. G = 25 Ω, Ig = 1 ma, I = 2A, S = ?



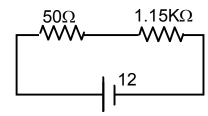
Potential across A B is same

$25 \times 10^{-3} = (2 - 10^{-3})S$

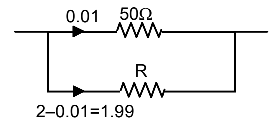
$\Rightarrow S = \frac{25 \times 10^{-3}}{2 - 10^{-3}} = \frac{25 \times 10^{-3}}{1.999}$

$= 12.5 \times 10^{-3} = 1.25 \times 10^{-2}$.

52. $R_{eff} = (1150 + 50)\Omega = 1200\ \Omega$

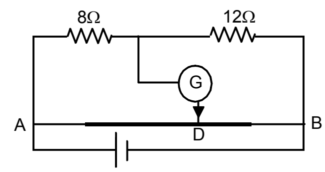


$i = (12 / 1200)A = 0.01\ A.$
(The resistor of 50 Ω can tolerate)
Let R be the resistance of sheet used.
The potential across both the resistors is same.
$0.01 \times 50 = 1.99 \times R$



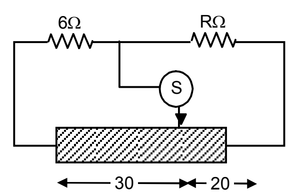
$\Rightarrow R = \frac{0.01 \times 50}{1.99} = \frac{50}{199} = 0.251\ \Omega.$

53. If the wire is connected to the potentiometer wire so that $\frac{R_{AD}}{R_{DB}} = \frac{8}{12}$, then according to wheat stone's bridge no current will flow through galvanometer.



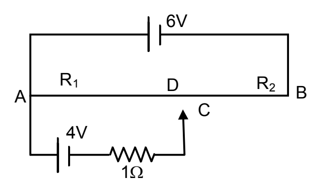
$\frac{R_{AB}}{R_{DB}} = \frac{L_{AB}}{L_B} = \frac{8}{12} = \frac{2}{3}$ (Acc. To principle of potentiometer).
$I_{AB} + I_{DB} = 40$ cm
$\Rightarrow I_{DB}\ 2/3 + I_{DB} = 40$ cm
$\Rightarrow (2/3 + 1)I_{DB} = 40$ cm
$\Rightarrow 5/3\ I_{DB} = 40 \Rightarrow L_{DB} = \frac{40 \times 3}{5} = 24$ cm.
$I_{AB} = (40 – 24)$ cm = 16 cm.

54. The deflections does not occur in galvanometer if the condition is a balanced wheatstone bridge.



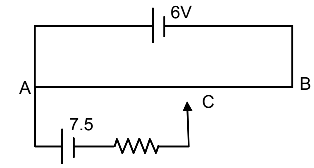
Let Resistance / unit length = r.
Resistance of 30 m length = 30 r.
Resistance of 20 m length = 20 r.
For balanced wheatstones bridge = $\frac{6}{R} = \frac{30r}{20r}$
$\Rightarrow 30\ R = 20 \times 6 \Rightarrow R = \frac{20 \times 6}{30} = 4\ \Omega.$

55. a) Potential difference between A and B is 6 V.



B is at 0 potential.
Thus potential of A point is 6 V.
The potential difference between Ac is 4 V.
$V_A – V_C = 0.4$
$V_C = V_A – 4 = 6 – 4 = 2$ V.

b) The potential at D = 2V, $V_{AD} = 4$ V ; V_{BD} = OV
Current through the resisters R_1 and R_2 are equal.
Thus, $\frac{4}{R_1} = \frac{2}{R_2}$
$\Rightarrow \frac{R_1}{R_2} = 2$
$\Rightarrow \frac{l_1}{l_2} = 2$ (Acc. to the law of potentiometer)
$l_1 + l_2 = 100$ cm
$\Rightarrow l_1 + \frac{l_1}{2} = 100 \text{ cm} \Rightarrow \frac{3l_1}{2} = 100$ cm
$\Rightarrow l_1 = \frac{200}{3}$ cm = 66.67 cm.
AD = 66.67 cm



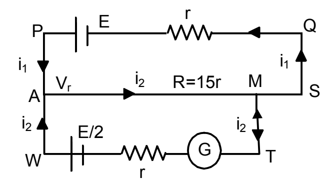
c) When the points C and D are connected by a wire current flowing through it is 0 since the points are equipotential.

d) Potential at A = 6 v

Potential at C = 6 – 7.5 = –1.5 V

The potential at B = 0 and towards A potential increases.

Thus –ve potential point does not come within the wire.



56. Resistance per unit length = $\frac{15r}{6}$

For length x, Rx = $\frac{15r}{6} \times x$

a) For the loop PASQ $(i_1 + i_2)\frac{15}{6}rx + \frac{15}{6}(6 - x)i_1 + i_1R = E$ …(1)

For the loop AWTM, $-i_2.R - \frac{15}{6}rx\,(i_1 + i_2) = E/2$

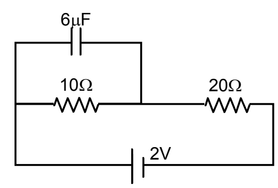
$\Rightarrow i_2R + \frac{15}{6}r \times (i_1 + i_2) = E/2$ …(2)

For zero deflection galvanometer $i_2 = 0 \Rightarrow \frac{15}{6}rx \,.\, i_1 = E/2 = i_1 = \frac{E}{5x \cdot r}$

Putting $i_1 = \frac{E}{5x \cdot r}$ and $i_2 = 0$ in equation (1), we get x = 320 cm.

b) Putting x = 5.6 and solving equation (1) and (2) we get $i_2 = \frac{3E}{22r}$.

57. In steady stage condition no current flows through the capacitor.



$R_{eff} = 10 + 20 = 30\ \Omega$

$i = \frac{2}{30} = \frac{1}{15}A$

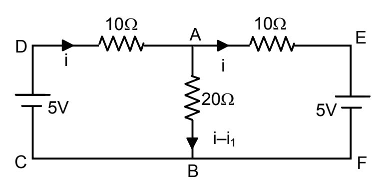
Voltage drop across 10 Ω resistor = i × R

$= \frac{1}{15} \times 10 = \frac{10}{15} = \frac{2}{3}V$

Charge stored on the capacitor (Q) = CV

$= 6 \times 10^{-6} \times 2/3 = 4 \times 10^{-6}$ C = 4 μC.

58. Taking circuit, ABCDA,



$10i + 20(i - i_1) - 5 = 0$

$\Rightarrow 10i + 20i - 20i_1 - 5 = 0$

$\Rightarrow 30i - 20i_1 - 5 = 0$ …(1)

Taking circuit ABFEA,

$20(i - i_1) - 5 - 10i_1 = 0$

$\Rightarrow 10i - 20i_1 - 10i_1 - 5 = 0$

$\Rightarrow 20i - 30i_1 - 5 = 0$ …(2)

From (1) and (2)

$(90 - 40)i_1 = 0$

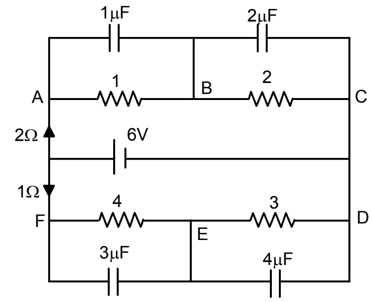
$\Rightarrow i_1 = 0$

$30i - 5 = 0$

$\Rightarrow i = 5/30 = 0.16$ A

Current through 20 Ω is 0.16 A.

59. At steady state no current flows through the capacitor.



$R_{eq} = \frac{3 \times 6}{3 + 6} = 2\ \Omega.$

$i = \frac{6}{2} = 3.$

Since current is divided in the inverse ratio of the resistance in each branch, thus 2Ω will pass through 1, 2 Ω branch and 1 through 3, 3Ω branch

$V_{AB} = 2 \times 1 = 2V.$

Q on 1 μF capacitor = 2 × 1 μc = 2 μC

$V_{BC} = 2 \times 2 = 4V.$

Q on 2 μF capacitor = 4 × 2 μc = 8 μC

$V_{DE} = 1 \times 3 = 2V.$

Q on 4 μF capacitor = 3 × 4 μc = 12 μC

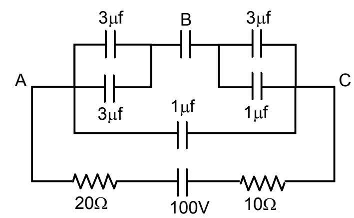
$V_{FE} = 3 \times 1 = V.$

Q across 3 μF capacitor = 3 × 3 μc = 9 μC.

60. C_{eq} = [(3 μf p 3 μf) s (1 μf p 1 μf)] p (1 μf)

= [(3 + 3)μf s (2μf)] p 1 μf

= 3/2 + 1 = 5/2 μf



V = 100 V

Q = CV = 5/2 × 100 = 250 μc

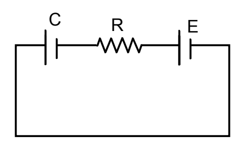
Charge stored across 1 μf capacitor = 100 μc

C_{eq} between A and B is 6 μf = C

Potential drop across AB = V = Q/C = 25 V

Potential drop across BC = 75 V.

61. a) Potential difference = E across resistor



b) Current in the circuit = E/R

c) Pd. Across capacitor = E/R

d) Energy stored in capacitor = $\frac{1}{2}CE^2$

e) Power delivered by battery = E × I = $E \times \frac{E}{R} = \frac{E^2}{R}$

f) Power converted to heat = $\frac{E^2}{R}$

62. $A = 20\ cm^2 = 20 \times 10^{-4}\ m^2$

$d = 1\ mm = 1 \times 10^{-3}\ m$; R = 10 KΩ

$C = \frac{E_0 A}{d} = \frac{8.85 \times 10^{-12} \times 20 \times 10^{-4}}{1 \times 10^{-3}}$

$= \frac{8.85 \times 10^{-12} \times 2 \times 10^{-3}}{10^{-3}} = 17.7 \times 10^{-2}$ Farad.

Time constant = $CR = 17.7 \times 10^{-2} \times 10 \times 10^3$

$= 17.7 \times 10^{-8} = 0.177 \times 10^{-6}\ s = 0.18\ \mu s.$

63. $C = 10\ \mu F = 10^{-5}\ F$, emf = 2 V

$t = 50\ ms = 5 \times 10^{-2}\ s,\ q = Q(1 - e^{-t/RC})$

$Q = CV = 10^{-5} \times 2$

$q = 12.6 \times 10^{-6}\ F$

$\Rightarrow 12.6 \times 10^{-6} = 2 \times 10^{-5}\ (1 - e^{-5 \times 10^{-2} / R \times 10^{-5}})$

$\Rightarrow \dfrac{12.6\times10^{-6}}{2\times10^{-5}} = 1 - e^{-5\times10^{-2}/R\times10^{-5}}$

$\Rightarrow$ 1 – 0.63 = $e^{-5\times10^{3}/R}$

$\Rightarrow \dfrac{-5000}{R} = \ln 0.37$

$\Rightarrow$ R = $\dfrac{5000}{0.9942}$ = 5028 Ω = 5.028 × 10^3Ω = 5 KΩ.

64. C = 20 × 10^{-6} F, E = 6 V, R = 100 Ω
t = 2 × 10^{-3} sec
q = EC (1 – $e^{-t/RC}$)
= $6\times20\times10^{-6}\left(1-e^{\frac{-2\times10^{-3}}{100\times20\times10^{-6}}}\right)$
= 12 x 10^{-5} (1 – e^{-1}) = 7.12 × 0.63 × 10^{-5} = 7.56 × 10^{-5}
= 75.6 × 10^{-6} = 76 μc.

65. C = 10 μF, Q = 60 μC, R = 10 Ω
a) at t = 0, q = 60 μc
b) at t = 30 μs, q = $Qe^{-t/RC}$
= 60 × 10^{-6} × $e^{-0.3}$ = 44 μc
c) at t = 120 μs, q = 60 × 10^{-6} × $e^{-1.2}$ = 18 μc
d) at t = 1.0 ms, q = 60 × 10^{-6} × e^{-10} = 0.00272 = 0.003 μc.

66. C = 8 μF, E = 6V, R = 24 Ω
a) I = $\dfrac{V}{R} = \dfrac{6}{24} = 0.25A$
b) q = Q(1 – $e^{-t/RC}$)
= (8 × 10^{-6} × 6) [1 – c^{-1}] = 48 × 10^{-6} × 0.63 = 3.024 × 10^{-5}
V = $\dfrac{Q}{C} = \dfrac{3.024\times10^{-5}}{8\times10^{-6}} = 3.78$
E = V + iR
$\Rightarrow$ 6 = 3.78 + i24
$\Rightarrow$ i = 0.09 Å

67. A = 40 m^2 = 40 × 10^{-4}
d = 0.1 mm = 1 × 10^{-4} m
R = 16 Ω ; emf = 2 V
C = $\dfrac{E_0A}{d} = \dfrac{8.85\times10^{-12}\times40\times10^{-4}}{1\times10^{-4}}$ = 35.4 × 10^{-11} F
Now, E = $\dfrac{Q}{AE_0}(1-e^{-t/RC}) = \dfrac{CV}{AE_0}(1-e^{-t/RC})$
= $\dfrac{35.4\times10^{-11}\times2}{40\times10^{-4}\times8.85\times10^{-12}}(1-e^{-1.76})$
= 1.655 × 10^{-4} = 1.7 × 10^{-4} V/m.

68. A = 20 cm^2, d = 1 mm, K = 5, e = 6 V
R = 100 × 10^3 Ω, t = 8.9 × 10^{-5} s
C = $\dfrac{KE_0A}{d} = \dfrac{5\times8.85\times10^{-12}\times20\times10^{-4}}{1\times10^{-3}}$
= $\dfrac{10\times8.85\times10^{-3}\times10^{-12}}{10^{-3}}$ = 88.5 × 10^{-12}

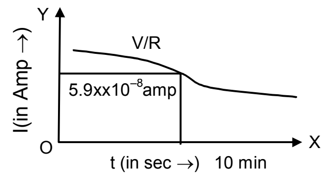
$q = EC(1 - e^{-t/RC})$

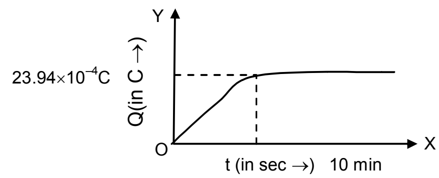
$= 6 \times 88.5 \times 10^{-12} \left(1 - e^{\frac{-89\times10^{-6}}{88.5\times10^{-12}\times10^{4}}}\right) = 530.97$

Energy = $\frac{1}{2} \times \frac{500.97 \times 530}{88.5 \times 10^{-12}}$

$= \frac{530.97 \times 530.97}{88.5 \times 2} \times 10^{12}$

69. Time constant RC = $1 \times 10^6 \times 100 \times 10^6$ = 100 sec





a) $q = VC(1 - e^{-t/CR})$

I = Current = dq/dt = VC.(-) $e^{-t/RC}$, (–1)/RC

$= \frac{V}{R} e^{-t/RC} = \frac{V}{R \cdot e^{t/RC}} = \frac{24}{10^6} \cdot \frac{1}{e^{t/100}}$

$= 24 \times 10^{-6}\ 1/e^{t/100}$

t = 10 min, 600 sec.

$Q = 24 \times 10{+}{-}4 \times (1 - e^{-6}) = 23.99 \times 10^{-4}$

I = $\frac{24}{10^6} \cdot \frac{1}{e^6} = 5.9 \times 10^{-8}$ Amp. .

b) $q = VC(1 - e^{-t/CR})$

70. $Q/2 = Q(1 - e^{-t/CR})$

$\Rightarrow \frac{1}{2} = (1 - e^{-t/CR})$

$\Rightarrow e^{-t/CR} = ½$

$\Rightarrow \frac{t}{RC} = \log 2 \Rightarrow n = 0.69.$

71. $q = Qe^{-t/RC}$

q = 0.1 % Q RC ⇒ Time constant

$= 1 \times 10^{-3}$ Q

So, 1×10^{-3} Q = Q × $e^{-t/RC}$

$\Rightarrow e^{-t/RC} = \ln 10^{-3}$

$\Rightarrow t/RC = -(-6.9) = 6.9$

72. $q = Q(1 - e^{-n})$

$\frac{1}{2}\frac{Q^2}{C} =$ Initial value ; $\frac{1}{2}\frac{q^2}{c} =$ Final value

$\frac{1}{2}\frac{q^2}{c} \times 2 = \frac{1}{2}\frac{Q^2}{C}$

$\Rightarrow q^2 = \frac{Q^2}{2} \Rightarrow q = \frac{Q}{\sqrt{2}}$

$\frac{Q}{\sqrt{2}} = Q(1 - e^{-n})$

$\Rightarrow \frac{1}{\sqrt{2}} = 1 - e^{-n} \Rightarrow e^{-n} = 1 - \frac{1}{\sqrt{2}}$

$\Rightarrow n = \log\left(\frac{\sqrt{2}}{\sqrt{2} - 1}\right) = 1.22$

73. Power = $CV^2 = Q \times V$

Now, $\frac{QV}{2} = QV \times e^{-t/RC}$

$\Rightarrow$ ½ = $e^{-t/RC}$

$\Rightarrow \dfrac{t}{RC} = -\ln 0.5$

$\Rightarrow$ –(–0.69) = 0.69

74. Let at any time t, q = EC (1 – $e^{-t/CR}$)

E = Energy stored = $\dfrac{q^2}{2c} = \dfrac{E^2C^2}{2c}(1-e^{-t/CR})^2 = \dfrac{E^2C}{2}(1-e^{-t/CR})^2$

R = rate of energy stored = $\dfrac{dE}{dt} = \dfrac{-E^2C}{2}\left(\dfrac{-1}{RC}\right)^2(1-e^{-t/RC})e^{-t/RC} = \dfrac{E^2}{CR}\cdot e^{-t/RC}\left(1-e^{-t/CR}\right)$

$\dfrac{dR}{dt} = \dfrac{E^2}{2R}\left[\dfrac{-1}{RC}e^{-t/CR}\cdot(1-e^{-t/CR}) + (-)\cdot e^{-t/CR(1-/RC)}\cdot e^{-t/CR}\right]$

$\dfrac{E^2}{2R} = \left(\dfrac{-e^{-t/CR}}{RC} + \dfrac{e^{-2t/CR}}{RC} + \dfrac{1}{RC}\cdot e^{-2t/CR}\right) = \dfrac{E^2}{2R}\left(\dfrac{2}{RC}\cdot e^{-2t/CR} - \dfrac{e^{-t/CR}}{RC}\right)$ …(1)

For R_{max} dR/dt = 0 $\Rightarrow$ 2.$e^{-t/RC}$ –1 = 0 $\Rightarrow$ $e^{-t/CR}$ = 1/2

$\Rightarrow$ –t/RC = –$\ln^2$ $\Rightarrow$ t = RC ln 2

$\therefore$ Putting t = RC ln 2 in equation (1) We get $\dfrac{dR}{dt} = \dfrac{E^2}{4R}$.

75. C = 12.0 μF = 12 × 10^{-6}

emf = 6.00 V, R = 1 Ω

t = 12 μc, i = i_0 $e^{-t/RC}$

= $\dfrac{CV}{T} \times e^{-t/RC} = \dfrac{12\times10^{-6}\times6}{12\times10^{-6}}\times e^{-1}$

= 2.207 = 2.1 A

b) Power delivered by battery

We known, V = V_0 $e^{-t/RC}$ (where V and V_0 are potential VI)

VI = V_0I $e^{-t/RC}$

$\Rightarrow$ VI = V_0I × e^{-1} = 6 × 6 × e^{-1} = 13.24 W

c) U = $\dfrac{CV^2}{T}(e^{-t/RC})^2$ [$\dfrac{CV^2}{T}$ = energy drawing per unit time]

= $\dfrac{12\times10^{-6}\times36}{12\times10^{-6}}\times\left(e^{-1}\right)^2$ = 4.872.

76. Energy stored at a part time in discharging = $\dfrac{1}{2}CV^2(e^{-t/RC})^2$

Heat dissipated at any time

= (Energy stored at t = 0) – (Energy stored at time t)

= $\dfrac{1}{2}CV^2 - \dfrac{1}{2}CV^2(-e^{-1})^2 = \dfrac{1}{2}CV^2(1-e^{-2})$

77. $\int i^2Rdt = \int i_0^2 Re^{-2t/RC}\,dt = i_0^2R\int e^{-2t/RC}dt$

= $i_0^2R(-RC/2)e^{-2t/RC} = \dfrac{1}{2}Ci_0^2R^2e^{-2t/RC} = \dfrac{1}{2}CV^2$ (Proved).

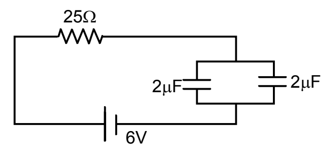
78. Equation of discharging capacitor

= $q_0e^{-t/RC} = \dfrac{K\in_0 AV}{d}e^{\frac{-1}{(\rho dK\in_0 A)/Ad}} = \dfrac{K\in_0 AV}{d}e^{-t/\rho K\in_0}$

$\therefore$ τ = ρK$\in_0$

$\therefore$ Time constant is ρK$\in_0$ is independent of plate area or separation between the plate.

79. $q = q_0(1 - e^{-t/RC})$

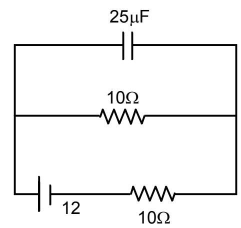


$= 25(2 + 2) \times 10^{-6} \left(1 - e^{\frac{-0.2\times10^{-3}}{25\times4\times10^{-6}}}\right)$

$= 24 \times 10^{-6} (1 - e^{-2}) = 20.75$

Charge on each capacitor = 20.75/2 = 10.3

80. In steady state condition, no current passes through the 25 μF capacitor,



$\therefore$ Net resistance = $\frac{10\Omega}{2} = 5\Omega$.

Net current = $\frac{12}{5}$

Potential difference across the capacitor = 5

Potential difference across the 10 Ω resistor

= 12/5 × 10 = 24 V

$q = Q(e^{-t/RC}) = V \times C(e^{-t/RC}) = 24 \times 25 \times 10^{-6} \left[e^{-1\times10^{-3}/10\times25\times10^{-4}}\right]$

$= 24 \times 25 \times 10^{-6}\, e^{-4} = 24 \times 25 \times 10^{-6} \times 0.0183 = 10.9 \times 10^{-6}$ C

Charge given by the capacitor after time t.

Current in the 10 Ω resistor = $\frac{10.9\times10^{-6}\,C}{1\times10^{-3}\,sec} = 11mA$.

81. C = 100 μF, emf = 6 V, R = 20 KΩ, t = 4 S.

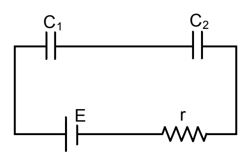
Charging : $Q = CV(1 - e^{-t/RC})$ $\left[\frac{-t}{RC} = \frac{4}{2\times10^4\times10^{-4}}\right]$

$= 6 \times 10^{-4}(1 - e^{-2}) = 5.187 \times 10^{-4}$ C = Q

Discharging : $q = Q(e^{-t/RC}) = 5.184 \times 10^{-4} \times e^{-2}$

$= 0.7 \times 10^{-4}$ C = 70 μc.

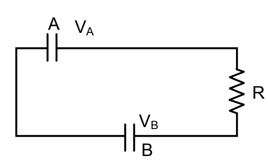
82. $C_{eff} = \frac{C_1C_2}{C_1 + C_2}$



$Q = C_{eff}\, E(1 - e^{-t/RC}) = \frac{C_1C_2}{C_1 + C_2}\, E(1 - e^{-t/RC})$

83. Let after time t charge on plate B is +Q.

Hence charge on plate A is Q – q.



$V_A = \frac{Q-q}{C}$, $V_B = \frac{q}{C}$

$V_A - V_B = \frac{Q-q}{C} - \frac{q}{C} = \frac{Q-2q}{C}$

Current = $\frac{V_A - V_B}{R} = \frac{Q-2q}{CR}$

Current = $\frac{dq}{dt} = \frac{Q-2q}{CR}$

$\Rightarrow \frac{dq}{Q-2q} = \frac{1}{RC}\cdot dt \quad \Rightarrow \int_0^q \frac{dq}{Q-2q} = \frac{1}{RC}\cdot\int_0^t dt$

$\Rightarrow -\frac{1}{2}[\ln(Q-2q) - \ln Q] = \frac{1}{RC}\cdot t \Rightarrow \ln\frac{Q-2q}{Q} = \frac{-2}{RC}\cdot t$

$\Rightarrow Q - 2q = Q\, e^{-2t/RC} \Rightarrow 2q = Q(1 - e^{-2t/RC})$

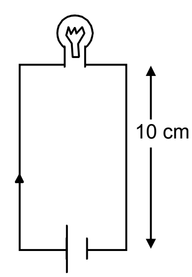
$\Rightarrow q = \frac{Q}{2}(1 - e^{-2t/RC})$

84. The capacitor is given a charge Q. It will discharge and the capacitor will be charged up when connected with battery.

Net charge at time t = $Qe^{-t/RC} + Q(1 - e^{-t/RC})$.

CHAPTER – 33
THERMAL AND CHEMICAL EFFECTS OF ELECTRIC CURRENT

1. $i = 2$ A, $r = 25\ \Omega$,
   $t = 1$ min $= 60$ sec
   Heat developed $= i^2 RT = 2 \times 2 \times 25 \times 60 = 6000$ J
2. $R = 100\ \Omega$, $E = 6$ v
   Heat capacity of the coil = 4 J/k $\Delta T = 15°c$
   Heat liberate $\Rightarrow \frac{E^2}{Rt} = 4 \text{ J/K} \times 15$
   $\Rightarrow \frac{6 \times 6}{100} \times t = 60 \Rightarrow t = 166.67$ sec $= 2.8$ min
3. (a) The power consumed by a coil of resistance R when connected across a supply v is $P = \frac{v^2}{R}$
   The resistance of the heater coil is, therefore $R = \frac{v^2}{P} = \frac{(250)^2}{500} = 125\ \Omega$
   (b) If P = 1000 w then $R = \frac{v^2}{P} = \frac{(250)^2}{1000} = 62.5\ \Omega$
4. $f = 1 \times 10^{-6}\ \Omega$m $P = 500$ W $E = 250$ v
   (a) $R = \frac{V^2}{P} = \frac{250 \times 250}{500} = 125\ \Omega$
   (b) $A = 0.5$ mm$^2 = 0.5 \times 10^{-6}$ m$^2 = 5 \times 10^{-7}$ m^2
   $R = \frac{f l}{A} = l = \frac{RA}{f} = \frac{125 \times 5 \times 10^{-7}}{1 \times 10^{-6}} = 625 \times 10^{-1} = 62.5$ m
   (c) $62.5 = 2\pi r \times n$, $62.5 = 3 \times 3.14 \times 4 \times 10^{-3} \times n$
   $\Rightarrow n = \frac{62.5}{2 \times 3.14 \times 4 \times 10^{3}} \Rightarrow n = \frac{62.5 \times 10^{-3}}{8 \times 3.14} \approx 2500$ turns
5. $V = 250$ V $P = 100$ w
   $R = \frac{v^2}{P} = \frac{(250)^2}{100} = 625\ \Omega$
   Resistance of wire $R = \frac{f l}{A} = 1.7 \times 10^{-8} \times \frac{10}{5 \times 10^{-6}} = 0.034\ \Omega$
   $\therefore$ The effect in resistance = 625.034 Ω
   $\therefore$ The current in the conductor $= \frac{V}{R} = \left(\frac{220}{625.034}\right)$ A
   $\therefore$ The power supplied by one side of connecting wire $= \left(\frac{220}{625.034}\right)^2 \times 0.034$
   $\therefore$ The total power supplied $= \left(\frac{220}{625.034}\right)^2 \times 0.034 \times 2 = 0.0084$ w $= 8.4$ mw



6. $E = 220$ v $P = 60$ w
   $R = \frac{V^2}{P} = \frac{220 \times 220}{60} = \frac{220 \times 11}{3}\ \Omega$
   (a) $E = 180$ v $P = \frac{V^2}{R} = \frac{180 \times 180 \times 3}{220 \times 11} = 40.16 \approx 40$ w

(b) E = 240 v $\quad P = \frac{V^2}{R} = \frac{240 \times 240 \times 3}{220 \times 11} = 71.4 \approx 71$ w

7. Output voltage = 220 ± 1% $\quad$ 1% of 220 V = 2.2 v

The resistance of bulb $R = \frac{V^2}{P} = \frac{(220)^2}{100} = 484\ \Omega$

(a) For minimum power consumed $V_1 = 220 - 1\% = 220 - 2.2 = 217.8$

$\therefore i = \frac{V_1}{R} = \frac{217.8}{484} = 0.45$ A

Power consumed = $i \times V_1 = 0.45 \times 217.8 = 98.01$ W

(b) for maximum power consumed $V_2 = 220 + 1\% = 220 + 2.2 = 222.2$

$\therefore i = \frac{V_2}{R} = \frac{222.2}{484} = 0.459$

Power consumed = $i \times V_2 = 0.459 \times 222.2 = 102$ W

8. V = 220 v $\quad$ P = 100 w

$R = \frac{V^2}{P} = \frac{220 \times 220}{100} = 484\ \Omega$

P = 150 w $\quad V = \sqrt{PR} = \sqrt{150 \times 22 \times 22} = 22\sqrt{150} = 269.4 \approx 270$ v

9. P = 1000 $\quad$ V = 220 v $\quad R = \frac{V^2}{P} = \frac{48400}{1000} = 48.4\ \Omega$

Mass of water = $\frac{1}{100} \times 1000 = 10$ kg

Heat required to raise the temp. of given amount of water = $ms\Delta t = 10 \times 4200 \times 25 = 1050000$

Now heat liberated is only 60%. So $\frac{V^2}{R} \times T \times 60\% = 1050000$

$\Rightarrow \frac{(220)^2}{48.4} \times \frac{60}{100} \times T = 1050000 \Rightarrow T = \frac{10500}{6} \times \frac{1}{60}$ nub = 29.16 min.

10. Volume of water boiled = 4 × 200 cc = 800 cc

$T_1 = 25°C \quad T_2 = 100°C \quad \Rightarrow T_2 - T_1 = 75°C$

Mass of water boiled = 800 × 1 = 800 gm = 0.8 kg

Q(heat req.) = $MS\Delta\theta = 0.8 \times 4200 \times 75 = 252000$ J.

1000 watt – hour = 1000 × 3600 watt-sec = 1000× 3600 J

No. of units = $\frac{252000}{1000 \times 3600} = 0.07$ = 7 paise

(b) $Q = mS\Delta T = 0.8 \times 4200 \times 95$ J

No. of units = $\frac{0.8 \times 4200 \times 95}{1000 \times 3600} = 0.0886 \approx 0.09$

Money consumed = 0.09 Rs = 9 paise.

11. P = 100 w $\quad$ V = 220 v

Case I : Excess power = 100 – 40 = 60 w

Power converted to light = $\frac{60 \times 60}{100} = 36$ w

Case II : Power = $\frac{(220)^2}{484} = 82.64$ w

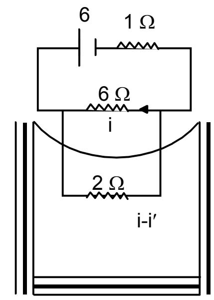
Excess power = 82.64 – 40 = 42.64 w

Power converted to light = $42.64 \times \frac{60}{100} = 25.584$ w

$\Delta P = 36 - 25.584 = 10.416$

Required % = $\frac{10.416}{36} \times 100 = 28.93 \approx 29\%$

12. $R_{eff} = \frac{12}{8} + 1 = \frac{5}{2}$ $\quad i = \frac{6}{(5/2)} = \frac{12}{5}$ Amp.



$i' \, 6 = (i - i')2 \Rightarrow i' \, 6 = \frac{12}{5} \times 2 - 2i$

$8i' = \frac{24}{5} \Rightarrow i' = \frac{24}{5 \times 8} = \frac{3}{5}$ Amp

$i - i' = \frac{12}{5} - \frac{3}{5} = \frac{9}{5}$ Amp

(a) Heat = $i^2 RT = \frac{9}{5} \times \frac{9}{5} \times 2 \times 15 \times 60 = 5832$

2000 J of heat raises the temp. by 1K

5832 J of heat raises the temp. by 2.916K.

(b) When 6Ω resistor get burnt $R_{eff} = 1 + 2 = 3\ \Omega$

$i = \frac{6}{3} = 2$ Amp.

Heat = 2 × 2 × 2 ×15 × 60 = 7200 J

2000 J raises the temp. by 1K

7200 J raises the temp by 3.6k

13. $\theta = 0.001°C$ $\quad a = -46 \times 10^{-6}$ v/deg, $\quad b = -0.48 \times 10^{-6}$ v/deg^2

Emf = $a_{BlAg}\,\theta + (1/2)\, b_{BlAg}\, \theta^2 = -46 \times 10^{-6} \times 0.001 - (1/2) \times 0.48 \times 10^{-6} (0.001)^2$

$= -46 \times 10^{-9} - 0.24 \times 10^{-12} = -46.00024 \times 10^{-9} = -4.6 \times 10^{-8}$ V

14. $E = a_{AB}\theta + b_{AB}\theta^2$ $\quad a_{CuAg} = a_{CuPb} - b_{AgPb} = 2.76 - 2.5 = 0.26\ \mu v/°C$

$b_{CuAg} = b_{CuPb} - b_{AgPb} = 0.012 - 0.012\ \mu vc = 0$

$E = a_{AB}\theta = (0.26 \times 40)\ \mu V = 1.04 \times 10^{-5}$ V

15. $\theta = 0°C$

$a_{Cu,Fe} = a_{Cu,Pb} - a_{Fe,Pb} = 2.76 - 16.6 = -13.8\ \mu v/°C$

$B_{Cu,Fe} = b_{Cu,Pb} - b_{Fe,Pb} = 0.012 + 0.030 = 0.042\ \mu v/°C^2$

Neutral temp. on $-\frac{a}{b} = \frac{13.8}{0.042}$ °C = 328.57°C

16. (a) 1eq. mass of the substance requires 96500 coulombs

Since the element is monoatomic, thus eq. mass = mol. Mass

6.023×10^{23} atoms require 96500 C

1 atoms require $\frac{96500}{6.023 \times 10^{23}}$ C = 1.6×10^{-19} C

(b) Since the element is diatomic eq.mass = (1/2) mol.mass

$\therefore$ (1/2) × 6.023×10^{23} atoms 2eq. 96500 C

$\Rightarrow$ 1 atom require = $\frac{96500 \times 2}{6.023 \times 10^{23}} = 3.2 \times 10^{-19}$ C

17. At Wt. At = 107.9 g/mole

I = 0.500 A

E_{Ag} = 107.9 g [As Ag is monoatomic]

$Z_{Ag} = \frac{E}{f} = \frac{107.9}{96500} = 0.001118$

M = Zit = 0.001118 × 0.5 × 3600 = 2.01

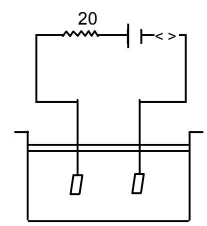
18. t = 3 min = 180 sec w = 2 g
E.C.E = 1.12×10^{-6} kg/c
$\Rightarrow 3 \times 10^{-3} = 1.12 \times 10^{-6} \times i \times 180$
$\Rightarrow i = \dfrac{3 \times 10^{-3}}{1.12 \times 10^{-6} \times 180} = \dfrac{1}{6.72} \times 10^2 \approx 15$ Amp.

19. $\dfrac{H_2}{22.4L} \rightarrow 2g$ $1L \rightarrow \dfrac{2}{22.4}$
m = Zit $\dfrac{2}{22.4} = \dfrac{1}{96500} \times 5 \times T \Rightarrow T = \dfrac{2}{22.4} \times \dfrac{96500}{5} = 1732.21$ sec ≈ 28.7 min ≈ 29 min.

20. w_1 = Zit $\Rightarrow 1 = \dfrac{mm}{3 \times 96500} \times 2 \times 1.5 \times 3600 \Rightarrow mm = \dfrac{3 \times 96500}{2 \times 1.5 \times 3600} = 26.8$ g/mole
$\dfrac{E_1}{E_2} = \dfrac{w_1}{w_2} \Rightarrow \dfrac{107.9}{\left(\dfrac{mm}{3}\right)} = \dfrac{w_1}{1} \Rightarrow w_1 = \dfrac{107.9 \times 3}{26.8} = 12.1$ gm

21. I = 15 A Surface area = 200 cm^2, Thickness = 0.1 mm
Volume of Ag deposited = 200 × 0.01 = 2 cm^3 for one side
For both sides, Mass of Ag = 4 × 10.5 = 42 g
$Z_{Ag} = \dfrac{E}{F} = \dfrac{107.9}{96500}$ m = ZIT
$\Rightarrow 42 = \dfrac{107.9}{96500} \times 15 \times T \Rightarrow T = \dfrac{42 \times 96500}{107.9 \times 15} = 2504.17$ sec = 41.73 min ≈ 42 min

22. w = Zit



$2.68 = \dfrac{107.9}{96500} \times i \times 10 \times 60$
$\Rightarrow I = \dfrac{2.68 \times 965}{107.9 \times 6} = 3.99 \approx 4$ Amp
Heat developed in the 20 Ω resister = $(4)^2 \times 20 \times 10 \times 60$ = 192000 J = 192 KJ

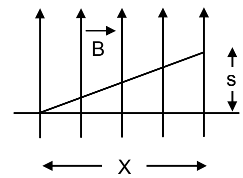
23. For potential drop, t = 30 min = 180 sec
$V_i = V_f + iR \Rightarrow 12 = 10 + 2i \Rightarrow i = 1$ Amp
m = Zit = $\dfrac{107.9}{96500} \times 1 \times 30 \times 60 = 2.01$ g ≈ 2 g

24. A= 10 cm^2× 10^{-4}cm^2
t = 10m = 10×10^{-6}
Volume = A(2t) = $10 \times 10^{-4} \times 2 \times 10 \times 10^{-6} = 2 \times 10^2 \times 10^{-10} = 2 \times 10^{-8}$ m^3
Mass = $2 \times 10^{-8} \times 9000 = 18 \times 10^{-5}$ kg
W = Z × C $\Rightarrow 18 \times 10^{-5} = 3 \times 10^{-7} \times C$
$\Rightarrow q = \dfrac{18 \times 10^{-5}}{3 \times 10^{-7}} = 6 \times 10^2$
$V = \dfrac{W}{q} = \Rightarrow W = Vq = 12 \times 6 \times 10^2 = 76 \times 10^2 = 7.6$ KJ

♣ ♣ ♣ ♣ ♣

CHAPTER – 34
MAGNETIC FIELD

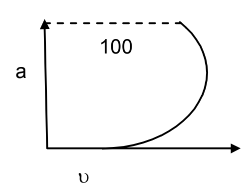
1. $q = 2 \times 1.6 \times 10^{-19}$ C, $\upsilon = 3 \times 10^4$ km/s $= 3 \times 10^7$ m/s
   B = 1 T, $F = qB\upsilon = 2 \times 1.6 \times 10^{-19} \times 3 \times 10^7 \times 1 = 9.6\ 10^{-12}$ N. towards west.
2. KE = 10 Kev = 1.6×10^{-15} J, $\vec{B} = 1 \times 10^{-7}$ T
   (a) The electron will be deflected towards left
   (b) $(1/2)\ mv^2 = KE \Rightarrow V = \sqrt{\frac{KE \times 2}{m}}$ F = qVB & accln = $\frac{qVB}{m_e}$



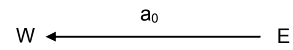
   Applying $s = ut + (1/2)\ at^2 = \frac{1}{2} \times \frac{qVB}{m_e} \times \frac{x^2}{V^2} = \frac{qBx^2}{2m_eV}$

   $$= \frac{qBx^2}{2m_e\sqrt{\frac{KE \times 2}{m}}} = \frac{1}{2} \times \frac{1.6 \times 10^{-19} \times 1 \times 10^{-7} \times 1^2}{9.1 \times 10^{-31} \times \sqrt{\frac{1.6 \times 10^{-15} \times 2}{9.1 \times 10^{-31}}}}$$

   By solving we get, s = 0.0148 ≈ 1.5×10^{-2} cm
3. $B = 4 \times 10^{-3}$ T $(\hat{K})$
   $F = [4\hat{i} + 3\hat{j} \times 10^{-10}]$ N. $F_X = 4 \times 10^{-10}$ N $F_Y = 3 \times 10^{-10}$ N
   $Q = 1 \times 10^{-9}$ C.
   Considering the motion along x-axis :–
   $F_X = quV_YB \Rightarrow V_Y = \frac{F}{qB} = \frac{4 \times 10^{-10}}{1 \times 10^{-9} \times 4 \times 10^{-3}} = 100$ m/s
   Along y-axis
   $F_Y = qV_XB \Rightarrow V_X = \frac{F}{qB} = \frac{3 \times 10^{-10}}{1 \times 10^{-9} \times 4 \times 10^{-3}} = 75$ m/s
   Velocity = $(-75\hat{i} + 100\hat{j})$ m/s
4. $\vec{B} = (7.0\ i - 3.0\ j) \times 10^{-3}$ T
   $\vec{a}$ = acceleration = $(---i + 7j) \times 10^{-6}$ m/s^2
   Let the gap be x.
   Since $\vec{B}$ and $\vec{a}$ are always perpendicular
   $\vec{B} \times \vec{a} = 0$
   $\Rightarrow (7x \times 10^{-3} \times 10^{-6} - 3 \times 10^{-3}\ 7 \times 10^{-6}) = 0$
   $\Rightarrow 7x - 21 = 0 \Rightarrow x = 3$
5. m = 10 g = 10×10^{-3} kg
   q = 400 mc = 400×10^{-6} C
   υ = 270 m/s, B = 500 μt = 500×10^{-6} Tesla
   Force on the particle = $quB = 4 \times 10^{-6} \times 270 \times 500 \times 10^{-6} = 54 \times 10^{-8}$ (K)
   Acceleration on the particle = 54×10^{-6} m/s^2 (K)
   Velocity along $\hat{i}$ and acceleration along $\hat{k}$
   along x-axis the motion is uniform motion and
   along y-axis it is accelerated motion.
   Along – X axis $100 = 270 \times t \Rightarrow t = \frac{10}{27}$
   Along – Z axis $s = ut + (1/2)\ at^2$
   $\Rightarrow s = \frac{1}{2} \times 54 \times 10^{-6} \times \frac{10}{27} \times \frac{10}{27} = 3.7 \times 10^{-6}$



6. $q_P = e$, $m_P = m$, $F = q_P \times E$



or $ma_0 = eE$ or, $E = \dfrac{ma_0}{e}$ towards west

The acceleration changes from a_0 to $3a_0$

Hence net acceleration produced by magnetic field $\vec{B}$ is $2a_0$.

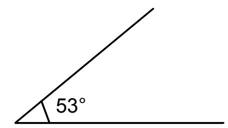
Force due to magnetic field

$= \vec{F_B} = m \times 2a_0 = e \times V_0 \times B$

$\Rightarrow B = \dfrac{2ma_0}{eV_0}$ downwards

7. $l = 10$ cm $= 10 \times 10^{-3}$ m $= 10^{-1}$ m

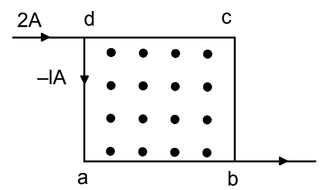
$i = 10$ A, $B = 0.1$ T, $\theta = 53°$



$|F| = iL\,B \sin\theta = 10 \times 10^{-1} \times 0.1 \times 0.79 = 0.0798 \approx 0.08$

direction of F is along a direction $\perp$r to both l and B.

8. $\vec{F} = ilB = 1 \times 0.20 \times 0.1 = 0.02$ N



For $\vec{F} = il \times B$

So, For

da & cb $\rightarrow l \times B = l\,B \sin 90°$ towards left

Hence $\vec{F}$ 0.02 N towards left

For

dc & ab $\rightarrow \vec{F} = 0.02$ N downward

9. $F = ilB \sin\theta$

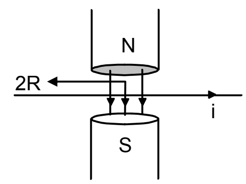
$= ilB \sin 90°$

$= i\,2RB$

$= 2 \times (8 \times 10^{-2}) \times 1$

$= 16 \times 10^{-2}$

$= 0.16$ N.



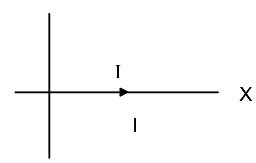
10. Length = l, Current = $l\,\hat{i}$

$\vec{B} = B_0(\hat{i}+\hat{j}+\hat{k})T = B_0\hat{i}+B_0\hat{j}+B_0\hat{k}T$

$F = I\,l \times \vec{B} = I\,l\,\hat{i} \times B_0\hat{i}+B_0\hat{j}+B_0\hat{k}$

$= I\,l\,B_0\,\hat{i}\times\hat{i} + lB_0\,\hat{i}\times\hat{j} + lB_0\,\hat{i}\times\hat{k} = I\,l\,B_0\,\hat{K} - I\,l\,B_0\,\hat{j}$

or, $|\vec{F}| = \sqrt{2I^2l^2{B_0}^2} = \sqrt{2}\,I\,l\,B_0$



11. $i = 5$ A, $l = 50$ cm $= 0.5$ m

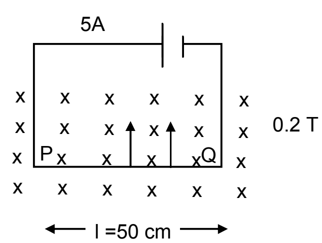
$B = 0.2$ T,

$F = ilB \sin\theta = ilB \sin 90°$

$= 5 \times 0.5 \times 0.2$

$= 0.05$ N

$(\hat{j})$



12. $l = 2\pi a$

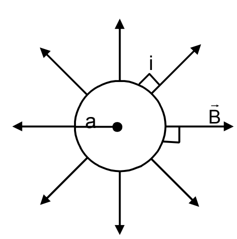
Magnetic field = $\vec{B}$ radially outwards

Current $\Rightarrow$ 'i'

$F = i\,l \times B$

$= i \times (2\pi a \times \vec{B})$

$\otimes = 2\pi ai\,B$ perpendicular to the plane of the figure going inside.

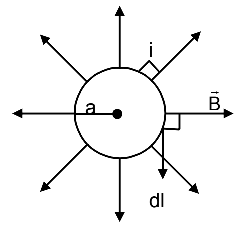


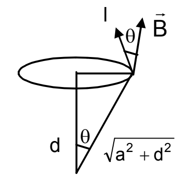
13. $\vec{B} = B_0 \vec{e_r}$

$\vec{e_r}$ = Unit vector along radial direction

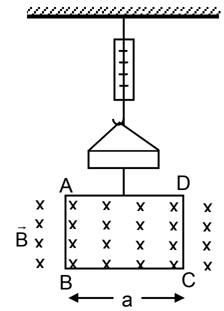
$F = i(\vec{l} \times \vec{B}) = ilB \sin\theta$

$= \frac{i(2\pi a)B_0 a}{\sqrt{a^2+d^2}} = \frac{i2\pi a^2 B_0}{\sqrt{a^2+d^2}}$

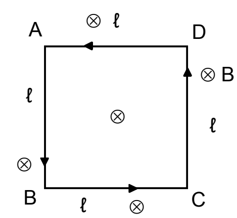




14. Current anticlockwise
Since the horizontal Forces have no effect.
Let us check the forces for current along AD & BC [Since there is no $\vec{B}$]
In AD, F = 0
For BC
F = iaB upward
Current clockwise
Similarly, F = – iaB downwards
Hence change in force = change in tension
= iaB – (–iaB) = 2 iaB

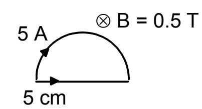


15. F_1 = Force on AD = iℓB inwards
F_2 = Force on BC = iℓB inwards
They cancel each other
F_3 = Force on CD = iℓB inwards
F_4 = Force on AB = iℓB inwards
They also cancel each other.
So the net force on the body is 0.



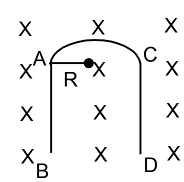
16. For force on a current carrying wire in an uniform magnetic field
We need, $l \rightarrow$ length of wire
$i \rightarrow$ Current
$B \rightarrow$ Magnitude of magnetic field
Since $\vec{F} = i\ell B$
Now, since the length of the wire is fixed from A to B, so force is independent of the shape of the wire.

17. Force on a semicircular wire
= 2iRB
= 2 × 5 × 0.05 × 0.5
= 0.25 N



18. Here the displacement vector $\vec{dl} = \lambda$
So magnetic for $i \rightarrow t\, \vec{dl} \times \vec{B} = i \times \lambda B$

19. Force due to the wire AB and force due to wire CD are equal and opposite to each other. Thus they cancel each other.
Net force is the force due to the semicircular loop = 2iRB



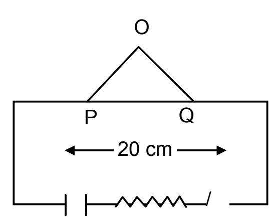
20. Mass = 10 mg = 10^{-5} kg
Length = 1 m
I = 2 A, B = ?
Now, Mg = ilB

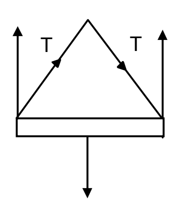
$\Rightarrow B = \frac{mg}{il} = \frac{10^{-5} \times 9.8}{2 \times 1} = 4.9 \times 10^{-5}$ T

21. (a) When switch S is open
2T Cos 30° = mg

$\Rightarrow T = \frac{mg}{2\cos 30°}$

$= \frac{200 \times 10^{-3} \times 9.8}{2\sqrt{(3/2)}} = 1.13$





(b) When the switch is closed and a current passes through the circuit = 2 A

Then

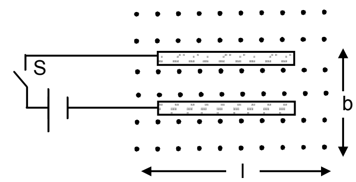
$\Rightarrow$ 2T Cos 30° = mg + ilB

= 200 × 10^{-3} 9.8 + 2 × 0.2 × 0.5 = 1.96 + 0.2 = 2.16

$\Rightarrow 2T = \frac{2.16 \times 2}{\sqrt{3}} = 2.49$

$\Rightarrow T = \frac{2.49}{2} = 1.245 \approx 1.25$

22. Let 'F' be the force applied due to magnetic field on the wire and 'x' be the dist covered.

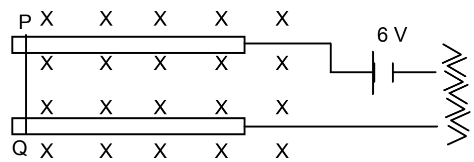


So, F × l = μmg × x

$\Rightarrow$ ibBl = μmgx

$\Rightarrow x = \frac{ibBl}{\mu mg}$

23. μR = F

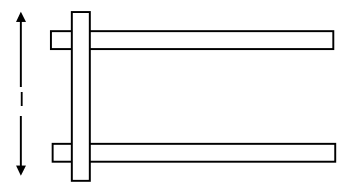


$\Rightarrow \mu \times m \times g = ilB$

$\Rightarrow \mu \times 10 \times 10^{-3} \times 9.8 = \frac{6}{20} \times 4.9 \times 10^{-2} \times 0.8$

$\Rightarrow \mu = \frac{0.3 \times 0.8 \times 10^{-2}}{2 \times 10^{-2}} = 0.12$

24. Mass = m



length = l

Current = i

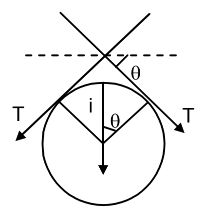
Magnetic field = B = ?

friction Coefficient = μ

iBl = μmg

$\Rightarrow B = \frac{\mu mg}{il}$

25. (a) F_{dl} = i × dl × B   towards centre. (By cross product rule)



(b) Let the length of subtends an small angle of 20 at the centre.

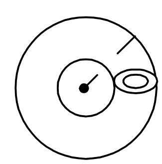
Here 2T sin θ = i × dl × B

$\Rightarrow 2T\theta = i \times a \times 2\theta \times B$ [As $\theta \to 0$, Sin $\theta \approx 0$]

$\Rightarrow$ T = i × a × B   dl = a × 2θ

Force of compression on the wire = i a B

26. $Y = \frac{\text{Stress}}{\text{Strain}} = \frac{\left(\frac{F}{\pi r^2}\right)}{\left(\frac{dl}{L}\right)}$



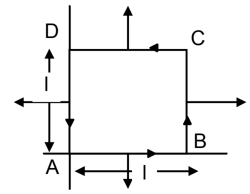
$\Rightarrow \frac{dl}{L} Y = \frac{F}{\pi r^2} \Rightarrow dl = \frac{F}{\pi r^2} \times \frac{L}{Y}$

$= \frac{iaB}{\pi r^2} \times \frac{2\pi a}{Y} = \frac{2\pi a^2 iB}{\pi r^2 Y}$

So, $dp = \frac{2\pi a^2 iB}{\pi r^2 Y}$ (for small cross sectional circle)

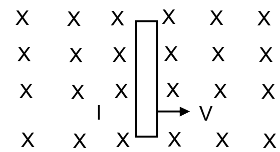
$dr = \frac{2\pi a^2 iB}{\pi r^2 Y} \times \frac{1}{2\pi} = \frac{a^2 iB}{\pi r^2 Y}$

27. $\vec{B} = B_0\left(1+\frac{x}{l}\right)\hat{K}$



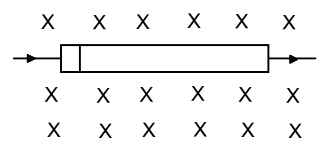
f_1 = force on AB = $iB_0[1 + 0]l = iB_0l$
f_2 = force on CD = $iB_0[1 + 0]l = iB_0l$
f_3 = force on AD = $iB_0[1 + 0/1]l = iB_0l$
f_4 = force on AB = $iB_0[1 + 1/1]l = 2iB_0l$
Net horizontal force = $F_1 - F_2 = 0$
Net vertical force = $F_4 - F_3 = iB_0l$

28. (a) Velocity of electron = υ



Magnetic force on electron
$F = e\upsilon B$
(b) $F = qE$; $F = e\upsilon B$
or, $qE = e\upsilon B$
$\Rightarrow eE = e\upsilon B$ or, $\vec{E} = \upsilon B$
(c) $E = \frac{dV}{dr} = \frac{V}{l}$
$\Rightarrow V = lE = l\upsilon B$

29. (a) $i = V_0nAe$



$\Rightarrow V_0 = \frac{i}{nae}$
(b) $F = ilB = \frac{iBl}{nA} = \frac{iB}{nA}$ (upwards)
(c) Let the electric field be E
$Ee = \frac{iB}{An} \Rightarrow E = \frac{iB}{Aen}$
(d) $\frac{dv}{dr} = E \Rightarrow dV = Edr$
$= E\times d = \frac{iB}{Aen}d$

30. $q = 2.0 \times 10^{-8}$ C $\vec{B} = 0.10$ T
$m = 2.0 \times 10^{-10}$ g $= 2 \times 10^{-13}$ g
$\upsilon = 2.0 \times 10^{3}$ m/′
$R = \frac{m\upsilon}{qB} = \frac{2\times10^{-13}\times2\times10^{3}}{2\times10^{-8}\times10^{-1}} = 0.2$ m = 20 cm
$T = \frac{2\pi m}{qB} = \frac{2\times3.14\times2\times10^{-13}}{2\times10^{-8}\times10^{-1}} = 6.28 \times 10^{-4}$ s

31. $r = \frac{mv}{qB}$
$0.01 = \frac{mv}{e0.1}$ …(1)
$r = \frac{4m\times V}{2e\times0.1}$ …(2)
(2) ÷ (1)
$\Rightarrow \frac{r}{0.01} = \frac{4mVe\times0.1}{2e\times0.1\times mv} = \frac{4}{2} = 2$ $\Rightarrow r = 0.02$ m = 2 cm.

32. KE = 100ev = 1.6×10^{-17} J
$(1/2) \times 9.1 \times 10^{-31} \times V^2 = 1.6 \times 10^{-17}$ J
$\Rightarrow V^2 = \frac{1.6\times10^{-17}\times2}{9.1\times10^{-31}} = 0.35 \times 10^{14}$

or, V = 0.591 × 10^7 m/s

Now r = $\frac{m\upsilon}{qB} \Rightarrow \frac{9.1\times10^{-31}\times0.591\times10^{7}}{1.6\times10^{-19}\times B} = \frac{10}{100}$

$\Rightarrow$ B = $\frac{9.1\times0.591}{1.6}\times\frac{10^{-23}}{10^{-19}}$ = 3.3613 × 10^{-4} T ≈ 3.4 × 10^{-4} T

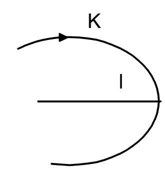
T = $\frac{2\pi m}{qB} = \frac{2\times3.14\times9.1\times10^{-31}}{1.6\times10^{-19}\times3.4\times10^{-4}}$

No. of Cycles per Second f = $\frac{1}{T}$

= $\frac{1.6\times3.4}{2\times3.14\times9.1}\times\frac{10^{-19}\times10^{-4}}{10^{-31}}$ = 0.0951 × 10^8 ≈ 9.51 × 10^6

Note: ∴ Puttig $\vec{B}$ 3.361 × 10^{-4} T We get f = 9.4 × 10^6

33. Radius = l, K.E = K



L = $\frac{mV}{qB} \Rightarrow$ l = $\frac{\sqrt{2mk}}{qB}$

$\Rightarrow$ B = $\frac{\sqrt{2mk}}{ql}$

34. V = 12 KV   E = $\frac{V}{l}$ Now, F = qE = $\frac{qV}{l}$ or, a = $\frac{F}{m} = \frac{qV}{ml}$

υ = 1 × 10^6 m/s

or V = $\sqrt{2\times\frac{qV}{ml}\times l} = \sqrt{2\times\frac{q}{m}\times12\times10^{3}}$

or 1 × 10^6 = $\sqrt{2\times\frac{q}{m}\times12\times10^{3}}$

$\Rightarrow$ 10^{12} = 24 × 10^3 × $\frac{q}{m}$

$\Rightarrow$ $\frac{m}{q} = \frac{24\times10^{3}}{10^{12}}$ = 24 × 10^{-9}

r = $\frac{mV}{qB} = \frac{24\times10^{-9}\times1\times10^{6}}{2\times10^{-1}}$ = 12 × 10^{-2} m = 12 cm

35. V = 10 Km/′ = 10^4 m/s
B = 1 T, q = 2e.
(a) F = qVB = 2 × 1.6 × 10^{-19} × 10^4 × 1 = 3.2 × 10^{-15} N

(b) r = $\frac{mV}{qB} = \frac{4\times1.6\times10^{-27}\times10^{4}}{2\times1.6\times10^{-19}\times1}$ = 2 × $\frac{10^{-23}}{10^{-19}}$ = 2 × 10^{-4} m

(c)Time taken = $\frac{2\pi r}{V} = \frac{2\pi mv}{qB\times v} = \frac{2\pi\times4\times1.6\times10^{-27}}{2\times1.6\times10^{-19}\times1}$

= 4π × 10^{-8} = 4 × 3.14 × 10^{-8} = 12.56 × 10^{-8} = 1.256 × 10^{-7} sex.

36. υ = 3 × 10^6 m/s,   B = 0.6 T,   m = 1.67 × 10^{-27} kg
F = qυB q_P = 1.6 × 10^{-19} C

or, $\vec{a} = \frac{F}{m} = \frac{q\upsilon B}{m}$

= $\frac{1.6\times10^{-19}\times3\times10^{6}\times10^{-1}}{1.67\times10^{-27}}$

= 17.245 × 10^{13} = 1.724 × 10^4 m/s^2

37. (a) R = 1 n, B= 0.5 T, $r = \frac{m\upsilon}{qB}$

$\Rightarrow 1 = \frac{9.1\times10^{-31}\times\upsilon}{1.6\times10^{-19}\times0.5}$

$\Rightarrow \upsilon = \frac{1.6\times0.5\times10^{-19}}{9.1\times10^{-31}}$ = 0.0879 × 10^{10} ≈ 8.8 × 10^{10} m/s

No, it is not reasonable as it is more than the speed of light.

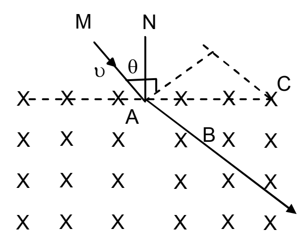
(b) $r = \frac{m\upsilon}{qB}$

$\Rightarrow 1 = \frac{1.6\times10^{-27}\times v}{1.6\times10^{-19}\times0.5}$

$\Rightarrow \upsilon = \frac{1.6\times10^{-19}\times0.5}{1.6\times10^{-27}}$ = 0.5 × 10^8 = 5 × 10^7 m/s.

38. (a) Radius of circular arc = $\frac{m\upsilon}{qB}$

(b) Since MA is tangent to are ABC, described by the particle.
Hence ∠MAO = 90°
Now, ∠NAC = 90° [∵ NA is ⊥r]
∴∠OAC = ∠OCA = θ [By geometry]
Then ∠AOC = 180 – (θ + θ) = π – 2θ



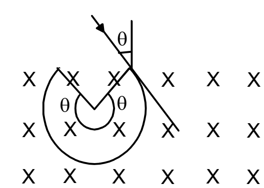
(c) Dist. Covered $l = r\theta = \frac{m\upsilon}{qB}(\pi-2\theta)$

$t = \frac{l}{\upsilon} = \frac{m}{qB}(\pi-2\theta)$

(d) If the charge 'q' on the particle is negative. Then

(i) Radius of Circular arc = $\frac{m\upsilon}{qB}$

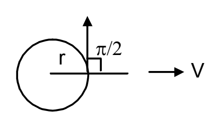
(ii) In such a case the centre of the arc will lie with in the magnetic field, as seen in the fig. Hence the angle subtended by the major arc = π + 2θ

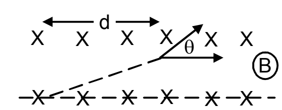


(iii) Similarly the time taken by the particle to cover the same path = $\frac{m}{qB}(\pi+2\theta)$

39. Mass of the particle = m, Charge = q, Width = d

(a) If $d = \frac{mV}{qB}$

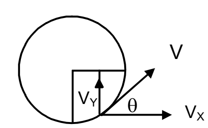




The d is equal to radius. θ is the angle between the radius and tangent which is equal to π/2 (As shown in the figure)

(b) If ≈ $\frac{mV}{2qB}$ distance travelled = (1/2) of radius

Along x-directions $d = V_X t$ [Since acceleration in this direction is 0. Force acts along $\hat{j}$ directions]

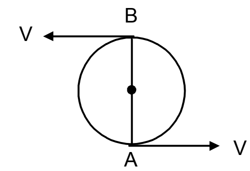


$t = \frac{d}{V_X}$ …(1)

$V_Y = u_Y + a_Y t = \frac{0+qu_XBt}{m} = \frac{qu_XBt}{m}$

From (1) putting the value of t, $V_Y = \frac{qu_XBd}{mV_X}$

$\text{Tan } \theta = \frac{V_Y}{V_X} = \frac{qBd}{mV_X} = \frac{qBmV_X}{2qBmV_X} = \frac{1}{2}$

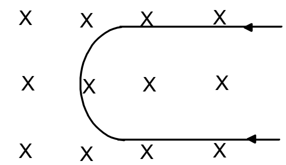


$\Rightarrow \theta = \tan^{-1}\left(\frac{1}{2}\right) = 26.4 \approx 30° = \pi/6$

(c) $d \approx \frac{2mu}{qB}$

Looking into the figure, the angle between the initial direction and final direction of velocity is π.

40. $u = 6\times 10^4$ m/s, B = 0.5 T, $r_1 = 3/2 = 1.5$ cm, $r_2 = 3.5/2$ cm



$r_1 = \frac{mv}{qB} = \frac{A\times(1.6\times10^{-27})\times6\times10^4}{1.6\times10^{-19}\times0.5}$

$\Rightarrow 1.5 = A \times 12 \times 10^{-4}$

$\Rightarrow A = \frac{1.5}{12\times10^{-4}} = \frac{15000}{12}$

$r_2 = \frac{mu}{qB} \Rightarrow \frac{3.5}{2} = \frac{A'\times(1.6\times10^{-27})\times6\times10^4}{1.6\times10^{-19}\times0.5}$

$\Rightarrow A' = \frac{3.5\times0.5\times10^{-19}}{2\times6\times10^4\times10^{-27}} = \frac{3.5\times0.5\times10^4}{12}$

$\frac{A}{A'} = \frac{1.5}{12\times10^{-4}}\times\frac{12\times10^{-4}}{3.5\times0.5} = \frac{6}{7}$

Taking common ration = 2 (For Carbon). The isotopes used are C^{12} and C^{14}

41. V = 500 V B = 20 mT = (2×10^{-3}) T

$E = \frac{V}{d} = \frac{500}{d} \Rightarrow F = \frac{q500}{d} \Rightarrow a = \frac{q500}{dm}$

$\Rightarrow u^2 = 2ad = 2 \times \frac{q500}{dm} \times d \Rightarrow u^2 = \frac{1000\times q}{m} \Rightarrow u = \sqrt{\frac{1000\times q}{m}}$

$r_1 = \frac{m_1\sqrt{1000\times q_1}}{q_1\sqrt{m_1}B} = \frac{\sqrt{m_1}\sqrt{1000}}{\sqrt{q_1}\,B} = \frac{\sqrt{57\times1.6\times10^{-27}\times10^3}}{\sqrt{1.6\times10^{-19}}\times2\times10^{-3}} = 1.19 \times 10^{-2}\text{ m} = 119\text{ cm}$

$r_1 = \frac{m_2\sqrt{1000\times q_2}}{q_2\sqrt{m_2}B} = \frac{\sqrt{m_2}\sqrt{1000}}{\sqrt{q_2}\,B} = \frac{\sqrt{1000\times58\times1.6\times10^{-27}}}{\sqrt{1.6\times10^{-19}}\times20\times10^{-3}} = 1.20 \times 10^{-2}\text{ m} = 120\text{ cm}$

42. For K – 39 : $m = 39 \times 1.6 \times 10^{-27}$ kg, $B = 5 \times 10^{-1}$ T, $q = 1.6 \times 10^{-19}$ C, K.E = 32 KeV.
Velocity of projection : $= (1/2) \times 39 \times (1.6 \times 10^{-27})\, v^2 = 32 \times 10^3 \times 1.6 \times 10^{-27} \Rightarrow v = 4.050957468 \times 10^5$
Through out ht emotion the horizontal velocity remains constant.

$t = \frac{0.01}{40.5095746\,8\times10^5} = 24 \times 10^{-19}$ sec. [Time taken to cross the magnetic field]

Accln. In the region having magnetic field $= \frac{qvB}{m}$

$= \frac{1.6\times10^{-19}\times4.050957468\times10^5\times0.5}{39\times1.6\times10^{-27}} = 5193.535216 \times 10^8 \text{ m/s}^2$

V(in vertical direction) = at $= 5193.535216 \times 10^8 \times 24 \times 10^{-9} = 12464.48452$ m/s.

Total time taken to reach the screen $= \frac{0.965}{40.5095746\,8\times10^5} = 0.000002382$ sec.

Time gap $= 2383 \times 10^{-9} - 24 \times 10^{-9} = 2358 \times 10^{-9}$ sec.

Distance moved vertically (in the time) $= 12464.48452 \times 2358\times 10^{-9} = 0.0293912545$ m

$V^2 = 2as \Rightarrow (12464.48452)^2 = 2 \times 5193.535216 \times 10^8 \times S \Rightarrow S = 0.1495738143 \times 10^{-3}$ m.

Net displacement from line = 0.0001495738143 + 0.0293912545 = 0.0295408283143 m

For K – 41 : $(1/2) \times 41 \times 1.6 \times 10^{-27}$ $v = 32 \times 10^3\ 1.6 \times 10^{-19} \Rightarrow v = 39.50918387$ m/s.

$a = \frac{qvB}{m} = \frac{1.6\times10^{-19}\times395091.8387\times0.5}{41\times1.6\times10^{-27}}$ = 4818.193154 × 10^8 m/s^2

t = (time taken for coming outside from magnetic field) = $\frac{00.1}{39501.8387}$ = 25 × 10^{-9} sec.

V = at (Vertical velocity) = 4818.193154 × 10^8 × 10^8 25 × 10^{-9} = 12045.48289 m/s.

(Time total to reach the screen) = $\frac{0.965}{395091.8387}$ = 0.000002442

Time gap = 2442 × 10^{-9} – 25 × 10^{-9} = 2417 × 10^{-9}

Distance moved vertically = 12045.48289 × 2417 × 10^{-9} = 0.02911393215

Now, V^2 = 2as ⇒ $(12045.48289)^2$ = 2 × 4818.193151 × S ⇒ S = 0.0001505685363 m

Net distance travelled = 0.0001505685363 + 0.02911393215 = 0.0292645006862

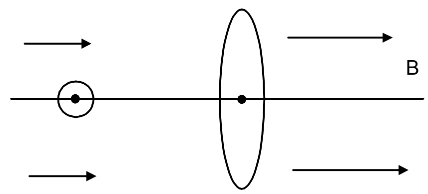
Net gap between K– 39 and K– 41 = 0.0295408283143 – 0.0292645006862

= 0.0001763276281 m ≈ 0.176 mm

43. The object will make a circular path, perpendicular to the plance of paper

Let the radius of the object be r

$\frac{mv^2}{r} = qvB \Rightarrow r = \frac{mV}{qB}$



Here object distance K = 18 cm.

$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$ (lens eqn.) ⇒ $\frac{1}{v}-\left(\frac{1}{-18}\right)=\frac{1}{12}$ ⇒ v = 36 cm.

Let the radius of the circular path of image = r′

So magnification = $\frac{v}{u}=\frac{r'}{r}$(magnetic path = $\frac{\text{image height}}{\text{object height}}$) ⇒ r′ = $\frac{v}{u}r$ ⇒ r′ = $\frac{36}{18}\times4$ = 8 cm.

Hence radius of the circular path in which the image moves is 8 cm.

44. Given magnetic field = B, Pd = V, mass of electron = m, Charge =q,

Let electric field be 'E' ∴E = $\frac{V}{R}$, Force Experienced = eE

Acceleration = $\frac{eE}{m} = \frac{eE}{Rm}$ Now, V^2 = 2 × a × S [∵ x = 0]

$V = \sqrt{\frac{2\times e\times V\times R}{Rm}} = \sqrt{\frac{2eV}{m}}$

Time taken by particle to cover the arc = $\frac{2\pi m}{qB} = \frac{2\pi m}{eB}$

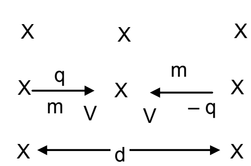
Since the acceleration is along 'Y' axis.

Hence it travels along x axis in uniform velocity

Therefore, ′ = υ × t = $\sqrt{\frac{2em}{m}}\times\frac{2\pi m}{eB} = \sqrt{\frac{8\pi^2 mV}{eB^2}}$

45. (a) The particulars will not collide if

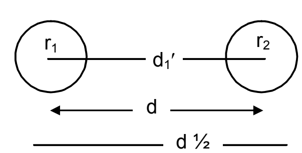
$d = r_1 + r_2$



$\Rightarrow d = \frac{mV_m}{qB}+\frac{mV_m}{qB}$

$\Rightarrow d = \frac{2mV_m}{qB} \Rightarrow V_m = \frac{qBd}{2m}$

(b) $V = \frac{V_m}{2}$



$d_1' = r_1 + r_2 = 2\left(\frac{m\times qBd}{2\times2m\times qB}\right) = \frac{d}{2}$ (min. dist.)

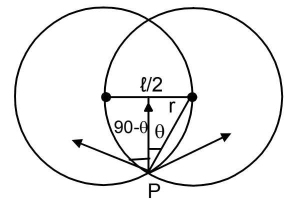
Max. distance $d_2' = d + 2r = d + \frac{d}{2} = \frac{3d}{2}$

(c) $V = 2V_m$

$r_1' = \frac{m_2 V_m}{qB} = \frac{m \times 2 \times qBd}{2n \times qB}$, $r_2 = d$ $\therefore$ The arc is 1/6

(d) $V_m = \frac{qBd}{2m}$

The particles will collide at point P. At point p, both the particles will have motion m in upward direction. Since the particles collide inelastically the stick together.



Distance l between centres = d, $\sin\theta = \frac{l}{2r}$

Velocity upward = $v \cos 90 - \theta = V \sin\theta = \frac{Vl}{2r}$

$\frac{mv^2}{r} = qvB \Rightarrow r = \frac{mv}{qB}$

$V \sin\theta = \frac{vl}{2r} = \frac{vl}{2\frac{mv}{qb}} = \frac{qBd}{2m} = V_m$

Hence the combined mass will move with velocity V_m

46. B = 0.20 T, υ = ? m = 0.010g = 10^{-5} kg, q = 1×10^{-5} C

Force due to magnetic field = Gravitational force of attraction

So, $q\upsilon B = mg$

$\Rightarrow 1 \times 10^{-5} \times \upsilon \times 2 \times 10^{-1} = 1 \times 10^{-5} \times 9.8$

$\Rightarrow \upsilon = \frac{9.8 \times 10^{-5}}{2 \times 10^{-6}} = 49$ m/s.

47. r = 0.5 cm = 0.5×10^{-2} m

B = 0.4 T, E = 200 V/m

The path will straighten, if $qE = quB \Rightarrow E = \frac{rqB \times B}{m}$ $[\because r = \frac{mv}{qB}]$

$\Rightarrow E = \frac{rqB^2}{m} \Rightarrow \frac{q}{m} = \frac{E}{B^2 r} = \frac{200}{0.4 \times 0.4 \times 0.5 \times 10^{-2}} = 2.5 \times 10^5$ c/kg

48. $M_P = 1.6 \times 10^{-27}$ Kg

$\upsilon = 2 \times 10^5$ m/s r = 4 cm = 4×10^{-2} m

Since the proton is undeflected in the combined magnetic and electric field. Hence force due to both the fields must be same.

i.e. $qE = q\upsilon B \Rightarrow E = \upsilon B$

Won, when the electricfield is stopped, then if forms a circle due to force of magnetic field

<u>We know</u> $r = \frac{m\upsilon}{qB}$

$\Rightarrow 4 \times 10^2 = \frac{1.6 \times 10^{-27} \times 2 \times 10^5}{1.6 \times 10^{-19} \times B}$

$\Rightarrow B = \frac{1.6 \times 10^{-27} \times 2 \times 10^5}{4 \times 10^2 \times 1.6 \times 10^{-19}} = 0.5 \times 10^{-1} = 0.005$ T

$E = \upsilon B = 2 \times 10^5 \times 0.05 = 1 \times 10^4$ N/C

49. q = 5 μF = 5×10^{-6} C, m = 5×10^{-12} kg, V = 1 km/s = 10^3 m/′

$\theta = \sin^{-1}(0.9)$, B = 5×10^{-3} T

We have $mv'^2 = qv'B$ $r = \frac{mv'}{qB} = \frac{mv\sin\theta}{qB} = \frac{5 \times 10^{-12} \times 10^3 \times 9}{5 \times 10^{-6} + 5 \times 10^3 + 10} = 0.18$ metre

Hence dimeter = 36 cm.,

Pitch = $\frac{2\pi r}{v\sin\theta} v\cos\theta = \frac{2\times 3.1416\times 0.1\times\sqrt{1-0.51}}{0.9}$ = 0.54 metre = 54 mc.

The velocity has a x-component along with which no force acts so that the particle, moves with uniform velocity.

The velocity has a y-component with which is accelerates with acceleration a. with the Vertical component it moves in a circular crosssection. Thus it moves in a helix.

50. $\vec{B}$ = 0.020 T $\quad\quad$ M_P= 1.6 × 10^{-27} Kg

Pitch = 20 cm = 2 × 10^{-1} m

Radius = 5 cm = 5 × 10^{-2} m

We know for a helical path, the velocity of the proton has got two components $\theta_\perp$ & θ_H

Now, $r = \frac{m\theta_\perp}{qB} \Rightarrow 5\times 10^{-2} = \frac{1.6\times 10^{-27}\times\theta_\perp}{1.6\times 10^{-19}\times 2\times 10^{-2}}$

$\Rightarrow \theta_\perp = \frac{5\times 10^{-2}\times 1.6\times 10^{-19}\times 2\times 10^{-2}}{1.6\times 10^{-27}} = 1\times 10^5$ m/s

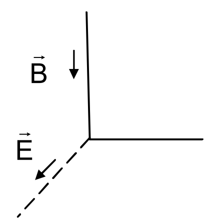
However, θ_H remains constant

$T = \frac{2\pi m}{qB}$

Pitch = θ_H ×T or, $\theta_H = \frac{\text{Pitch}}{T}$

$\theta_H = \frac{2\times 10^{-1}}{2\times 3.14\times 1.6\times 10^{-27}}\times 1.6\times 10^{-19}\times 2\times 10^{-2} = 0.6369\times 10^5 \approx 6.4\times 10^4$ m/s

51. Velocity will be along x – z plane



$\vec{B} = -B_0\hat{j}$ $\quad\quad$ $\vec{E} = E_0\hat{k}$

$F = q\left(\vec{E}+\vec{V}\times\vec{B}\right) = q\left[E_0\hat{k}+(u_x\hat{i}+u_x\hat{k})(-B_0\hat{j})\right] = (qE_0)\hat{k}-(u_xB_0)\hat{k}+(u_zB_0)\hat{i}$

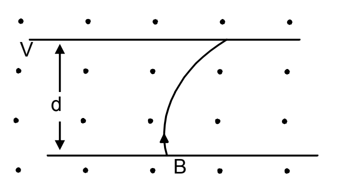
$F_z = (qE_0 - u_xB_0)$

Since $u_x = 0$, $F_z = qE_0$

$\Rightarrow a_z = \frac{qE_0}{m}$, So, $v^2 = u^2 + 2as \Rightarrow v^2 = 2\frac{qE_0}{m}Z$ [distance along Z direction be z]

$\Rightarrow V = \sqrt{\frac{2qE_0Z}{m}}$

52. The force experienced first is due to the electric field due to the capacitor



$E = \frac{V}{d}$ $\quad\quad$ F= eE

$a = \frac{eE}{m_e}$ [Where e→ charge of electron m_e → mass of electron]

$\upsilon^2 = u^2 + 2as \Rightarrow \upsilon^2 = 2\times\frac{eE}{m_e}\times d = \frac{2\times e\times V\times d}{dm_e}$

or $\upsilon = \sqrt{\frac{2eV}{m_e}}$

Now, The electron will fail to strike the upper plate only when d is greater than radius of the are thus formed.

or, $d > \frac{m_e\times\sqrt{\frac{2eV}{m_e}}}{eB} \Rightarrow d > \frac{\sqrt{2m_eV}}{eB^2}$

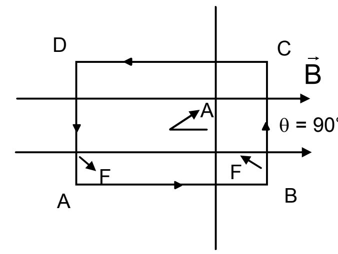
53. $\tau = ni\vec{A}\times\vec{B}$

$\Rightarrow \tau$ = ni AB Sin 90° $\Rightarrow$ 0.2 = 100 × 2 × 5 × 4 × 10^{-4} × B

$\Rightarrow B = \frac{0.2}{100\times 2\times 5\times 4\times 10^{-4}}$ = 0.5 Tesla

54. n = 50, r = 0.02 m
$A = \pi \times (0.02)^2$, B = 0.02 T
i = 5 A, $\mu = niA = 50 \times 5 \times \pi \times 4 \times 10^{-4}$
τ is max. when $\theta = 90°$
$\tau = \mu \times B = \mu B \sin 90° = \mu B = 50 \times 5 \times 3.14 \times 4 \times 10^{-4} \times 2 \times 10^{-1} = 6.28 \times 10^{-2}$ N-M
Given $\tau = (1/2)\,\tau_{max}$
$\Rightarrow \sin\theta = (1/2)$
or, $\theta = 30°$ = Angle between area vector & magnetic field.
$\Rightarrow$ Angle between magnetic field and the plane of the coil = 90° – 30° = 60°

55. $l = 20\text{ cm} = 20 \times 10^{-2}$ m
$B = 10\text{ cm} = 10 \times 10^{-2}$ m
i = 5 A, B = 0.2 T
(a) There is no force on the sides AB and CD. But the force on the sides AD and BC are opposite. So they cancel each other.
(b) Torque on the loop



$\tau = ni\,\vec{A} \times \vec{B} = niAB \sin 90°$
$= 1 \times 5 \times 20 \times 10^{-2} \times 10 \times 10^{-2}\ 0.2 = 2 \times 10^{-2} = 0.02$ N-M
Parallel to the shorter side.

56. n = 500, r = 0.02 m, $\theta = 30°$
i = 1A, $B = 4 \times 10^{-1}$ T
$i = \mu \times B = \mu B \sin 30° = ni AB \sin 30°$
$= 500 \times 1 \times 3.14 \times 4 \times 10^{-4} \times 4 \times 10^{-1} \times (1/2) = 12.56 \times 10^{-2} = 0.1256 \approx 0.13$ N-M

57. (a) radius = r
Circumference = $L = 2\pi r$

$\Rightarrow r = \dfrac{L}{2\pi}$

$\Rightarrow \pi r^2 = \dfrac{\pi L^2}{4\pi^2} = \dfrac{L^2}{4\pi}$

$\tau = i\,\vec{A} \times \vec{B} = \dfrac{iL^2B}{4\pi}$

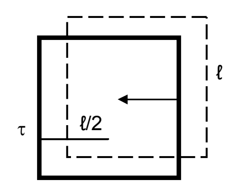
(b) Circumfernce = L

$4S = L \Rightarrow S = \dfrac{L}{4}$

Area $= S^2 = \left(\dfrac{L}{4}\right)^2 = \dfrac{L^2}{16}$

$\tau = i\,\vec{A} \times \vec{B} = \dfrac{iL^2B}{16}$

58. Edge = l, Current = i Turns= n, mass = M
Magnetic filed = B
$\tau = \mu B \sin 90° = \mu B$
Min Torque produced must be able to balance the torque produced due to weight
Now, $\tau B = \tau$ Weight



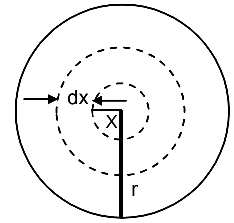
$\mu B = \mu g\left(\dfrac{l}{2}\right) \Rightarrow n \times i \times l^2\, B = \mu g\left(\dfrac{l}{2}\right) \quad \Rightarrow B = \dfrac{\mu g}{2nil}$

59. (a) $i = \dfrac{q}{t} = \dfrac{q}{(2\pi/\omega)} = \dfrac{q\omega}{2\pi}$

(b) $\mu = n\,ia = i\,A\ [\because n = 1] = \dfrac{q\omega\pi r^2}{2\pi} = \dfrac{q\omega r^2}{2}$

(c) $\mu = \dfrac{q\omega r^2}{2}$, $L = I\omega = mr^2\,\omega$, $\dfrac{\mu}{L} = \dfrac{q\omega r^2}{2mr^2\omega} = \dfrac{q}{2m} \Rightarrow \mu = \left(\dfrac{q}{2m}\right)L$

60. dp on the small length dx is $\frac{q}{\pi r^2} 2\pi x\ dx$.



$$di = \frac{q2\pi \times dx}{\pi r^2 t} = \frac{q2\pi x dx\,\omega}{\pi r^2 q 2\pi} = \frac{q\omega}{\pi r^2} x dx$$

$$d\mu = n\ di\ A = di\ A = \frac{q\omega x dx}{\pi r^2} \pi x^2$$

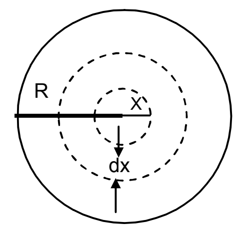
$$\mu = \int_0^\mu d\mu = \int_0^r \frac{q\omega}{r^2} x^3 dx = \frac{q\omega}{r^2}\left[\frac{x^4}{4}\right]^r = \frac{q\omega r^4}{r^2 \times 4} = \frac{q\omega r^2}{4}$$

$I = I\ \omega = (1/2)\ mr^2\ \omega$ [$\therefore$ M.I. for disc is $(1/2)\ mr^2$]

$$\frac{\mu}{I} = \frac{q\omega r^2}{4 \times \left(\frac{1}{2}\right) mr^2 \omega} \Rightarrow \frac{\mu}{I} = \frac{q}{2m} \Rightarrow \mu = \frac{q}{2m} I$$

61. Considering a strip of width dx at a distance x from centre,

$$dq = \frac{q}{\left(\frac{4}{3}\right)\pi R^3} 4\pi x^2 dx$$



$$di = \frac{dq}{dt} = \frac{q4\pi x^2 dx}{\left(\frac{4}{3}\right)\pi R^3 t} = \frac{3qx^2 dx\omega}{R^3 2\pi}$$

$$d\mu = di \times A = \frac{3qx^2 dx\omega}{R^3 2\pi} \times 4\pi x^2 = \frac{6q\omega}{R^3} x^4\ dx$$

$$\mu = \int_0^\mu d\mu = \int_0^R \frac{6q\omega}{R^3} x^4\ dx = \frac{6q\omega}{R^3}\left[\frac{x^5}{5}\right]_0^R = \frac{6q\omega}{R^3}\frac{R^5}{5} = \frac{6}{5} q\omega R^2$$

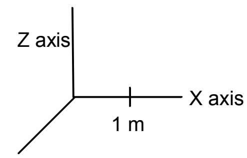
♣ ♣ ♣ ♣ ♣

CHAPTER – 35
MAGNETIC FIELD DUE TO CURRENT

1. $F = q\vec{\upsilon} \times \vec{B}$ or, $B = \frac{F}{q\upsilon} = \frac{F}{IT\upsilon} = \frac{N}{A.sec./sec.} = \frac{N}{A-m}$

   $B = \frac{\mu_0 I}{2\pi r}$ or, $\mu_0 = \frac{2\pi r B}{I} = \frac{m \times N}{A - m \times A} = \frac{N}{A^2}$

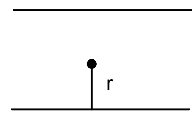
2. i = 10 A, d = 1 m



   $B = \frac{\mu_0 i}{2\pi r} = \frac{10^{-7} \times 4\pi \times 10}{2\pi \times 1} = 20 \times 10^{-6}$ T = 2 μT

   Along +ve Y direction.

3. d = 1.6 mm

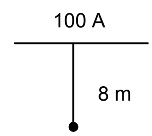


   So, r = 0.8 mm = 0.0008 m

   i = 20 A

   $\vec{B} = \frac{\mu_0 i}{2\pi r} = \frac{4\pi \times 10^{-7} \times 20}{2 \times \pi \times 8 \times 10^{-4}} = 5 \times 10^{-3}$ T = 5 mT

4. i = 100 A, d = 8 m

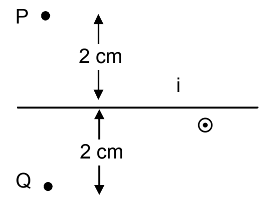


   $B = \frac{\mu_0 i}{2\pi r}$

   $= \frac{4\pi \times 10^{-7} \times 100}{2 \times \pi \times 8} = 2.5\ \mu T$

5. $\mu_0 = 4\pi \times 10^{-7}$ T-m/A

   r = 2 cm = 0.02 m, I = 1 A, $\vec{B} = 1 \times 10^{-5}$ T



   We know: Magnetic field due to a long straight wire carrying current = $\frac{\mu_0 I}{2\pi r}$

   $\vec{B}$ at P = $\frac{4\pi \times 10^{-7} \times 1}{2\pi \times 0.02} = 1 \times 10^{-5}$ T upward

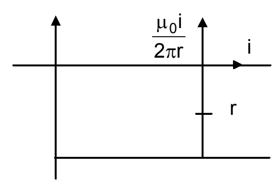
   net B = $2 \times 1 \times 10^{-7}$ T = 20 μT

   B at Q = 1×10^{-5} T downwards

   Hence net $\vec{B}$ = 0

6. (a) The maximum magnetic field is $B + \frac{\mu_0 I}{2\pi r}$ which are along the left keeping the sense along the direction of traveling current.

   (b)The minimum $B - \frac{\mu_0 I}{2\pi r}$

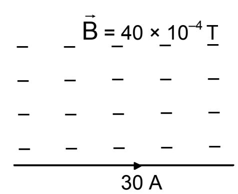


   If r = $\frac{\mu_0 I}{2\pi B}$ B net = 0

   r < $\frac{\mu_0 I}{2\pi B}$ B net = 0

   r > $\frac{\mu_0 I}{2\pi B}$ B net = $B - \frac{\mu_0 I}{2\pi r}$

7. $\mu_0 = 4\pi \times 10^{-7}$ T-m/A, I = 30 A, B = 4.0×10^{-4} T Parallel to current.



   $\vec{B}$ due to wore at a pt. 2 cm

   $= \frac{\mu_0 I}{2\pi r} = \frac{4\pi \times 10^{-7} \times 30}{2\pi \times 0.02} = 3 \times 10^{-4}$ T

   net field = $\sqrt{\left(3 \times 10^{-4}\right)^2 + \left(4 \times 10^{-4}\right)^2} = 5 \times 10^{-4}$ T

8. i = 10 A. $(\hat{K})$

B = 2 × 10^{-3} T South to North $(\hat{J})$

To cancel the magnetic field the point should be choosen so that the net magnetic field is along $-\hat{J}$ direction.

∴ The point is along $-\hat{i}$ direction or along west of the wire.

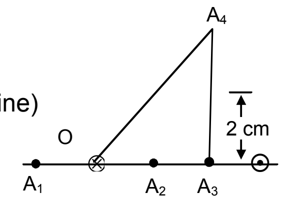
$B = \frac{\mu_0 I}{2\pi r}$

$\Rightarrow 2 \times 10^{-3} = \frac{4\pi \times 10^{-7} \times 10}{2\pi \times r}$

$\Rightarrow r = \frac{2 \times 10^{-7}}{2 \times 10^{-3}} = 10^{-3}$ m = 1 mm.

9. Let the tow wires be positioned at O & P

$R = OA, = \sqrt{(0.02)^2 + (0.02)^2} = \sqrt{8 \times 10^{-4}} = 2.828 \times 10^{-2}$ m



(a) $\vec{B}$ due to Q, at $A_1 = \frac{4\pi \times 10^{-7} \times 10}{2\pi \times 0.02} = 1 \times 10^{-4}$ T (⊥r towards up the line)

$\vec{B}$ due to P, at $A_1 = \frac{4\pi \times 10^{-7} \times 10}{2\pi \times 0.06} = 0.33 \times 10^{-4}$ T (⊥r towards down the line)

net $\vec{B} = 1 \times 10^{-4} - 0.33 \times 10^{-4} = 0.67 \times 10^{-4}$ T

(b) $\vec{B}$ due to O at $A_2 = \frac{2 \times 10^{-7} \times 10}{0.01} = 2 \times 10^{-4}$ T ⊥r down the line

$\vec{B}$ due to P at $A_2 = \frac{2 \times 10^{-7} \times 10}{0.03} = 0.67 \times 10^{-4}$ T ⊥r down the line

net $\vec{B}$ at $A_2 = 2 \times 10^{-4} + 0.67 \times 10^{-4} = 2.67 \times 10^{-4}$ T

(c) $\vec{B}$ at A_3 due to O = 1×10^{-4} T ⊥r towards down the line

$\vec{B}$ at A_3 due to P = 1×10^{-4} T ⊥r towards down the line

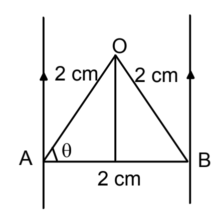
Net $\vec{B}$ at $A_3 = 2 \times 10^{-4}$ T

(d) $\vec{B}$ at A_4 due to O = $\frac{2 \times 10^{-7} \times 10}{2.828 \times 10^{-2}} = 0.7 \times 10^{-4}$ T towards SE

$\vec{B}$ at A_4 due to P = 0.7×10^{-4} T towards SW

Net $\vec{B} = \sqrt{\left(0.7 \times 10^{-4}\right)^2 + \left(0.7 \times 10^{-4}\right)^2} = 0.989 \times 10^{-4} \approx 1 \times 10^{-4}$ T

10. Cos θ = ½ , θ = 60° & ∠AOB = 60°



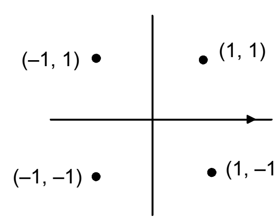
$B = \frac{\mu_0 I}{2\pi r} = \frac{10^{-7} \times 2 \times 10}{2 \times 10^{-2}} = 10^{-4}$ T

So net is $[(10^{-4})^2 + (10^{-4})^2 + 2(10^{-8}) \text{ Cos } 60°]^{1/2}$

$= 10^{-4}[1 + 1 + 2 \times ½]^{1/2} = 10^{-4} \times \sqrt{3}$ T = 1.732×10^{-4} T

11. (a) $\vec{B}$ for X = $\vec{B}$ for Y

Both are oppositely directed hence net $\vec{B}$ = 0



(b) $\vec{B}$ due to X = $\vec{B}$ due to X both directed along Z–axis

Net $\vec{B} = \frac{2 \times 10^{-7} \times 2 \times 5}{1} = 2 \times 10^{-6}$ T = 2 μT

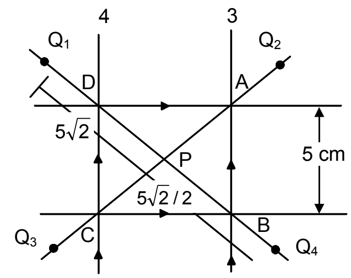
(c) $\vec{B}$ due to X = $\vec{B}$ due to Y both directed opposite to each other.

Hence Net $\vec{B}$ = 0

(d) $\vec{B}$ due to X = $\vec{B}$ due to Y = 1×10^{-6} T both directed along (–) ve Z–axis

Hence Net $\vec{B} = 2 \times 1.0 \times 10^{-6} = 2$ μT

12. (a) For each of the wire



Magnitude of magnetic field

$= \frac{\mu_0 i}{4\pi r}(\text{Sin}45° + \text{Sin}45°) = \frac{\mu_0 \times 5}{4\pi \times (5/2)} \frac{2}{\sqrt{2}}$

For AB ⊙ for BC ⊙ For CD ⊗ and for DA ⊗.

The two ⊙ and 2⊗ fields cancel each other. Thus $B_{net} = 0$

(b) At point Q_1

due to (1) $B = \frac{\mu_0 i}{2\pi \times 2.5 \times 10^{-2}} = \frac{4\pi \times 5 \times 2 \times 10^{-7}}{2\pi \times 5 \times 10^{-2}} = 4 \times 10^{-5}$ ⊙

due to (2) $B = \frac{\mu_0 i}{2\pi \times (15/2) \times 10^{-2}} = \frac{4\pi \times 5 \times 2 \times 10^{-7}}{2\pi \times 15 \times 10^{-2}} = (4/3) \times 10^{-5}$ ⊙

due to (3) $B = \frac{\mu_0 i}{2\pi \times (5 + 5/2) \times 10^{-2}} = \frac{4\pi \times 5 \times 2 \times 10^{-7}}{2\pi \times 15 \times 10^{-2}} = (4/3) \times 10^{-5}$ ⊙

due to (4) $B = \frac{\mu_0 i}{2\pi \times 2.5 \times 10^{-2}} = \frac{4\pi \times 5 \times 2 \times 10^{-7}}{2\pi \times 5 \times 10^{-2}} = 4 \times 10^{-5}$ ⊙

$B_{net} = [4 + 4 + (4/3) + (4/3)] \times 10^{-5} = \frac{32}{3} \times 10^{-5} = 10.6 \times 10^{-5} \approx 1.1 \times 10^{-4}$ T

At point Q_2

due to (1) $\frac{\mu_o i}{2\pi \times (2.5) \times 10^{-2}}$ ⊙

due to (2) $\frac{\mu_o i}{2\pi \times (15/2) \times 10^{-2}}$ ⊙

due to (3) $\frac{\mu_o i}{2\pi \times (2.5) \times 10^{-2}}$ ⊗

due to (4) $\frac{\mu_o i}{2\pi \times (15/2) \times 10^{-2}}$ ⊗

$B_{net} = 0$

At point Q_3

due to (1) $\frac{4\pi \times 10^{-7} \times 5}{2\pi \times (15/2) \times 10^{-2}} = 4/3 \times 10^{-5}$ ⊗

due to (2) $\frac{4\pi \times 10^{-7} \times 5}{2\pi \times (5/2) \times 10^{-2}} = 4 \times 10^{-5}$ ⊗

due to (3) $\frac{4\pi \times 10^{-7} \times 5}{2\pi \times (5/2) \times 10^{-2}} = 4 \times 10^{-5}$ ⊗

due to (4) $\frac{4\pi \times 10^{-7} \times 5}{2\pi \times (15/2) \times 10^{-2}} = 4/3 \times 10^{-5}$ ⊗

$B_{net} = [4 + 4 + (4/3) + (4/3)] \times 10^{-5} = \frac{32}{3} \times 10^{-5} = 10.6 \times 10^{-5} \approx 1.1 \times 10^{-4}$ T

For Q_4

due to (1) $4/3 \times 10^{-5}$ ⊗

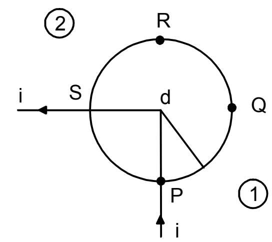
due to (2) 4×10^{-5} ⊗

due to (3) $4/3 \times 10^{-5}$ ⊗

due to (4) 4×10^{-5} ⊗

$B_{net} = 0$

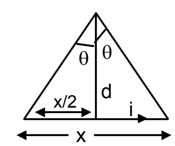
13. Since all the points lie along a circle with radius = 'd'
Hence 'R' & 'Q' both at a distance 'd' from the wire.
So, magnetic field $\vec{B}$ due to are same in magnitude.
As the wires can be treated as semi infinite straight current carrying conductors. Hence magnetic field $\vec{B} = \frac{\pi_0 i}{4\pi d}$



At P
B$_1$ due to 1 is 0
B$_2$ due to 2 is $\frac{\pi_0 i}{4\pi d}$
At Q
B$_1$ due to 1 is $\frac{\pi_0 i}{4\pi d}$
B$_2$ due to 2 is 0
At R
B$_1$ due to 1 is 0
B$_2$ due to 2 is $\frac{\pi_0 i}{4\pi d}$
At S
B$_1$ due to 1 is $\frac{\pi_0 i}{4\pi d}$
B$_2$ due to 2 is 0

14. $B = \frac{\pi_0 i}{4\pi d} 2 \sin\theta$

$$= \frac{\pi_0 i}{4\pi d} \frac{2 \times x}{2 \times \sqrt{d^2 + \frac{x^2}{4}}} = \frac{\mu_0 i x}{4\pi d\sqrt{d^2 + \frac{x^2}{4}}}$$



(a) When d >> x
Neglecting x w.r.t. d

$$B = \frac{\mu_0 i x}{\mu\pi d\sqrt{d^2}} = \frac{\mu_0 i x}{\mu\pi d^2}$$

$$\therefore B \propto \frac{1}{d^2}$$

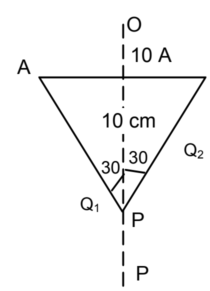
(b) When x >> d, neglecting d w.r.t. x

$$B = \frac{\mu_0 i x}{4\pi d x/2} = \frac{2\mu_0 i}{4\pi d}$$

$$\therefore B \propto \frac{1}{d}$$

15. $I = 10$ A, a = 10 cm = 0.1 m

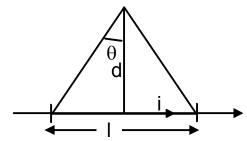
$r = OP = \frac{\sqrt{3}}{2} \times 0.1$ m



$$B = \frac{\mu_0 I}{4\pi r}(\sin\phi_1 + \sin\phi_2)$$

$$= \frac{10^{-7} \times 10 \times 1}{\frac{\sqrt{3}}{2} \times 0.1} = \frac{2 \times 10^{-5}}{1.732} = 1.154 \times 10^{-5}\text{ T} = 11.54\ \mu\text{T}$$

16. $B_1 = \frac{\mu_0 i}{2\pi d}$, $B_2 = \frac{\mu_0 i}{4\pi d}(2\times \text{Sin}\theta) = \frac{\mu_0 i}{4\pi d}\frac{2\times\ell}{2\sqrt{d^2+\frac{\ell^2}{4}}} = \frac{\mu_0 i\ell}{4\pi d\sqrt{d^2+\frac{\ell^2}{4}}}$



$B_1 - B_2 = \frac{1}{100}B_2 \Rightarrow \frac{\mu_0 i}{2\pi d} - \frac{\mu_0 i\ell}{4\pi d\sqrt{d^2+\frac{\ell^2}{4}}} = \frac{\mu_0 i}{200\pi d}$

$\Rightarrow \frac{\mu_0 i\ell}{4\pi d\sqrt{d^2+\frac{\ell^2}{4}}} = \frac{\mu_0 i}{\pi d}\left(\frac{1}{2}-\frac{1}{200}\right)$

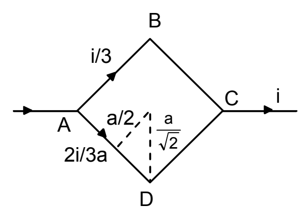
$\Rightarrow \frac{\ell}{4\sqrt{d^2+\frac{\ell^2}{4}}} = \frac{99}{200}$ $\Rightarrow \frac{\ell^2}{d^2+\frac{\ell^2}{4}} = \left(\frac{99\times 4}{200}\right)^2 = \frac{156816}{40000} = 3.92$

$\Rightarrow \ell^2 = 3.92\ d^2 + \frac{3.92}{4}\ell^2$

$\left(\frac{1-3.92}{4}\right)\ell^2 = 3.92\ d^2 \Rightarrow 0.02\ \ell^2 = 3.92\ d^2 \Rightarrow \frac{d^2}{\ell^2} = \frac{0.02}{3.92} = \frac{d}{\ell} = \sqrt{\frac{0.02}{3.92}} = 0.07$

17. As resistances vary as r & 2r

Hence Current along ABC = $\frac{i}{3}$ & along ADC = $\frac{2}{3i}$

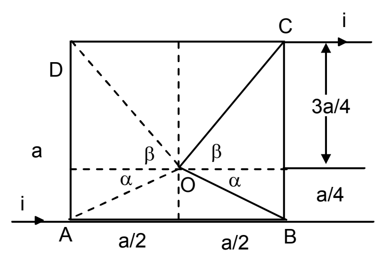


Now,

$\vec{B}$ due to ADC = $2\left[\frac{\mu_0 i\times 2\times 2\times\sqrt{2}}{4\pi 3a}\right] = \frac{2\sqrt{2}\mu_0 i}{3\pi a}$

$\vec{B}$ due to ABC = $2\left[\frac{\mu_0 i\times 2\times\sqrt{2}}{4\pi 3a}\right] = \frac{2\sqrt{2}\mu_0 i}{6\pi a}$

Now $\vec{B} = \frac{2\sqrt{2}\mu_0 i}{3\pi a} - \frac{2\sqrt{2}\mu_0 i}{6\pi a} = \frac{\sqrt{2}\mu_0 i}{3\pi a}$ $\otimes$



18. $A_0 = \sqrt{\frac{a^2}{16}+\frac{a^2}{4}} = \sqrt{\frac{5a^2}{16}} = \frac{a\sqrt{5}}{4}$

$D_0 = \sqrt{\left(\frac{3a}{4}\right)^2+\left(\frac{a}{2}\right)^2} = \sqrt{\frac{9a^2}{16}+\frac{a^2}{4}} = \sqrt{\frac{13a^2}{16}} = \frac{a\sqrt{13}}{4}$

Magnetic field due to AB

$B_{AB} = \frac{\mu_0}{4\pi}\times\frac{i}{2(a/4)}(\text{Sin }(90 - i) + \text{Sin }(90 - \alpha))$

$= \frac{\mu_0\times 2i}{4\pi a}2\text{Cos}\alpha = \frac{\mu_0\times 2i}{4\pi a}\times 2\times\frac{(a/2)}{a(\sqrt{5}/4)} = \frac{2\mu_0 i}{\pi\sqrt{5}}$

Magnetic field due to DC

$B_{DC} = \frac{\mu_0}{4\pi}\times\frac{i}{2(3a/4)}2\text{Sin }(90° - B)$

$= \frac{\mu_0 i\times 4\times 2}{4\pi\times 3a}\text{Cos}\beta = \frac{\mu_0 i}{\pi\times 3a}\times\frac{(a/2)}{(\sqrt{13a}/4)} = \frac{2\mu_0 i}{\pi a3\sqrt{13}}$

The magnetic field due to AD & BC are equal and appropriate hence cancle each other.

Hence, net magnetic field is $\frac{2\mu_0 i}{\pi\sqrt{5}} - \frac{2\mu_0 i}{\pi a3\sqrt{13}} = \frac{2\mu_0 i}{\pi a}\left[\frac{1}{\sqrt{5}}-\frac{1}{3\sqrt{13}}\right]$

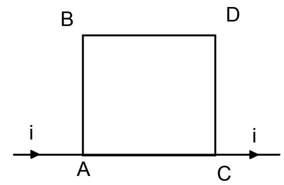
19. $\vec{B}$ due t BC &

$\vec{B}$ due to AD at Pt 'P' are equal ore Opposite

Hence net $\vec{B}$ = 0

Similarly, due to AB & CD at P = 0

$\therefore$ The net $\vec{B}$ at the Centre of the square loop = zero.



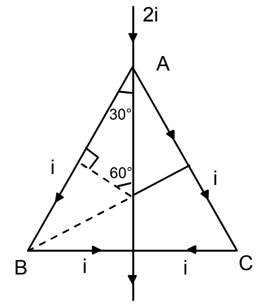
20. For AB B is along $\odot$ $B = \frac{\mu_0 i}{4\pi r}(\text{Sin}60° + \text{Sin}60°)$

For AC B $\otimes$ $B = \frac{\mu_0 i}{4\pi r}(\text{Sin}60° + \text{Sin}60°)$

For BD B $\odot$ $B = \frac{\mu_0 i}{4\pi r}(\text{Sin}60°)$

For DC B $\otimes$ $B = \frac{\mu_0 i}{4\pi r}(\text{Sin}60°)$

$\therefore$ Net B = 0



21. (a) ΔABC is Equilateral

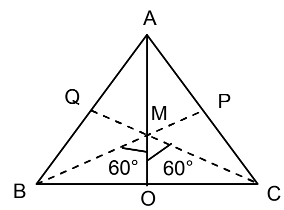
AB = BC = CA = ℓ/3

Current = i

$AO = \frac{\sqrt{3}}{2}a = \frac{\sqrt{3}\times\ell}{2\times 3} = \frac{\ell}{2\sqrt{3}}$

$\phi_1 = \phi_2 = 60°$

So, $MO = \frac{\ell}{6\sqrt{3}}$ as AM : MO = 2 : 1



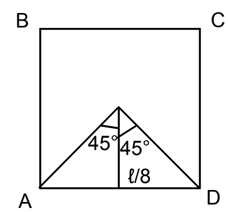
$\vec{B}$ due to BC at <.

$= \frac{\mu_0 i}{4\pi r}(\text{Sin}\phi_1 + \text{Sin}\phi_2) = \frac{\mu_0 i}{4\pi}\times i \times 6\sqrt{3}\times\sqrt{3} = \frac{\mu_0 i \times 9}{2\pi\ell}$

net $\vec{B} = \frac{9\mu_0 i}{2\pi\ell}\times 3 = \frac{27\mu_0 i}{2\pi\ell}$

(b) $\vec{B}$ due to AD $= \frac{\mu_0 i \times 8}{4\pi\times\ell}\sqrt{2} = \frac{8\sqrt{2}\mu_0 i}{4\pi\ell}$

Net $\vec{B} = \frac{8\sqrt{2}\mu_0 i}{4\pi\ell}\times 4 = \frac{8\sqrt{2}\mu_0 i}{\pi\ell}$



22. Sin (α/2) = $\frac{r}{x}$

$\Rightarrow$ r = x Sin (α/2)

Magnetic field B due to AR

$\frac{\mu_0 i}{4\pi r}\left[\text{Sin}(180-(90-(\alpha/2)))+1\right]$

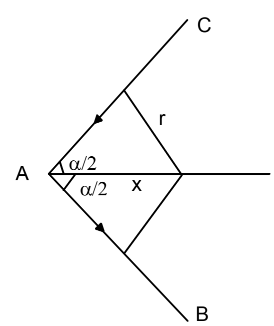
$\Rightarrow \frac{\mu_0 i[\text{Sin}(90-(\alpha/2))+1]}{4\pi\times\text{Sin}(\alpha/2)}$

$= \frac{\mu_0 i(\text{Cos}(\alpha/2)+1)}{4\pi\times\text{Sin}(\alpha/2)}$

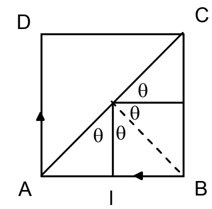
$= \frac{\mu_0 i 2\text{Cos}^4(\alpha/4)}{4\pi\times 2\text{Sin}(\alpha/4)\text{Cos}(\alpha/4)} = \frac{\mu_0 i}{4\pi x}\text{Cot}(\alpha/4)$

The magnetic field due to both the wire.

$\frac{2\mu_0 i}{4\pi x}\text{Cot}(\alpha/4) = \frac{\mu_0 i}{2\pi x}\text{Cot}(\alpha/4)$



23. $\vec{B}AB$



$$\frac{\mu_0 i \times 2}{4\pi b} \times 2\text{Sin}\theta = \frac{\mu_0 i \text{Sin}\theta}{\pi b}$$

$$= \frac{\mu_0 i \ell}{\pi b\sqrt{\ell^2 + b^2}} = \vec{B}DC \qquad \therefore \text{Sin } (\ell^2 + b) = \frac{(\ell/2)}{\sqrt{\ell^2/4 + b^2/4}} = \frac{\ell}{\sqrt{\ell^2 + b^2}}$$

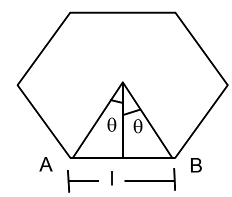
$\vec{B}BC$

$$\frac{\mu_0 i \times 2}{4\pi \ell} \times 2 \times 2\text{Sin}\theta' = \frac{\mu_0 i \text{Sin}\theta'}{\pi \ell} \qquad \therefore \text{Sin } \theta' = \frac{(b/2)}{\sqrt{\ell^2/4 + b^2/4}} = \frac{b}{\sqrt{\ell^2 + b^2}}$$

$$= \frac{\mu_0 i b}{\pi \ell\sqrt{\ell^2 + b^2}} = \vec{B}AD$$

$$\text{Net } \vec{B} = \frac{2\mu_0 i \ell}{\pi b\sqrt{\ell^2 + b^2}} + \frac{2\mu_0 i b}{\pi \ell\sqrt{\ell^2 + b^2}} = \frac{2\mu_0 i(\ell^2 + b^2)}{\pi \ell b\sqrt{\ell^2 + b^2}} = \frac{2\mu_0 i\sqrt{\ell^2 + b^2}}{\pi \ell b}$$

24. $2\theta = \frac{2\pi}{n} \Rightarrow \theta = \frac{\pi}{n}$, $\qquad \ell = \frac{2\pi r}{n}$



$$\text{Tan } \theta = \frac{\ell}{2x} \Rightarrow x = \frac{\ell}{2\text{Tan}\theta}$$

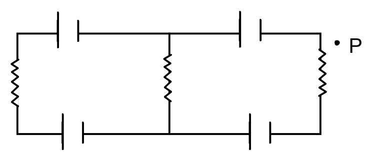
$$\frac{\ell}{2} = \frac{\pi r}{n}$$

$$B_{AB} = \frac{\mu_0 i}{4\pi(x)}(\text{Sin}\theta + \text{Sin}\theta) = \frac{\mu_0 i 2\text{Tan}\theta \times 2\text{Sin}\theta}{4\pi\ell}$$

$$= \frac{\mu_0 i 2\text{Tan}(\pi/n) 2\text{Sin}(\pi/n) n}{4\pi 2\pi r} = \frac{\mu_0 i n\text{Tan}(\pi/n)\text{Sin}(\pi/n)}{2\pi^2 r}$$

For n sides, $B_{net} = \frac{\mu_0 i n\text{Tan}(\pi/n)\text{Sin}(\pi/n)}{2\pi^2 r}$

25. Net current in circuit = 0
Hence the magnetic field at point P = 0
[Owing to wheat stone bridge principle]

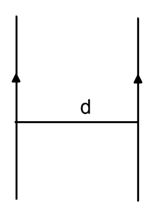


26. Force acting on 10 cm of wire is 2×10^{-5} N

$$\frac{dF}{dl} = \frac{\mu_0 i_1 i_2}{2\pi d}$$

$$\Rightarrow \frac{2 \times 10^{-5}}{10 \times 10^{-2}} = \frac{\mu_0 \times 20 \times 20}{2\pi d}$$

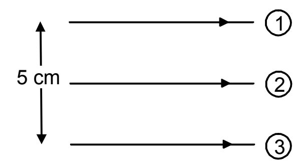
$$\Rightarrow d = \frac{4\pi \times 10^{-7} \times 20 \times 20 \times 10 \times 10^{-2}}{2\pi \times 2 \times 10^{-5}} = 400 \times 10^{-3} = 0.4 \text{ m} = 40 \text{ cm}$$



27. i = 10 A
Magnetic force due to two parallel Current Carrying wires.

$$F = \frac{\mu_0 I_1 I_2}{2\pi r}$$

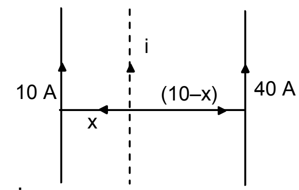
So, $\vec{F}$ or 1 = $\vec{F}$ by 2 + $\vec{F}$ by 3



$$= \frac{\mu_0 \times 10 \times 10}{2\pi \times 5 \times 10^{-2}} + \frac{\mu_0 \times 10 \times 10}{2\pi \times 10 \times 10^{-2}}$$

$$= \frac{4\pi \times 10^{-7} \times 10 \times 10}{2\pi \times 5 \times 10^{-2}} + \frac{4\pi \times 10^{-7} \times 10 \times 10}{2\pi \times 10 \times 10^{-2}}$$

$$= \frac{2 \times 10^{-3}}{5} + \frac{10^{-3}}{5} = \frac{3 \times 10^{-3}}{5} = 6 \times 10^{-4} \text{ N} \quad \text{towards middle wire}$$

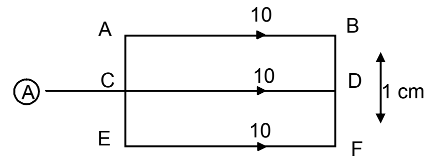


28. $\dfrac{\mu_0 10i}{2\pi x} = \dfrac{\mu_0 i40}{2\pi(10-x)}$

$\Rightarrow \dfrac{10}{x} = \dfrac{40}{10-x} \Rightarrow \dfrac{1}{x} = \dfrac{4}{10-x}$

$\Rightarrow 10 - x = 4x \Rightarrow 5x = 10 \Rightarrow x = 2$ cm

The third wire should be placed 2 cm from the 10 A wire and 8 cm from 40 A wire.



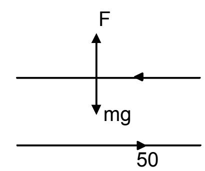
29. $F_{AB} = F_{CD} + F_{EF}$

$= \dfrac{\mu_0 \times 10 \times 10}{2\pi \times 1 \times 10^{-2}} + \dfrac{\mu_0 \times 10 \times 10}{2\pi \times 2 \times 10^{-2}}$

$= 2\times 10^{-3} + 10^{-3} = 3 \times 10^{-3}$ downward.

$F_{CD} = F_{AB} + F_{EF}$

As F_{AB} & F_{EF} are equal and oppositely directed hence F = 0



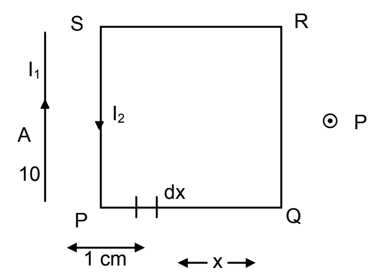
30. $\dfrac{\mu_0 i_1 i_2}{2\pi d}$ = mg (For a portion of wire of length 1m)

$\Rightarrow \dfrac{\mu_0 \times 50 \times i_2}{2\pi \times 5 \times 10^{-3}} = 1 \times 10^{-4} \times 9.8$

$\Rightarrow \dfrac{4\pi \times 10^{-7} \times 5 \times i_2}{2\pi \times 5 \times 10^{-3}} = 9.8 \times 10^{-4}$

$\Rightarrow 2 \times i_2 \times 10^{-3} = 9.3 \times 10^{-3} \times 10^{-1}$

$\Rightarrow i_2 = \dfrac{9.8}{2} \times 10^{-1} = 0.49$ A



31. $I_2 = 6$ A

$I_1 = 10$ A

F_{PQ}

'F' on dx $= \dfrac{\mu_0 i_1 i_2}{2\pi x} dx = \dfrac{\mu_0 i_1 i_2}{2\pi}\dfrac{dx}{x} = \dfrac{\mu_0 \times 30}{\pi}\dfrac{dx}{x}$

$\vec{F}_{PQ} = \dfrac{\mu_0 \times 30}{x}\int_1 \dfrac{dx}{x} = 30 \times 4 \times 10^{-7} \times [\log x]_1^2$

$= 120 \times 10^{-7}$ [log 3 – log 1]

Similarly force of $\vec{F}_{RS} = 120 \times 10^{-7}$ [log 3 – log 1]

So, $\vec{F}_{PQ} = \vec{F}_{RS}$

$\vec{F}_{PS} = \dfrac{\mu_0 \times i_1 i_2}{2\pi \times 1 \times 10^{-2}} - \dfrac{\mu_0 \times i_1 i_2}{2\pi \times 2 \times 10^{-2}}$

$= \dfrac{2 \times 6 \times 10 \times 10^{-7}}{10^{-2}} - \dfrac{2 \times 10^{-7} \times 6 \times 6}{2 \times 10^{-2}} = 8.4 \times 10^{-4}$ N (Towards right)

$\vec{F}_{RQ} = \dfrac{\mu_0 \times i_1 i_2}{2\pi \times 3 \times 10^{-2}} - \dfrac{\mu_0 \times i_1 i_2}{2\pi \times 2 \times 10^{-2}}$

$= \dfrac{4\pi \times 10^{-7} \times 6 \times 10}{2\pi \times 3 \times 10^{-2}} - \dfrac{4\pi \times 10^{-7} \times 6 \times 6}{2\pi \times 2 \times 10^{-2}} = 4 \times 10^{-4} + 36 \times 10^{-5} = 7.6 \times 10^{-4}$ N

Net force towards down

$= (8.4 + 7.6) \times 10^{-4} = 16 \times 10^{-4}$ N

32. B = 0.2 mT, i = 5 A, n = 1, r = ?

$B = \dfrac{n\mu_0 i}{2r}$

$\Rightarrow r = \dfrac{n \times \mu_0 i}{2B} = \dfrac{1 \times 4\pi \times 10^{-7} \times 5}{2 \times 0.2 \times 10^{-3}} = 3.14 \times 5 \times 10^{-3}$ m $= 15.7 \times 10^{-3}$ m $= 15.7 \times 10^{-1}$ cm = 1.57 cm

33. $B = \frac{n\mu_0 i}{2r}$

$n = 100$, $r = 5$ cm $= 0.05$ m

$\vec{B} = 6 \times 10^{-5}$ T

$i = \frac{2rB}{n\mu_0} = \frac{2 \times 0.05 \times 6 \times 10^{-5}}{100 \times 4\pi \times 10^{-7}} = \frac{3}{6.28} \times 10^{-1} = 0.0477 \approx 48$ mA

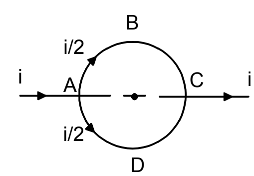
34. 3×10^5 revolutions in 1 sec.

1 revolutions in $\frac{1}{3 \times 10^5}$ sec

$i = \frac{q}{t} = \frac{1.6 \times 10^{-19}}{\left(\frac{1}{3 \times 10^5}\right)}$ A

$B = \frac{\mu_0 i}{2r} = \frac{4\pi \times 10^{-7} . 16 \times 10^{-19} 3 \times 10^5}{2 \times 0.5 \times 10^{-10}} \;\; \frac{2\pi \times 1.6 \times 3}{0.5} \times 10^{-11} = 6.028 \times 10^{-10} \approx 6 \times 10^{-10}$ T

35. I = i/2 in each semicircle



ABC = $\vec{B} = \frac{1}{2} \times \frac{\mu_0 (i/2)}{2a}$ downwards

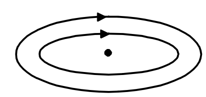
ADC = $\vec{B} = \frac{1}{2} \times \frac{\mu_0 (i/2)}{2a}$ upwards

Net $\vec{B} = 0$

36. $r_1 = 5$ cm $\quad r_2 = 10$ cm

$n_1 = 50 \quad n_2 = 100$

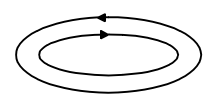
$i = 2$ A



(a) $B = \frac{n_1 \mu_0 i}{2r_1} + \frac{n_2 \mu_0 i}{2r_2}$

$= \frac{50 \times 4\pi \times 10^{-7} \times 2}{2 \times 5 \times 10^{-2}} + \frac{100 \times 4\pi \times 10^{-7} \times 2}{2 \times 10 \times 10^{-2}}$

$= 4\pi \times 10^{-4} + 4\pi \times 10^{-4} = 8\pi \times 10^{-4}$

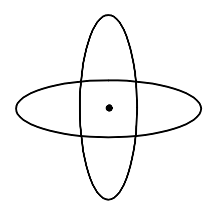


(b) $B = \frac{n_1 \mu_0 i}{2r_1} - \frac{n_2 \mu_0 i}{2r_2} = 0$

37. Outer Circle

$n = 100$, $r = 100$m $= 0.1$ m

$i = 2$ A



$\vec{B} = \frac{n\mu_0 i}{2a} = \frac{100 \times 4\pi \times 10^{-7} \times 2}{2 \times 0.1} = 4\pi \times 10^{-4}$ horizontally towards West.

Inner Circle

$r = 5$ cm $= 0.05$ m, $n = 50$, $i = 2$ A

$\vec{B} = \frac{n\mu_0 i}{2r} = \frac{4\pi \times 10^{-7} \times 2 \times 50}{2 \times 0.05} = 4\pi \times 10^{-4}$ downwards

Net $B = \sqrt{(4\pi \times 10^{-4})^2 + (4\pi \times 10^{-4})^2} = \sqrt{32\pi^2 \times 10^{-8}} = 17.7 \times 10^{-4} \approx 18 \times 10^{-4} = 1.8 \times 10^{-3} = 1.8$ mT

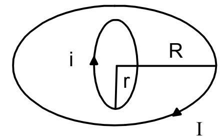
38. $r = 20$ cm, $\quad i = 10$ A, $\quad V = 2 \times 10^6$ m/s, $\quad \theta = 30°$

$F = e(\vec{V} \times \vec{B}) = eVB \sin\theta$

$= 1.6 \times 10^{-19} \times 2 \times 10^6 \times \frac{\mu_0 i}{2r} \sin 30°$

$= \frac{1.6 \times 10^{-19} \times 2 \times 10^6 \times 4\pi \times 10^{-7} \times 10}{2 \times 2 \times 20 \times 10^{-2}} = 16\pi \times 10^{-19}$ N

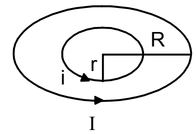
39. $\vec{B}$ Large loop = $\dfrac{\mu_0 I}{2R}$



'i' due to larger loop on the smaller loop

= i(A × B) = i AB Sin 90° = i × πr^2 × $\dfrac{\mu_0 I}{2r}$

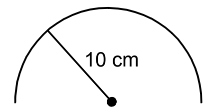
40. The force acting on the smaller loop



F = ilB Sin θ

$= \dfrac{i 2\pi r \mu_o I 1}{2R \times 2} = \dfrac{\mu_0 i I \pi r}{2R}$

41. i = 5 Ampere, r = 10 cm = 0.1 m

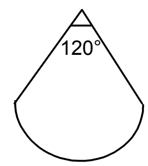


As the semicircular wire forms half of a circular wire,

So, $\vec{B} = \dfrac{1}{2}\dfrac{\mu_0 i}{2r} = \dfrac{1}{2} \times \dfrac{4\pi \times 10^{-7} \times 5}{2 \times 0.1}$

= 15.7 × 10^{-6} T ≈ 16 × 10^{-6} T = 1.6 × 10^{-5} T

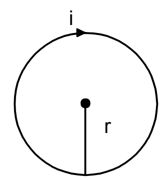
42. $B = \dfrac{\mu_0 i}{2R}\dfrac{\theta}{2\pi} = \dfrac{2\pi}{3 \times 2\pi} \times \dfrac{\mu_0 i}{2R}$



$= \dfrac{4\pi \times 10^{-7} \times 6}{6 \times 10^{t10^{-2}}}$ = 4π × 10^{-6}

= 4 × 3.14 × 10^{-6} = 12.56 × 10^{-6} = 1.26 × 10^{-5} T

43. $\vec{B}$ due to loop $\dfrac{\mu_0 i}{2r}$



Let the straight current carrying wire be kept at a distance R from centre. Given I = 4i

$\vec{B}$ due to wire = $\dfrac{\mu_0 I}{2\pi R} = \dfrac{\mu_0 \times 4i}{2\pi R}$

Now, the $\vec{B}$ due to both will balance each other

Hence $\dfrac{\mu_0 i}{2r} = \dfrac{\mu_0 4i}{2\pi R} \Rightarrow R = \dfrac{4r}{\pi}$

Hence the straight wire should be kept at a distance 4π/r from centre in such a way that the direction of current in it is opposite to that in the nearest part of circular wire. As a result the direction will $\vec{B}$ will be oppose.

44. n = 200, i = 2 A, r = 10 cm = 10 × 10^{-2}n

(a) $B = \dfrac{n\mu_0 i}{2r} = \dfrac{200 \times 4\pi \times 10^{-7} \times 2}{2 \times 10 \times 10^{-2}}$ = 2 × 4π × 10^{-4}

= 2 × 4 × 3.14 × 10^{-4} = 25.12 × 10^{-4} T = 2.512 mT

(b) $B = \dfrac{n\mu_0 ia^2}{2(a^2 + d^2)^{3/2}}$ $\Rightarrow \dfrac{n\mu_0 i}{4a} = \dfrac{n\mu_0 ia^2}{2(a^2 + d^2)^{3/2}}$

$\Rightarrow \dfrac{1}{2a} = \dfrac{a^2}{2(a^2 + d^2)^{3/2}}$ $\Rightarrow (a^2 + d^2)^{3/2}\ 2a^3$ $\Rightarrow a^2 + d^2 = (2a^3)^{2/3}$

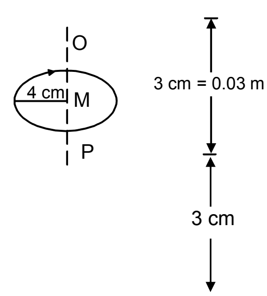
$\Rightarrow a^2 + d^2 = (2^{1/3} a)^2$ $\Rightarrow a^2 + d^2 = 2^{2/3} a^2$ $\Rightarrow (10^{-1})^2 + d^2 = 2^{2/3}\ (10^{-1})^2$

$\Rightarrow 10^{-2} + d^2 = 2^{2/3}\ 10^{-2}$ $\Rightarrow (10^{-2})(2^{2/3} - 1) = d^2$ $\Rightarrow (10^{-2})\ (4^{1/3} - 1) = d^2$

$\Rightarrow 10^{-2}(1.5874 - 1) = d^2$ $\Rightarrow d^2 = 10^{-2} \times 0.5874$

$\Rightarrow d = \sqrt{10^{-2} \times 0.5874}$ = 10^{-1} × 0.766 m = 7.66 × 10^{-2} = 7.66 cm.

45. At O P the $\vec{B}$ must be directed downwards



We Know B at the axial line at O & P

$= \dfrac{\mu_0 ia^2}{2(a^2 + d^2)^{3/2}}$ a = 4 cm = 0.04 m

$= \dfrac{4\pi \times 10^{-7} \times 5 \times 0.0016}{2((0.0025)^{3/2}}$ d = 3 cm = 0.03 m

= 40 × 10^{-6} = 4 × 10^{-5} T downwards in both the cases

46. $q = 3.14 \times 10^{-6}$ C, $r = 20$ cm $= 0.2$ m,

w = 60 rad/sec., $i = \frac{q}{t} = \frac{3.14 \times 10^{-6} \times 60}{2\pi \times 0.2} = 1.5 \times 10^{-5}$

$$\frac{\text{Electric field}}{\text{Magnetic field}} = \frac{\frac{xQ}{4\pi\varepsilon_0 (x^2 + a^2)^{3/2}}}{\frac{\mu_0 i a^2}{2(a^2 + x^2)^{3/2}}} = \frac{xQ}{4\pi\varepsilon_0 (x^2 + a^2)^{3/2}} \times \frac{2(x^2 + a^2)^{3/2}}{\mu_0 i a^2}$$

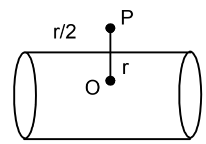
$$= \frac{9 \times 10^9 \times 0.05 \times 3.14 \times 10^{-6} \times 2}{4\pi \times 10^{-7} \times 15 \times 10^{-5} \times (0.2)^2}$$

$$= \frac{9 \times 5 \times 2 \times 10^3}{4 \times 13 \times 4 \times 10^{-12}} = \frac{3}{8}$$

47. (a) For inside the tube $\vec{B} = 0$

As, $\vec{B}$ inside the conducting tube = o

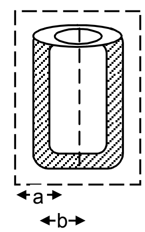
(b) For $\vec{B}$ outside the tube



$d = \frac{3r}{2}$

$$\vec{B} = \frac{\mu_0 i}{2\pi d} = \frac{\mu_0 i \times 2}{2\pi 3r} = \frac{\mu_0 i}{2\pi r}$$

48. (a) At a point just inside the tube the current enclosed in the closed surface = 0.

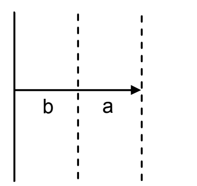


Thus $B = \frac{\mu_0 o}{A} = 0$

(b) Taking a cylindrical surface just out side the tube, from ampere's law.

$\mu_0 i = B \times 2\pi b \quad \Rightarrow B = \frac{\mu_0 i}{2\pi b}$

49. i is uniformly distributed throughout.



So, 'i' for the part of radius a = $\frac{i}{\pi b^2} \times \pi a^2 = \frac{ia^2}{b^2} = I$

Now according to Ampere's circuital law

$\oint B \times d\ell = B \times 2 \times \pi \times a = \mu_0 I$

$$\Rightarrow B = \mu_0 \frac{ia^2}{b^2} \times \frac{1}{2\pi a} = \frac{\mu_0 i a}{2\pi b^2}$$

50. (a) $r = 10$ cm $= 10 \times 10^{-2}$ m

$x = 2 \times 10^{-2}$ m, $i = 5$ A

i in the region of radius 2 cm

$$\frac{5}{\pi(10 \times 10^{-2})^2} \times \pi(2 \times 10^{-2})^2 = 0.2 \text{ A}$$

$B \times \pi\, (2 \times 10^{-2})^2 = \mu_0(0\text{-}2)$

$$\Rightarrow B = \frac{4\pi \times 10^{-7} \times 0.2}{\pi \times 4 \times 10^{-4}} = \frac{0.2 \times 10^{-7}}{10^{-4}} = 2 \times 10^{-4}$$

(b) 10 cm radius

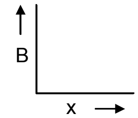
$B \times \pi\, (10 \times 10^{-2})^2 = \mu_0 \times 5$

$$\Rightarrow B = \frac{4\pi \times 10^{-7} \times 5}{\pi \times 10^{-2}} = 20 \times 10^{-5}$$

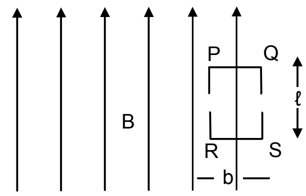
(c) x = 20 cm

$B \times \pi \times (20 \times 10^{-2})^2 = \mu_0 \times 5$

$$\Rightarrow B = \frac{\mu_0 \times 5}{\pi \times (20 \times 10^{-2})^2} = \frac{4\pi \times 10^{-7} \times 5}{\pi \times 400 \times 10^{-4}} = 5 \times 10^{-5}$$



51. We know, $\int B \times dl = \mu_0 i$. Theoritically B = 0 a t A



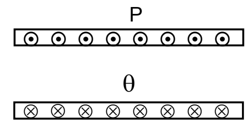
If, a current is passed through the loop PQRS, then

$B = \dfrac{\mu_0 i}{2(\ell + b)}$ will exist in its vicinity.

Now, As the $\vec{B}$ at A is zero. So there'll be no interaction

However practically this is not true. As a current carrying loop, irrespective of its near about position is always affected by an existing magnetic field.

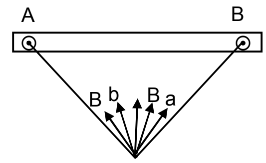
52. (a) At point P, i = 0, Thus B = 0

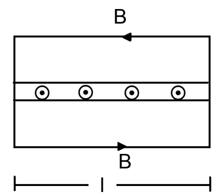


(b) At point R, i = 0, B = 0

(c) At point θ,

Applying ampere's rule to the above rectangle



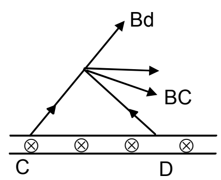


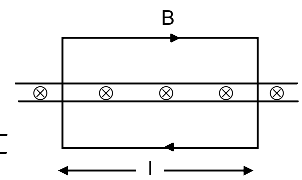
$B \times 2l = \mu_0 K_0 \int_o^l dl$

$\Rightarrow B \times 2l = \mu_0 kl \Rightarrow B = \dfrac{\mu_0 k}{2}$

$B \times 2l = \mu_0 K_0 \int_o^l dl$

$\Rightarrow B \times 2l = \mu_0 kl \Rightarrow B = \dfrac{\mu_0 k}{2}$





Since the $\vec{B}$ due to the 2 stripes are along the same direction, thus.

$B_{net} = \dfrac{\mu_0 k}{2} + \dfrac{\mu_0 k}{2} = \mu_0 k$

53. Charge = q, mass = m

We know radius described by a charged particle in a magnetic field B

$r = \dfrac{m\upsilon}{qB}$

Bit $B = \mu_0 K$ [according to Ampere's circuital law, where K is a constant]

$r = \dfrac{m\upsilon}{q\mu_0 k} \Rightarrow \upsilon = \dfrac{rq\mu_0 k}{m}$

54. i = 25 A, $B = 3.14 \times 10^{-2}$ T, n = ?

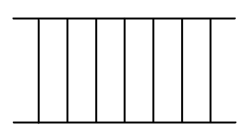
$B = \mu_0 ni$

$\Rightarrow 3.14 \times 10^{-2} = 4 \times \pi \times 10^{-7}\ n \times 5$

$\Rightarrow n = \dfrac{10^{-2}}{20 \times 10^{-7}} = \dfrac{1}{2} \times 10^4 = 0.5 \times 10^4 = 5000$ turns/m

55. r = 0.5 mm, i = 5 A, $B = \mu_0 ni$ (for a solenoid)

Width of each turn = 1 mm = 10^{-3} m



No. of turns 'n' = $\dfrac{1}{10^{-3}} = 10^3$

So, $B = 4\pi \times 10^{-7} \times 10^3 \times 5 = 2\pi \times 10^{-3}$ T

56. $\dfrac{R}{l} = 0.01\ \Omega$ in 1 m, r = 1.0 cm Total turns = 400, ℓ = 20 cm,

$B = 1 \times 10^{-2}$ T, $n = \dfrac{400}{20 \times 10^{-2}}$ turns/m

$i = \dfrac{E}{R_0} = \dfrac{E}{R_0 / l \times (2\pi r \times 400)} = \dfrac{E}{0.01 \times 2 \times \pi \times 0.01 \times 400}$

$B = \mu_0 ni$

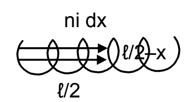
$\Rightarrow 10^2 = 4\pi \times 10^{-7} \times \dfrac{400}{20 \times 10^{-2}} \times \dfrac{E}{400 \times 2\pi \times 0.01 \times 10^{-2}}$

$\Rightarrow E = \dfrac{10^{-2} \times 20 \times 10^{-2} \times 400 \times 2\pi \times 10^{-2} 0.01}{4\pi \times 10^{-7} \times 400} = 1\ V$

57. Current at '0' due to the circular loop = $dB = \dfrac{\mu_0}{4\pi} \times \dfrac{a^2 i n dx}{\left[a^2 + \left(\dfrac{l}{2} - x\right)^2\right]^{3/2}}$

$\therefore$ for the whole solenoid $B = \int_0^B dB$

$= \int_0^{\ell} \dfrac{\mu_0 a^2 n i dx}{4\pi\left[a^2 + \left(\dfrac{\ell}{2} - x\right)^2\right]^{3/2}}$



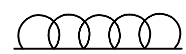
$= \dfrac{\mu_0 n i}{4\pi} \int_0^{\ell} \dfrac{a^2\, dx}{a^3\left[1 + \left(\ell - \dfrac{2x}{2a}\right)^2\right]^{3/2}} = \dfrac{\mu_0 n i}{4\pi a} \int_0^{\ell} \dfrac{dx}{\left[1 + \left(\ell - \dfrac{2x}{2a}\right)^2\right]^{3/2}} = 1 + \left(\ell - \dfrac{2x}{2a}\right)^2$

58. i = 2 a, $f = 10^{\,8}$ rev/sec, n= ?, $m_e = 9.1 \times 10^{-31}$ kg,

$q_e = 1.6 \times 10^{-19}$ c, $B = \mu_0 n i \Rightarrow n = \dfrac{B}{\mu_0 i}$

$f = \dfrac{qB}{2\pi m_e} \Rightarrow B = \dfrac{f 2\pi m_e}{q_e} \Rightarrow n = \dfrac{B}{\mu_0 i} = \dfrac{f 2\pi m_e}{q_e \mu_0 i} = \dfrac{10^8 \times 9.1 \times 10^{-31}}{1.6 \times 10^{-19} \times 2 \times 10^{-7} \times 2A} = 1421$ turns/m

59. No. of turns per unit length = n, radius of circle = r/2, current in the solenoid = i,
Charge of Particle = q, mass of particle = m $\therefore B = \mu_0 n i$



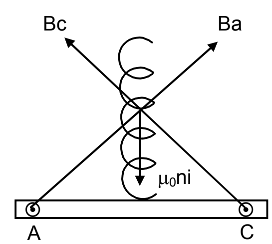
Again $\dfrac{mV^2}{r} = qVB \Rightarrow V = \dfrac{qBr}{m} = \dfrac{q\mu_0 n i r}{2m} = \dfrac{\mu_0 n i q r}{2m}$

60. No. of turns per unit length = ℓ
(a) As the net magnetic field = zero

$\therefore \vec{B}_{plate} = \vec{B}_{Solenoid}$

$\vec{B}_{plate} \times 2\ell = \mu_0 k d\ell = \mu_0 k \ell$

$\vec{B}_{plate} = \dfrac{\mu_0 k}{2}$ ...(1) $\vec{B}_{Solenoid} = \mu_0 n i$ …(2)



Equating both $i = \dfrac{\mu_0 k}{2}$

(b) $B_a \times \ell = \mu k \ell$ $\Rightarrow B_a = \mu_0 k$ BC = $\mu_0 k$

$B = \sqrt{{B_a}^2 + {B_c}^2} = \sqrt{2(\mu_0 k)^2} = \sqrt{2}\mu_0 k$

$2\,\mu_0 k = \mu_0 n i$ $i = \dfrac{\sqrt{2}k}{n}$

61. C = 100 μf, $Q = CV = 2 \times 10^{-3}$ C, t = 2 sec,
V = 20 V, V′ = 18 V, $Q' = CV = 1.8 \times 10^{-3}$ C,

$\therefore i = \dfrac{Q - Q'}{t} = \dfrac{2 \times 10^{-4}}{2} = 10^{-4}$ A n = 4000 turns/m.

$\therefore B = \mu_0 n i = 4\pi \times 10^{-7} \times 4000 \times 10^{-4} = 16\,\pi \times 10^{-7}$ T

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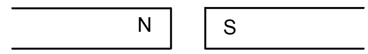
CHAPTER – 36
PERMANENT MAGNETS

1. m = 10 A-m, d = 5 cm = 0.05 m



$B = \frac{\mu_0}{4\pi}\frac{m}{r^2} = \frac{10^{-7} \times 10}{\left(5 \times 10^{-2}\right)^2} = \frac{10^{-2}}{25}$ = 4 × 10^{-4} Tesla

2. $m_1 = m_2$ = 10 A-m
r = 2 cm = 0.02 m
we know

Force exerted by tow magnetic poles on each other = $\frac{\mu_0}{4\pi}\frac{m_1 m_2}{r^2} = \frac{4\pi \times 10^{-7} \times 10^2}{4\pi \times 4 \times 10^{-4}}$ = 2.5 × 10^{-2} N

3. $B = -\frac{dv}{d\ell} \Rightarrow dv = -B\,d\ell = -0.2 \times 10^{-3} \times 0.5 = -0.1 \times 10^{-3}$ T-m

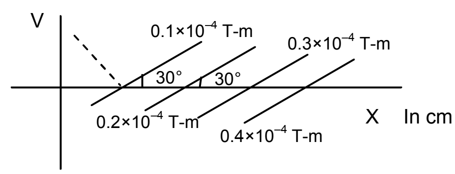
Since the sigh is –ve therefore potential decreases.

4. Here dx = 10 sin 30° cm = 5 cm

$\frac{dV}{dx} = B = \frac{0.1 \times 10^{-4}\ \text{T}-\text{m}}{5 \times 10^{-2}\ \text{m}}$

Since B is perpendicular to equipotential surface.
Here it is at angle 120° with (+ve) x-axis and B = 2 × 10^{-4} T

5. B = 2 × 10^{-4} T
d = 10 cm = 0.1 m
(a) if the point at end-on postion.



$B = \frac{\mu_0}{4\pi}\frac{2M}{d^3} \Rightarrow 2 \times 10^{-4} = \frac{10^{-7} \times 2M}{(10^{-1})^3}$

$\Rightarrow \frac{2 \times 10^{-4} \times 10^{-3}}{10^{-7} \times 2} = M \Rightarrow M = 1\ \text{Am}^2$

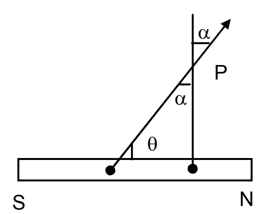
(b) If the point is at broad-on position

$\frac{\mu_0}{4\pi}\frac{M}{d^3} \Rightarrow 2 \times 10^{-4} = \frac{10^{-7} \times M}{(10^{-1})^3} \Rightarrow M = 2\ \text{Am}^2$

6. Given :

$\theta = \tan^{-1}\sqrt{2} \Rightarrow \tan\theta = \sqrt{2} \Rightarrow 2 = \tan^2\theta$

$\Rightarrow \tan\theta = 2\cot\theta \Rightarrow \frac{\tan\theta}{2} = \cot\theta$



We know $\frac{\tan\theta}{2} = \tan\alpha$

Comparing we get, $\tan\alpha = \cot\theta$
or, $\tan\alpha = \tan(90 - \theta)$ or $\alpha = 90 - \theta$ or $\theta + \alpha = 90$
Hence magnetic field due to the dipole is ⊥r to the magnetic axis.

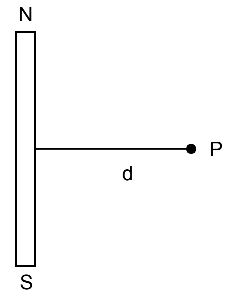
7. Magnetic field at the broad side on position :

$B = \frac{\mu_0}{4\pi}\frac{M}{\left(d^2 + \ell^2\right)^{3/2}}$ 2ℓ = 8 cm d = 3 cm

$\Rightarrow 4 \times 10^{-6} = \frac{10^{-7} \times m \times 8 \times 10^{-2}}{\left(9 \times 10^{-4} + 16 \times 10^{-4}\right)^{3/2}} \Rightarrow 4 \times 10^{-6} = \frac{10^{-9} \times m \times 8}{\left(10^{-4}\right)^{3/2} + (25)^{3/2}}$

$\Rightarrow m = \frac{4 \times 10^{-6} \times 125 \times 10^{-8}}{8 \times 10^{-9}}$ = 62.5 × 10^{-5} A-m

8. We know for a magnetic dipole with its north pointing the north, the neutral point in the broadside on position.



Again $\vec{B}$ in this case = $\frac{\mu_0 M}{4\pi d^3}$

$\therefore \frac{\mu_0 M}{4\pi d^3} = \overrightarrow{B_H}$ due to earth

$\Rightarrow \frac{10^{-7} \times 1.44}{d^3} = 18\ \mu T$

$\Rightarrow \frac{10^{-7} \times 1.44}{d^3} = 18 \times 10^{-6}$

$\Rightarrow d^3 = 8 \times 10^{-3}$

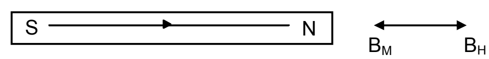
$\Rightarrow d = 2 \times 10^{-1}$ m = 20 cm

In the plane bisecting the dipole.

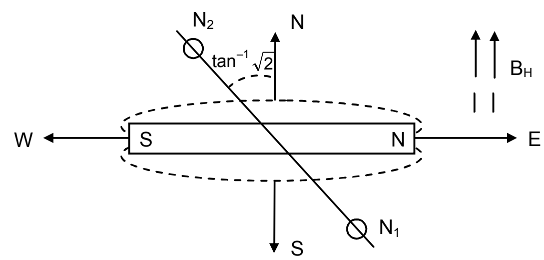
9. When the magnet is such that its North faces the geographic south of earth. The neutral point lies along the axial line of the magnet.

$\frac{\mu_0}{4\pi}\frac{2M}{d^3} = 18 \times 10^{-6} \Rightarrow \frac{10^{-7} \times 2 \times 0.72}{d^3} = 18 \times 10^{-6} \Rightarrow d^3 = \frac{2 \times 0.7 \times 10^{-7}}{18 \times 10^{-6}}$

$\Rightarrow d = \left(\frac{8 \times 10^{-9}}{10^{-6}}\right)^{1/3} = 2 \times 10^{-1}$ m = 20 cm



10. Magnetic moment = $0.72\sqrt{2}$ A-m^2 = M



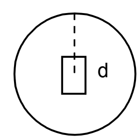
$B = \frac{\mu_0}{4\pi}\frac{M}{d^3}$ $\qquad B_H = 18\ \mu T$

$\Rightarrow \frac{4\pi \times 10^{-7} \times 0.72\sqrt{2}}{4\pi \times d^3} = 18 \times 10^{-6}$

$\Rightarrow d^3 = \frac{0.72 \times 1.414 \times 10^{-7}}{18 \times 10^{-6}} = 0.005656$

$\Rightarrow d \approx 0.2$ m = 20 cm

11. The geomagnetic pole is at the end on position of the earth.



$B = \frac{\mu_0}{4\pi}\frac{2M}{d^3} = \frac{10^{-7} \times 2 \times 8 \times 10^{22}}{(6400 \times 10^3)^3} \approx 60 \times 10^{-6}\ T = 60\ \mu T$

12. $\vec{B} = 3.4 \times 10^{-5}$ T

Given $\frac{\mu_0}{4\pi}\frac{M}{R^3} = 3.4 \times 10^{-5}$

$\Rightarrow M = \frac{3.4 \times 10^{-5} \times R^3 \times 4\pi}{4\pi \times 10^{-7}} = 3.4 \times 10^2\ R^3$

$\vec{B}$ at Poles = $\frac{\mu_0}{4\pi}\frac{2M}{R^3}$ = = 6.8×10^{-5} T

13. δ(dip) = 60°

$B_H = B \cos 60°$

$\Rightarrow B = 52 \times 10^{-6} = 52\ \mu T$

$B_V = B \sin \delta = 52 \times 10^{-6}\ \frac{\sqrt{3}}{2} = 44.98\ \mu T \approx 45\ \mu T$

14. If δ_1 and δ_2 be the apparent dips shown by the dip circle in the 2⊥r positions, the true dip δ is given by

$\text{Cot}^2\ \delta = \text{Cot}^2\ \delta_1 + \text{Cot}^2\ \delta_2$

$\Rightarrow \text{Cot}^2\ \delta = \text{Cot}^2\ 45° + \text{Cot}^2\ 53°$

$\Rightarrow \text{Cot}^2\ \delta = 1.56 \Rightarrow \delta = 38.6 \approx 39°$

15. We know $B_H = \frac{\mu_0 in}{2r}$

Give : $B_H = 3.6 \times 10^{-5}$ T $\quad \theta = 45°$

$i = 10$ mA $= 10^{-2}$ A $\quad \tan\theta = 1$

$n = ?$ $\quad r = 10$ cm $= 0.1$ m

$$n = \frac{B_H \tan\theta \times 2r}{\mu_0 i} = \frac{3.6 \times 10^{-5} \times 2 \times 1 \times 10^{-1}}{4\pi \times 10^{-7} \times 10^{-2}} = 0.5732 \times 10^3 \approx 573 \text{ turns}$$

16. $n = 50$ $\quad A = 2 \text{ cm} \times 2 \text{ cm} = 2 \times 2 \times 10^{-4}\ m^2$

$i = 20 \times 10^{-3}$ A $\quad B = 0.5$ T

$$\tau = ni(\vec{A} \times \vec{B}) = niAB \sin 90° = 50 \times 20 \times 10^{-3} \times 4 \times 10^{-4} \times 0.5 = 2 \times 10^{-4} \text{ N-M}$$

17. Given $\theta = 37°$ $\quad d = 10$ cm $= 0.1$ m

We know

$$\frac{M}{B_H} = \frac{4\pi}{\mu_0}\frac{(d^2 - \ell^2)^2}{2d}\tan\theta = \frac{4\pi}{\mu_0} \times \frac{d^4}{2d}\tan\theta \quad \text{[As the magnet is short]}$$

$$= \frac{4\pi}{4\pi \times 10^{-7}} \times \frac{(0.1)^3}{2} \times \tan 37° = 0.5 \times 0.75 \times 1 \times 10^{-3} \times 10^7 = 0.375 \times 10^4 = 3.75 \times 10^3 \text{ A-m}^2\text{ T}^{-1}$$

18. $\frac{M}{B_H}$ (found in the previous problem) $= 3.75 \times 10^3$ A-m^2 T^{-1}

$\theta = 37°$, $\quad d = ?$

$$\frac{M}{B_H} = \frac{4\pi}{\mu_0}(d^2 + \ell^2)^{3/2}\tan\theta$$

$\ell << d$ $\quad$ neglecting ℓ w.r.t.d

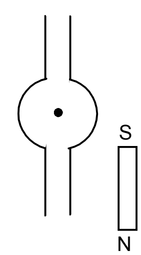
$$\Rightarrow \frac{M}{B_H} = \frac{4\pi}{\mu_0}d^3 \text{Tan}\theta \Rightarrow 3.75 \times 10^3 = \frac{1}{10^{-7}} \times d^3 \times 0.75$$

$$\Rightarrow d^3 = \frac{3.75 \times 10^3 \times 10^{-7}}{0.75} = 5 \times 10^{-4}$$

$\Rightarrow d = 0.079$ m $= 7.9$ cm

19. Given $\frac{M}{B_H} = 40$ A-m^2/T

Since the magnet is short 'ℓ' can be neglected



So, $\frac{M}{B_H} = \frac{4\pi}{\mu_0} \times \frac{d^3}{2} = 40$

$$\Rightarrow d^3 = \frac{40 \times 4\pi \times 10^{-7} \times 2}{4\pi} = 8 \times 10^{-6}$$

$\Rightarrow d = 2 \times 10^{-2}$ m $= 2$ cm

with the northpole pointing towards south.

20. According to oscillation magnetometer,

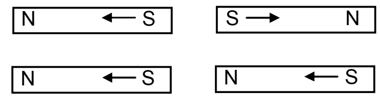
$$T = 2\pi\sqrt{\frac{I}{MB_H}}$$

$$\Rightarrow \frac{\pi}{10} = 2\pi\sqrt{\frac{1.2 \times 10^{-4}}{M \times 30 \times 10^{-6}}}$$

$$\Rightarrow \left(\frac{1}{20}\right)^2 = \frac{1.2 \times 10^{-4}}{M \times 30 \times 10^{-6}}$$

$$\Rightarrow M = \frac{1.2 \times 10^{-4} \times 400}{30 \times 10^{-6}} = 16 \times 10^2 \text{ A-m}^2 = 1600 \text{ A-m}^2$$

21. We know : $\upsilon = \dfrac{1}{2\pi}\sqrt{\dfrac{mB_H}{I}}$



For like poles tied together

$M = M_1 - M_2$

For unlike poles $M' = M_1 + M_2$

$\dfrac{\upsilon_1}{\upsilon_2} = \sqrt{\dfrac{M_1 - M_2}{M_1 + M_2}} \Rightarrow \left(\dfrac{10}{2}\right)^2 = \dfrac{M_1 - M_2}{M_1 + M_2} \Rightarrow 25 = \dfrac{M_1 - M_2}{M_1 + M_2}$

$\Rightarrow \dfrac{26}{24} = \dfrac{2M_1}{2M_2} \Rightarrow \dfrac{M_1}{M_2} = \dfrac{13}{12}$

22. $B_H = 24 \times 10^{-6}$ T $\qquad T_1 = 0.1'$

$B = B_H - B_{wire} = 2.4 \times 10^{-6} - \dfrac{\mu_o}{2\pi}\dfrac{i}{r} = 24 \times 10^{-6} - \dfrac{2 \times 10^{-7} \times 18}{0.2} = (24 - 10) \times 10^{-6} = 14 \times 10^{-6}$

$T = 2\pi\sqrt{\dfrac{I}{MB_H}} \qquad \dfrac{T_1}{T_2} = \sqrt{\dfrac{B}{B_H}}$

$\Rightarrow \dfrac{0.1}{T_2} = \sqrt{\dfrac{14 \times 10^{-6}}{24 \times 10^{-6}}} \Rightarrow \left(\dfrac{0.1}{T_2}\right)^2 = \dfrac{14}{24} \Rightarrow T_2^{\,2} = \dfrac{0.01 \times 14}{24} \Rightarrow T_2 = 0.076$

23. $T = 2\pi\sqrt{\dfrac{I}{MB_H}}$ $\qquad$ Here $I' = 2I$

$T_1 = \dfrac{1}{40}$ min $\qquad T_2 = ?$

$\dfrac{T_1}{T_2} = \sqrt{\dfrac{I}{I'}}$

$\Rightarrow \dfrac{1}{40T_2} = \sqrt{\dfrac{1}{2}} \Rightarrow \dfrac{1}{1600T_2^{\,2}} = \dfrac{1}{2} \Rightarrow T_2^{\,2} = \dfrac{1}{800} \Rightarrow T_2 = 0.03536$ min

For 1 oscillation Time taken = 0.03536 min.

For 40 Oscillation Time = 4 × 0.03536 = 1.414 = $\sqrt{2}$ min

24. γ_1 = 40 oscillations/minute

$B_H = 25\ \mu T$

m of second magnet = 1.6 A-m^2

d = 20 cm = 0.2 m

(a) For north facing north

$\gamma_1 = \dfrac{1}{2\pi}\sqrt{\dfrac{MB_H}{I}} \qquad \gamma_2 = \dfrac{1}{2\pi}\sqrt{\dfrac{M(B_H - B)}{I}}$

$B = \dfrac{\mu_0}{4\pi}\dfrac{m}{d^3} = \dfrac{10^{-7} \times 1.6}{8 \times 10^{-3}} = 20\ \mu T$

$\dfrac{\gamma_1}{\gamma_2} = \sqrt{\dfrac{B}{B_H - B}} \Rightarrow \dfrac{40}{\gamma_2} = \sqrt{\dfrac{25}{5}} \Rightarrow \gamma_2 = \dfrac{40}{\sqrt{5}} = 17.88 \approx 18$ osci/min

(b) For north pole facing south

$\gamma_1 = \dfrac{1}{2\pi}\sqrt{\dfrac{MB_H}{I}} \qquad \gamma_2 = \dfrac{1}{2\pi}\sqrt{\dfrac{M(B_H - B)}{I}}$

$\dfrac{\gamma_1}{\gamma_2} = \sqrt{\dfrac{B}{B_H - B}} \Rightarrow \dfrac{40}{\gamma_2} = \sqrt{\dfrac{25}{45}} \Rightarrow \gamma_2 = \dfrac{40}{\sqrt{\left(\dfrac{25}{45}\right)}} = 53.66 \approx 54$ osci/min

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CHAPTER – 37
MAGNETIC PROPERTIES OF MATTER

1. $B = \mu_0 ni$, $\quad H = \dfrac{B}{\mu_0}$

   $\Rightarrow H = ni$

   $\Rightarrow 1500$ A/m = n× 2

   $\Rightarrow n = 750$ turns/meter

   $\Rightarrow n = 7.5$ turns/cm

2. (a) H = 1500 A/m

   As the solenoid and the rod are long and we are interested in the magnetic intensity at the centre, the end effects may be neglected. There is no effect of the rod on the magnetic intensity at the centre.

   (b) $I = 0.12$ A/m

   We know $\vec{I} = X\vec{H}$ $\quad$ X = Susceptibility

   $\Rightarrow X = \dfrac{I}{H} = \dfrac{0.12}{1500} = 0.00008 = 8 \times 10^{-5}$

   (c) The material is paramagnetic

3. $B_1 = 2.5 \times 10^{-3}$, $\quad B_2 = 2.5$

   $A = 4 \times 10^{-4}\ m^2$, $\quad$ n = 50 turns/cm = 5000 turns/m

   (a) $B = \mu_0 ni$,

   $\Rightarrow 2.5 \times 10^{-3} = 4\pi \times 10^{-7} \times 5000 \times i$

   $\Rightarrow i = \dfrac{2.5 \times 10^{-3}}{4\pi \times 10^{-7} \times 5000} = 0.398\ A \approx 0.4\ A$

   (b) $I = \dfrac{B_2}{\mu_0} - H = \dfrac{2.5}{4\pi \times 10^{-7}} - (B_2 - B_1) = \dfrac{2.5}{4\pi \times 10^{-7}} - 2.497 = 1.99 \times 10^6 \approx 2 \times 10^6$

   (c) $I = \dfrac{M}{V} \Rightarrow I = \dfrac{m\ell}{A\ell} = \dfrac{m}{A}$

   $\Rightarrow m = IA = 2 \times 10^6 \times 4 \times 10^{-4} = 800$ A-m

4. (a) Given $\quad$ d = 15 cm = 0.15 m

   ℓ = 1 cm = 0.01 m

   A = 1.0 cm^2 = $1 \times 10^{-4}\ m^2$

   $B = 1.5 \times 10^{-4}$ T

   M = ?

   We Know $\vec{B} = \dfrac{\mu_0}{4\pi} \times \dfrac{2Md}{(d^2 - \ell^2)^2}$

   $\Rightarrow 1.5 \times 10^{-4} = \dfrac{10^{-7} \times 2 \times M \times 0.15}{(0.0225 - 0.0001)^2} = \dfrac{3 \times 10^{-8} M}{5.01 \times 10^{-4}}$

   $\Rightarrow M = \dfrac{1.5 \times 10^{-4} \times 5.01 \times 10^{-4}}{3 \times 10^{-8}} = 2.5\ A$

   (b) Magnetisation $I = \dfrac{M}{V} = \dfrac{2.5}{10^{-4} \times 10^{-2}} = 2.5 \times 10^6$ A/m

   (c) $H = \dfrac{m}{4\pi d^2} = \dfrac{M}{4\pi I d^2} = \dfrac{2.5}{4 \times 3.14 \times 0.01 \times (0.15)^2}$

   net $H = H_N + H_{'} = 2 \times 884.6 = 8.846 \times 10^2$

   $\vec{B} = \mu_0 (-H + I) = 4\pi \times 10^{-7} (2.5\times10^6 - 2 \times 884.6) \approx 3{,}14$ T

5. Permiability (μ) = $\mu_0(1 + x)$
Given susceptibility = 5500
$\mu = 4 \times 10^{-7} (1 + 5500)$
$= 4 \times 3.14 \times 10^{-7} \times 5501\ 6909.56 \times 10^{-7} \approx 6.9 \times 10^{-3}$

6. B = 1.6 T, H = 1000 A/m
μ = Permeability of material
$\mu = \frac{B}{H} = \frac{1.6}{1000} = 1.6 \times 10^{-3}$
$\mu r = \frac{\mu}{\mu_0} = \frac{1.6 \times 10^{-3}}{4\pi \times 10^{-7}} = 0.127 \times 10^4 \approx 1.3 \times 10^3$
$\mu = \mu_0 (1 + x)$
$\Rightarrow x = \frac{\mu}{\mu_0} - 1$
$= \mu_r - 1 = 1.3 \times 10^3 - 1 = 1300 - 1 = 1299 \approx 1.3 \times 10^3$

7. $x = \frac{C}{T} = \Rightarrow \frac{x_1}{x_2} = \frac{T_2}{T_1}$
$\Rightarrow \frac{1.2 \times 10^{-5}}{1.8 \times 10^{-5}} = \frac{T_2}{300}$
$\Rightarrow T_2 = \frac{12}{18} \times 300 = 200$ K.

8. $f = 8.52 \times 10^{28}$ atoms/m^3
For maximum 'I', Let us consider the no. of atoms present in 1 m^3 of volume.
Given: m per atom = $2 \times 9.27 \times 10^{-24}$ A–m^2
$I = \frac{\text{net m}}{V} = 2 \times 9.27 \times 10^{-24} \times 8.52 \times 10^{28} \approx 1.58 \times 10^6$ A/m
$B = \mu_0 (H + I) = \mu_0 I$ [$\therefore$ H = 0 in this case]
$= 4\pi \times 10^{-7} \times 1.58 \times 10^6 = 1.98 \times 10^{-1} \approx 2.0$ T

9. $B = \mu_0 ni$, $H = \frac{B}{\mu_0}$
Given n = 40 turn/cm = 4000 turns/m
$\Rightarrow$ H = ni
$H = 4 \times 10^4$ A/m
$\Rightarrow i = \frac{H}{n} = \frac{4 \times 10^4}{4000} = 10$ A.

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ELECTROMAGNETIC INDUCTION
CHAPTER - 38

1. (a) $\int E.dl = MLT^{-3}I^{-1} \times L = ML^2I^{-1}T^{-3}$

(b) $\vartheta BI = LT^{-1} \times MI^{-1}T^{-2} \times L = ML^2I^{-1}T^{-3}$

(c) $d\phi_s / dt = MI^{-1}T^{-2} \times L^2 = ML^2I^{-1}T^{-2}$

2. $\phi = at^2 + bt + c$

(a) a = $\left[\frac{\phi}{t^2}\right] = \left[\frac{\phi/t}{t}\right] = \frac{\text{Volt}}{\text{Sec}}$

b = $\left[\frac{\phi}{t}\right]$ = Volt

c = $[\phi]$ = Weber

(b) E = $\frac{d\phi}{dt}$ [a = 0.2, b = 0.4, c = 0.6, t = 2s]

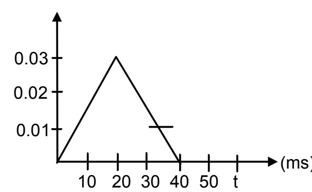
= 2at + b

= 2 × 0.2 × 2 + 0.4 = 1.2 volt

3. (a) ϕ_2 = B.A. = 0.01 × 2 × 10^{-3} = 2 × 10^{-5}.

ϕ_1 = 0

e = $-\frac{d\phi}{dt} = \frac{-2 \times 10^{-5}}{10 \times 10^{-3}}$ = – 2 mV



ϕ_3 = B.A. = 0.03 × 2 × 10^{-3} = 6 × 10^{-5}

$d\phi$ = 4 × 10^{-5}

e = $-\frac{d\phi}{dt}$ = –4 mV

ϕ_4 = B.A. = 0.01 × 2 × 10^{-3} = 2 × 10^{-5}

$d\phi$ = –4 × 10^{-5}

e = $-\frac{d\phi}{dt}$ = 4 mV

ϕ_5 = B.A. = 0

$d\phi$ = –2 × 10^{-5}

e = $-\frac{d\phi}{dt}$ = 2 mV

(b) emf is not constant in case of → 10 – 20 ms and 20 – 30 ms as –4 mV and 4 mV.

4. ϕ_1 = BA = 0.5 × $\pi(5 \times 10^{-2})^2$ = 5π 25 × 10^{-5} = 125 × 10^{-5}

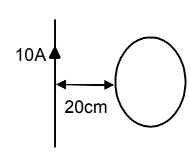
ϕ_2 = 0

E = $\frac{\phi_1 - \phi_2}{t} = \frac{125\pi \times 10^{-5}}{5 \times 10^{-1}}$ = 25π × 10^{-4} = 7.8 × 10^{-3}.

5. A = 1 mm^2 ; i = 10 A, d = 20 cm ; dt = 0.1 s

e = $\frac{d\phi}{dt} = \frac{BA}{dt} = \frac{\mu_0 i}{2\pi d} \times \frac{A}{dt}$

= $\frac{4\pi \times 10^{-7} \times 10}{2\pi \times 2 \times 10^{-1}} \times \frac{10^{-6}}{1 \times 10^{-1}} = 1 \times 10^{-10}\,V$.



6. (a) During removal,

ϕ_1 = B.A. = 1 × 50 × 0.5 × 0.5 – 25 × 0.5 = 12.5 Tesla-m^2

$\phi_2 = 0,\ \tau = 0.25$

e = $-\frac{d\phi}{dt} = \frac{\phi_2 - \phi_1}{dt} = \frac{12.5}{0.25} = \frac{125\times10^{-1}}{25\times10^{-2}} = 50V$

(b) During its restoration

$\phi_1 = 0$; $\phi_2 = 12.5$ Tesla-m^2 ; t = 0.25 s

E = $\frac{12.5 - 0}{0.25}$ = 50 V.

(c) During the motion

$\phi_1 = 0$, $\phi_2 = 0$

E = $\frac{d\phi}{dt} = 0$

7. R = 25 Ω

(a) e = 50 V, T = 0.25 s

i = e/R = 2A, H = i^2 RT

= 4 × 25 × 0.25 = 25 J

(b) e = 50 V, T = 0.25 s

i = e/R = 2A, H = i^2 RT = 25 J

(c) Since energy is a scalar quantity

Net thermal energy developed = 25 J + 25 J = 50 J.

8. A = 5 cm^2 = 5 × 10^{-4} m^2

B = B_0 sin ωt = 0.2 sin(300 t)

θ = 60°

a) Max emf induced in the coil

E = $-\frac{d\phi}{dt} = \frac{d}{dt}(BA\cos\theta)$

= $\frac{d}{dt}(B_0 \sin\omega t \times 5\times10^{-4}\times\frac{1}{2})$

= $B_0 \times \frac{5}{2}\times10^{-4}\frac{d}{dt}(\sin\omega t)$ = $\frac{B_0 5}{2}\times10^{-4}\cos\omega t\cdot\omega$

= $\frac{0.2\times5}{2}\times300\times10^{-4}\times\cos\omega t = 15\times10^{-3}\cos\omega t$

E_{max} = 15 × 10^{-3} = 0.015 V

b) Induced emf at t = (π/900) s

E = 15 × 10^{-3} × cos ωt

= 15 × 10^{-3} × cos (300 × π/900) = 15 × 10^{-3} × ½

= 0.015/2 = 0.0075 = 7.5 × 10^{-3} V

c) Induced emf at t = π/600 s

E = 15 × 10^{-3} × cos (300 × π/600)

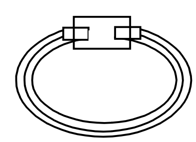
= 15 × 10^{-3} × 0 = 0 V.

9. $\vec{B}$ = 0.10 T

A = 1 cm^2 = 10^{-4} m^2

T = 1 s

φ = B.A. = 10^{-1} × 10^{-4} = 10^{-5}



e = $\frac{d\phi}{dt} = \frac{10^{-5}}{1} = 10^{-5}$ = 10 μV

10. E = 20 mV = 20 × 10^{-3} V

A = (2 × 10^{-2})^2 = 4 × 10^{-4}

Dt = 0.2 s, θ = 180°

$\phi_1 = BA$, $\phi_2 = -BA$

$d\phi = 2BA$

$E = \frac{d\phi}{dt} = \frac{2BA}{dt}$

$\Rightarrow 20 \times 10^{-3} = \frac{2 \times B \times 2 \times 10^{-4}}{2 \times 10^{-1}}$

$\Rightarrow 20 \times 10^{-3} = 4 \times B \times 10^{-3}$

$\Rightarrow B = \frac{20 \times 10^{-3}}{42 \times 10^{-3}} = 5T$

11. Area = A, Resistance = R, B = Magnetic field

$\phi = BA = Ba \cos 0° = BA$

$e = \frac{d\phi}{dt} = \frac{BA}{1}$; $i = \frac{e}{R} = \frac{BA}{R}$

$\phi = iT = BA/R$

12. $r = 2$ cm $= 2 \times 10^{-2}$ m

n = 100 turns / cm = 10000 turns/m

i = 5 A

$B = \mu_0 ni$

$= 4\pi \times 10^{-7} \times 10000 \times 5 = 20\pi \times 10^{-3} = 62.8 \times 10^{-3}$ T

$n_2 = 100$ turns

$R = 20\ \Omega$

$r = 1$ cm $= 10^{-2}$ m

Flux linking per turn of the second coil $= B\pi r^2 = B\pi \times 10^{-4}$

ϕ_1 = Total flux linking $= Bn_2\ \pi r^2 = 100 \times \pi \times 10^{-4} \times 20\pi \times 10^{-3}$

When current is reversed.

$\phi_2 = -\phi_1$

$d\phi = \phi_2 - \phi_1 = 2 \times 100 \times \pi \times 10^{-4} \times 20\pi \times 10^{-3}$

$E = -\frac{d\phi}{dt} = \frac{4\pi^2 \times 10^{-4}}{dt}$

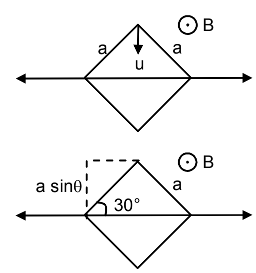
$I = \frac{E}{R} = \frac{4\pi^2 \times 10^{-4}}{dt \times 20}$

$q = Idt = \frac{4\pi^2 \times 10^{-4}}{dt \times 20} \times dt = 2 \times 10^{-4}$ C.

13. Speed = u

Magnetic field = B

Side = a



a) The perpendicular component i.e. a sinθ is to be taken which is ⊥r to velocity.

So, $l = a \sin\theta\ 30° = a/2$.

Net 'a' charge $= 4 \times a/2 = 2a$

So, induced emf $= B\vartheta l = 2auB$

b) Current $= \frac{E}{R} = \frac{2auB}{R}$

14. $\phi_1 = 0.35$ weber, $\phi_2 = 0.85$ weber

$D\phi = \phi_2 - \phi_1 = (0.85 - 0.35)$ weber $= 0.5$ weber

dt = 0.5 sec

$E = \frac{d\phi}{dt'} = \frac{0.5}{0.5}$ = 1 v.

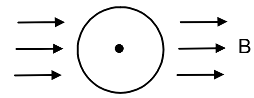
The induced current is anticlockwise as seen from above.

15. i = v(B × l)

= v B l cosθ

θ is angle between normal to plane and $\vec{B}$ = 90°.

= v B l cos 90° = 0.



16. u = 1 cm/′, B = 0.6 T

a) At t = 2 sec, distance moved = 2 × 1 cm/s = 2 cm

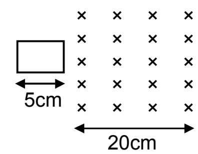
$E = \frac{d\phi}{dt} = \frac{0.6\times(2\times5-0)\times10^{-4}}{2}$ = 3 ×10^{-4} V

b) At t = 10 sec

distance moved = 10 × 1 = 10 cm

The flux linked does not change with time

∴ E = 0



c) At t = 22 sec

distance = 22 × 1 = 22 cm

The loop is moving out of the field and 2 cm outside.

$E = \frac{d\phi}{dt} = B\times\frac{dA}{dt}$

$= \frac{0.6\times(2\times5\times10^{-4})}{2}$ = 3 × 10^{-4} V

d) At t = 30 sec

The loop is total outside and flux linked = 0

∴ E = 0.

17. As heat produced is a scalar prop.

So, net heat produced = $H_a + H_b + H_c + H_d$

R = 4.5 mΩ = 4.5 ×10^{-3} Ω

a) e = 3 × 10^{-4} V

$i = \frac{e}{R} = \frac{3\times10^{-4}}{4.5\times10^{-3}} = 6.7\times10^{-2}$ Amp.

$H_a = (6.7\times10^{-2})^2 \times 4.5\times10^{-3}\times 5$

$H_b = H_d = 0$ [since emf is induced for 5 sec]

$H_c = (6.7\times10^{-2})^2 \times 4.5\times10^{-3}\times 5$

So Total heat = $H_a + H_c$

$= 2\times(6.7\times10^{-2})^2\times4.5\times10^{-3}\times5 = 2\times10^{-4}$ J.

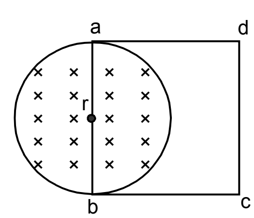
18. r = 10 cm, R = 4 Ω

$\frac{dB}{dt} = 0.010\ T/'$, $\frac{d\phi}{dt} = \frac{dB}{dt}A$

$E = \frac{d\phi}{dt} = \frac{dB}{dt}\times A = 0.01\left(\frac{\pi\times r^2}{2}\right)$

$= \frac{0.01\times3.14\times0.01}{2} = \frac{3.14}{2}\times10^{-4} = 1.57\times10^{-4}$

$i = \frac{E}{R} = \frac{1.57\times10^{-4}}{4}$ = 0.39 × 10^{-4} = 3.9 × 10^{-5}A

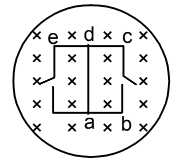


19. a) S_1 closed S_2 open

net R = 4 × 4 = 16 Ω

$e = \frac{d\phi}{dt} = A\frac{dB}{dt} = 10^{-4} \times 2 \times 10^{-2} = 2 \times 10^{-6}$ V

i through ad = $\frac{e}{R} = \frac{2 \times 10^{-6}}{16} = 1.25 \times 10^{-7}$ A along ad



b) R = 16 Ω

$e = A \times \frac{dB}{dt} = 2 \times 0^{-5}$ V

$i = \frac{2 \times 10^{-6}}{16} = 1.25 \times 10^{-7}$ A along d a

c) Since both S_1 and S_2 are open, no current is passed as circuit is open i.e. i = 0

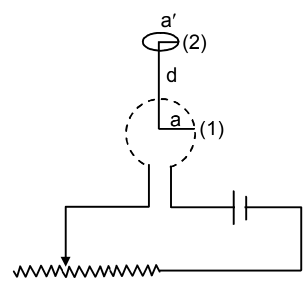
d) Since both S_1 and S_2 are closed, the circuit forms a balanced wheat stone bridge and no current will flow along ad i.e. i = 0.

20. Magnetic field due to the coil (1) at the center of (2) is $B = \frac{\mu_0 Nia^2}{2(a^2 + x^2)^{3/2}}$

Flux linked with the second,

$= B.A_{(2)} = \frac{\mu_0 Nia^2}{2(a^2 + x^2)^{3/2}} \pi a'^2$

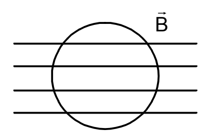
E.m.f. induced $\frac{d\phi}{dt} = \frac{\mu_0 Na^2 a'^2 \pi}{2(a^2 + x^2)^{3/2}} \frac{di}{dt}$



$= \frac{\mu_0 N\pi a^2 a'^2}{2(a^2 + x^2)^{3/2}} \frac{d}{dt} \frac{E}{((R/L)x + r)}$

$= \frac{\mu_0 N\pi a^2 a'^2}{2(a^2 + x^2)^{3/2}} E \frac{-1.R/L.v}{((R/L)x + r)^2}$

b) $= \frac{\mu_0 N\pi a^2 a'^2}{2(a^2 + x^2)^{3/2}} \frac{ERV}{L(R/2 + r)^2}$ (for x = L/2, R/L x = R/2)



a) For x = L

$E = \frac{\mu_0 N\pi a^2 a'^2 RvE}{2(a^2 + x^2)^{3/2}(R + r)^2}$

21. N = 50, $\vec{B}$ = 0.200 T ; r = 2.00 cm = 0.02 m

θ = 60°, t = 0.100 s

a) $e = \frac{Nd\phi}{dt} = \frac{N \times B.A}{T} = \frac{NBA\cos 60°}{T}$

$= \frac{50 \times 2 \times 10^{-1} \times \pi \times (0.02)^2}{0.1} = 5 \times 4 \times 10^{-3} \times \pi$

$= 2\pi \times 10^{-2}$ V $= 6.28 \times 10^{-2}$ V

b) $i = \frac{e}{R} = \frac{6.28 \times 10^{-2}}{4} = 1.57 \times 10^{-2}$ A

$Q = it = 1.57 \times 10^{-2} \times 10^{-1} = 1.57 \times 10^{-3}$ C

22. n = 100 turns, $B = 4 \times 10^{-4}$ T

$A = 25\ cm^2 = 25 \times 10^{-4}\ m^2$

a) When the coil is perpendicular to the field

$\phi = nBA$

When coil goes through half a turn

$\phi = BA \cos 18° = 0 - nBA$

$d\phi = 2nBA$

The coil undergoes 300 rev, in 1 min

300 × 2π rad/min = 10 π rad/sec

10π rad is swept in 1 sec.

π/π rad is swept 1/10π × π = 1/10 sec

$E = \frac{d\phi}{dt} = \frac{2nBA}{dt} = \frac{2 \times 100 \times 4 \times 10^{-4} \times 25 \times 10^{-4}}{1/10}$ = 2 × 10^{-3} V

b) ϕ_1 = nBA, ϕ_2 = nBA (θ = 360°)

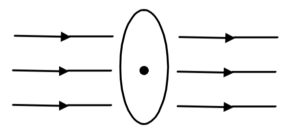
$d\phi = 0$

c) $i = \frac{E}{R} = \frac{2 \times 10^{-3}}{4} = \frac{1}{2} \times 10^{-3}$

= 0.5 × 10^{-3} = 5 × 10^{-4}

q = idt = 5 × 10^{-4} × 1/10 = 5 × 10^{-5} C.

23. r = 10 cm = 0.1 m



R = 40 Ω, N = 1000

θ = 180°, B_H = 3 × 10^{-5} T

ϕ = N(B.A) = NBA Cos 180° or = –NBA

= 1000 × 3 × 10^{-5} × π × 1 × 10^{-2} = 3π × 10^{-4} where

$d\phi$ = 2NBA = 6π ×10^{-4} weber

$e = \frac{d\phi}{dt} = \frac{6\pi \times 10^{-4} V}{dt}$

$i = \frac{6\pi \times 10^{-4}}{40dt} = \frac{4.71 \times 10^{-5}}{dt}$

$Q = \frac{4.71 \times 10^{-5} \times dt}{dt}$ = 4.71 × 10^{-5} C.

24. emf = $\frac{d\phi}{dt} = \frac{dB.A\cos\theta}{dt}$

= B A sin θ ω = –BA ω sin θ

(dq/dt = the rate of change of angle between arc vector and B = ω)

a) emf maximum = BAω = 0.010 × 25 × 10^{-4} × 80 × $\frac{2\pi \times \pi}{6}$

= 0.66 × 10^{-3} = 6.66 × 10^{-4} volt.

b) Since the induced emf changes its direction every time,

so for the average emf = 0

25. $H = \int_0^t i^2 R dt = \int_0^t \frac{B^2A^2\omega^2}{R^2}$ sin ωt R dt

$= \frac{B^2A^2\omega^2}{2R^2} \int_0^t (1 - \cos 2\omega t)dt$

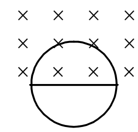
$= \frac{B^2A^2\omega^2}{2R} \left(t - \frac{\sin 2\omega t}{2\omega} \right)_0^{1\,\text{minute}}$

$= \frac{B^2A^2\omega^2}{2R} \left(60 - \frac{\sin 2 \times 8 - \times 2\pi / 60 \times 60}{2 \times 80 \times 2\pi / 60} \right)$

$= \frac{60}{200} \times \pi^2 r^4 \times B^2 \times \left(80^4 \times \frac{2\pi}{60} \right)^2$

$= \frac{60}{200} \times 10 \times \frac{64}{9} \times 10 \times 625 \times 10^{-8} \times 10^{-4} = \frac{625 \times 6 \times 64}{9 \times 2} \times 10^{-11}$ = 1.33 × 10^{-7} J.

26. ϕ_1 = BA, ϕ_2 = 0



$= \dfrac{2\times10^{-4}\times\pi(0.1)^2}{2} = \pi \times 10^{-5}$

$E = \dfrac{d\phi}{dt} = \dfrac{\pi\times10^{-6}}{2} = 1.57 \times 10^{-6}$ V

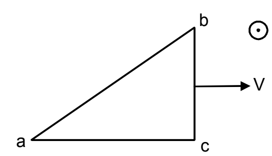
27. l = 20 cm = 0.2 m
v = 10 cm/s = 0.1 m/s
B = 0.10 T
a) $F = q\,v\,B = 1.6 \times 10^{-19} \times 1 \times 10^{-1} \times 1 \times 10^{-1} = 1.6 \times 10^{-21}$ N
b) qE = qvB
$\Rightarrow E = 1 \times 10^{-1} \times 1 \times 10^{-1} = 1 \times 10^{-2}$ V/m
This is created due to the induced emf.
c) Motional emf = $Bv\ell$
$= 0.1 \times 0.1 \times 0.2 = 2 \times 10^{-3}$ V

28. ℓ = 1 m, B = 0.2 T, v = 2 m/s, e = $B\ell v$
$= 0.2 \times 1 \times 2 = 0.4$ V

29. ℓ = 10 m, v = 3×10^{7} m/s, B = 3×10^{-10} T
Motional emf = $Bv\ell$
$= 3 \times 10^{-10} \times 3 \times 10^{7} \times 10 = 9 \times 10^{-3} = 0.09$ V

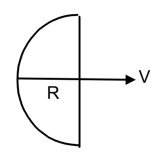
30. v = 180 km/h = 50 m/s
B = 0.2×10^{-4} T, L = 1 m
E = $Bv\ell$ = 0.2 I $10^{-4} \times 50 = 10^{-3}$ V
$\therefore$ The voltmeter will record 1 mv.

31. a) Zero as the components of ab are exactly opposite to that of bc. So they cancel each other. Because velocity should be perpendicular to the length.

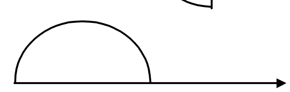


b) e = Bv × ℓ
= Bv (bc) +ve at C
c) e = 0 as the velocity is not perpendicular to the length.
d) e = Bv (bc) positive at 'a'.
i.e. the component of 'ab' along the perpendicular direction.

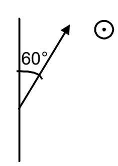
32. a) Component of length moving perpendicular to V is 2R
$\therefore$ E = B v 2R



b) Component of length perpendicular to velocity = 0
$\therefore$ E = 0

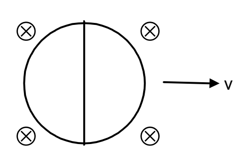


33. ℓ = 10 cm = 0.1 m ;
θ = 60° ; B = 1T
V = 20 cm/s = 0.2 m/s
E = $Bv\ell$ sin60°



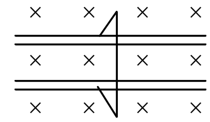
[As we have to take that component of length vector which is $\perp$r to the velocity vector]
$= 1 \times 0.2 \times 0.1 \times \sqrt{3}/2$
$= 1.732 \times 10^{-2} = 17.32 \times 10^{-3}$ V.

34. a) The e.m.f. is highest between diameter $\perp$r to the velocity. Because here length $\perp$r to velocity is highest.
E_{max} = VB2R
b) The length perpendicular to velocity is lowest as the diameter is parallel to the velocity E_{min} = 0.



35. $F_{magnetic} = i\ell B$

This force produces an acceleration of the wire.
But since the velocity is given to be constant.
Hence net force acting on the wire must be zero.

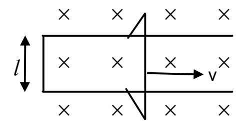


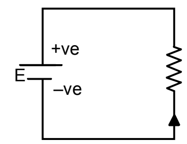
36. $E = Bv\ell$

Resistance = r × total length

$= r \times 2(\ell + vt) = 24(\ell + vt)$

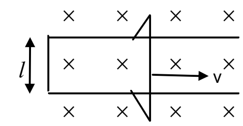
$i = \dfrac{Bv\ell}{2r(\ell + vt)}$





37. $e = Bv\ell$

$i = \dfrac{e}{R} = \dfrac{Bv\ell}{2r(\ell + vt)}$



a) $F = i\ell B = \dfrac{Bv\ell}{2r(\ell + vt)} \times \ell B = \dfrac{B^2\ell^2 v}{2r(\ell + vt)}$

b) Just after t = 0

$F_0 = i\,\ell\,B = \ell B\left(\dfrac{\ell Bv}{2r\ell}\right) = \dfrac{\ell B^2 v}{2r}$

$\dfrac{F_0}{2} = \dfrac{\ell B^2 v}{4r} = \dfrac{\ell^2 B^2 v}{2r(\ell + vt)}$

$\Rightarrow$ $2\ell = \ell + vt$

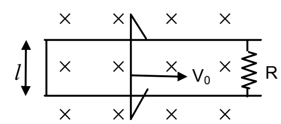
$\Rightarrow$ $T = \ell/v$

38. a) When the speed is V

Emf = $B\ell v$

Resistance = r + r

Current = $\dfrac{B\ell v}{r + R}$



b) Force acting on the wire = $i\ell B$

$= \dfrac{B\ell v \ell B}{R + r} = \dfrac{B^2\ell^2 v}{R + r}$

Acceleration on the wire = $\dfrac{B^2\ell^2 v}{m(R + r)}$

c) $v = v_0 + at = v_0 - \dfrac{B^2\ell^2 v}{m(R + r)}t$ [force is opposite to velocity]

$= v_0 - \dfrac{B^2\ell^2 x}{m(R + r)}$

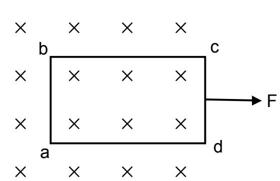
d) $a = v\dfrac{dv}{dx} = \dfrac{B^2\ell^2 v}{m(R + r)}$

$\Rightarrow dx = \dfrac{dvm(R + r)}{B^2\ell^2}$

$\Rightarrow x = \dfrac{m(R + r)v_0}{B^2\ell^2}$

39. R = 2.0 Ω , B = 0.020 T, l = 32 cm = 0.32 m

B = 8 cm = 0.08 m



a) $F = i\ell B = 3.2 \times 10^{-5}$ N

$= \dfrac{B^2\ell^2 v}{R} = 3.2 \times 10^5$

$\Rightarrow \dfrac{(0.020)^2 \times (0.08)^2 \times v}{2} = 3.2 \times 10^{-5}$

$\Rightarrow v = \dfrac{3.2 \times 10^{-5} \times 2}{6.4 \times 10^{-3} \times 4 \times 10^{-4}} = 25$ m/s

b) Emf $E = vB\ell = 25 \times 0.02 \times 0.08 = 4 \times 10^{-2}$ V

c) Resistance per unit length = $\dfrac{2}{0.8}$

Resistance of part ad/cb = $\dfrac{2 \times 0.72}{0.8} = 1.8\ \Omega$

$V_{ab} = iR = \dfrac{B\ell v}{2} \times 1.8 = \dfrac{0.02 \times 0.08 \times 25 \times 1.8}{2} = 0.036\ V = 3.6 \times 10^{-2}$ V

d) Resistance of cd = $\dfrac{2 \times 0.08}{0.8} = 0.2\ \Omega$

$V = iR = \dfrac{0.02 \times 0.08 \times 25 \times 0.2}{2} = 4 \times 10^{-3}$ V

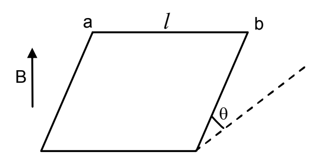
40. $\ell = 20$ cm $= 20 \times 10^{-2}$ m
$v = 20$ cm/s $= 20 \times 10^{-2}$ m/s
$B_H = 3 \times 10^{-5}$ T
$i = 2\ \mu A = 2 \times 10^{-6}$ A
$R = 0.2\ \Omega$

$i = \dfrac{B_v \ell v}{R}$

$\Rightarrow B_v = \dfrac{iR}{\ell v} = \dfrac{2 \times 10^{-6} \times 2 \times 10^{-1}}{20 \times 10^{-2} \times 20 \times 10^{-2}} = 1 \times 10^{-5}$ Tesla

$\tan\delta = \dfrac{B_v}{B_H} = \dfrac{1 \times 10^{-5}}{3 \times 10^{-5}} = \dfrac{1}{3} \Rightarrow \delta(\text{dip}) = \tan^{-1}(1/3)$

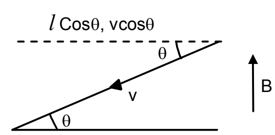
41. $I = \dfrac{B\ell v}{R} = \dfrac{B \times \ell\cos\theta \times v\cos\theta}{R}$



$= \dfrac{B\ell v}{R}\cos^2\theta$

$F = i\ell B = \dfrac{B\ell v\cos^2\theta \times \ell B}{R}$

Now, $F = mg\sin\theta$ [Force due to gravity which pulls downwards]

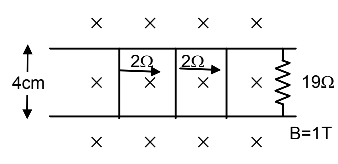


Now, $\dfrac{B^2\ell^2 v\cos^2\theta}{R} = mg\sin\theta$

$\Rightarrow B = \sqrt{\dfrac{Rmg\sin\theta}{\ell^2 v\cos^2\theta}}$

42. a) The wires constitute 2 parallel emf.

$\therefore$ Net emf $= B\ell v = 1 \times 4 \times 10^{-2} \times 5 \times 10^{-2} = 20 \times 10^{-4}$

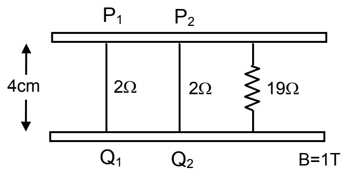


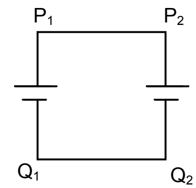
Net resistance = $\dfrac{2 \times 2}{2+2} + 19 = 20\ \Omega$

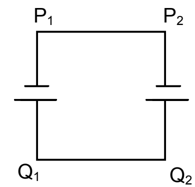
Net current = $\dfrac{20 \times 10^{-4}}{20} = 0.1$ mA.

b) When both the wires move towards opposite directions then not emf = 0

$\therefore$ Net current = 0

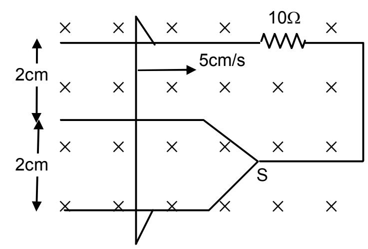
43. 





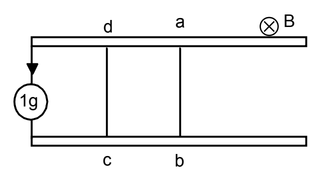
a) No current will pass as circuit is incomplete.
b) As circuit is complete
$VP_2Q_2 = B\ell v$
$= 1 \times 0.04 \times 0.05 = 2 \times 10^{-3}$ V
$R = 2\Omega$
$i = \dfrac{2\times10^{-3}}{2} = 1 \times 10^{-3}$ A = 1 mA.

44. B = 1 T, V = 5 I 10^{-2} m/′, R = 10 Ω

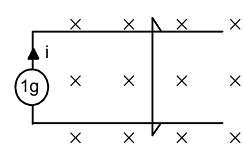


a) When the switch is thrown to the middle rail
$E = Bv\ell$
$= 1 \times 5 \times 10^{-2} \times 2 \times 10^{-2} = 10^{-3}$
Current in the 10 Ω resistor = E/R
$= \dfrac{10^{-3}}{10} = 10^{-4} = 0.1$ mA
b) The switch is thrown to the lower rail
$E = Bv\ell$
$= 1 \times 5 \times 10^{-2} \times 2 \times 10^{-2} = 20 \times 10^{-4}$
Current $= \dfrac{20\times10^{-4}}{10} = 2 \times 10^{-4} = 0.2$ mA

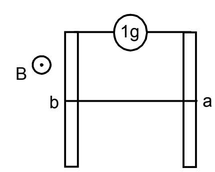
45. Initial current passing = i
Hence initial emf = ir
Emf due to motion of ab = $B\ell v$
Net emf = $ir - B\ell v$
Net resistance = 2r
Hence current passing = $\dfrac{ir - B\ell v}{2r}$



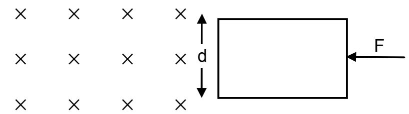
46. Force on the wire = $i\ell B$
Acceleration = $\dfrac{i\ell B}{m}$
Velocity = $\dfrac{i\ell Bt}{m}$



47. Given $B\ell v = mg$ …(1)
When wire is replaced we have
$2\,mg - B\ell v = 2\,ma$ [where a → acceleration]
$\Rightarrow a = \dfrac{2mg - B\ell v}{2m}$
Now, $s = ut + \dfrac{1}{2}at^2$
$\Rightarrow \ell = \dfrac{1}{2} \times \dfrac{2mg - B\ell v}{2m} \times t^2$ [∴ s = ℓ]
$\Rightarrow t = \sqrt{\dfrac{4ml}{2mg - B\ell v}} = \sqrt{\dfrac{4ml}{2mg - mg}} = \sqrt{2\ell / g}$. [from (1)]



48. a) emf developed = Bdv (when it attains a speed v)



Current = $\frac{Bdv}{R}$

Force = $\frac{Bd^2v^2}{R}$

This force opposes the given force

Net F = $F - \frac{Bd^2v^2}{R} = RF - \frac{Bd^2v^2}{R}$

Net acceleration = $\frac{RF - B^2d^2v}{mR}$

b) Velocity becomes constant when acceleration is 0.

$\frac{F}{m} - \frac{B^2d^2v_0}{mR} = 0$

$\Rightarrow \frac{F}{m} = \frac{B^2d^2v_0}{mR}$

$\Rightarrow V_0 = \frac{FR}{B^2d^2}$

c) Velocity at line t

a = $-\frac{dv}{dt}$

$\Rightarrow \int_0^v \frac{dv}{RF - l^2B^2v} = \int_0^t \frac{dt}{mR}$

$\Rightarrow \left[l_n[RF - l^2B^2v] \frac{1}{-l^2B^2} \right]_0^v \quad \left[\frac{t}{Rm} \right]_0^t$

$\Rightarrow \left[l_n(RF - l^2B^2v) \right]_0^v = \frac{-tl^2B^2}{Rm}$

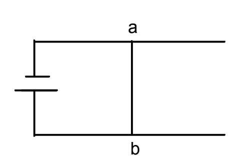
$\Rightarrow l_n(RF - l^2B^2v) - \ln(RF) = \frac{-t^2B^2t}{Rm}$

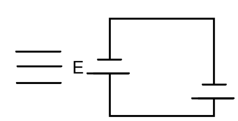
$\Rightarrow 1 - \frac{l^2B^2v}{RF} = e^{\frac{-l^2B^2t}{Rm}}$

$\Rightarrow \frac{l^2B^2v}{RF} = 1 - e^{\frac{-l^2B^2t}{Rm}}$

$\Rightarrow v = \frac{FR}{l^2B^2}\left(1 - e^{\frac{-l^2B^2v_0t}{Rv_0m}} \right) = v_0(1 - e^{-Fv_0m})$

49. Net emf = E – Bvℓ





I = $\frac{E - Bv\ell}{r}$ from b to a

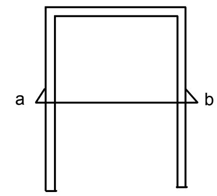
F = I ℓ B

= $\left(\frac{E - Bv\ell}{r} \right) \ell B$ = $\frac{\ell B}{r}(E - Bv\ell)$ towards right.

After some time when E = Bvℓ,

Then the wire moves constant velocity v

Hence v = E / Bℓ.



50. a) When the speed of wire is V
emf developed = $B \ell V$

b) Induced current is the wire = $\frac{B\ell v}{R}$ (from b to a)

c) Down ward acceleration of the wire

$= \frac{mg - F}{m}$ due to the current

$= mg - i\ell B/m = g - \frac{B^2\ell^2 V}{Rm}$

d) Let the wire start moving with constant velocity. Then acceleration = 0

$\frac{B^2\ell^2 v}{Rm} m = g$

$\Rightarrow V_m = \frac{gRm}{B^2\ell^2}$

e) $\frac{dV}{dt} = a$

$\Rightarrow \frac{dV}{dt} = \frac{mg - B^2\ell^2 v/R}{m}$

$\Rightarrow \frac{dv}{\frac{mg - B^2\ell^2 v/R}{m}} = dt$

$\Rightarrow \int_0^v \frac{mdv}{mg - \frac{B^2\ell^2 v}{R}} = \int_0^t dt$

$\Rightarrow \frac{m}{\frac{-B^2\ell^2}{R}} \left(\log(mg - \frac{B^2\ell^2 v}{R} \right)_0^v = t$

$\Rightarrow \frac{-mR}{B^2\ell^2} = \log\left[\log\left(mg - \frac{B^2\ell^2 v}{R} \right) - \log(mg) \right] = t$

$\Rightarrow \log\left[\frac{mg - \frac{B^2\ell^2 v}{R}}{mg} \right] = \frac{-tB^2\ell^2}{mR}$

$\Rightarrow \log\left[1 - \frac{B^2\ell^2 v}{Rmg} \right] = \frac{-tB^2\ell^2}{mR}$

$\Rightarrow 1 - \frac{B^2\ell^2 v}{Rmg} = e^{\frac{-tB^2\ell^2}{mR}}$

$\Rightarrow (1 - e^{-B^2\ell^2/mR}) = \frac{B^2\ell^2 v}{Rmg}$

$\Rightarrow v = \frac{Rmg}{B^2\ell^2}\left(1 - e^{-B^2\ell^2/mR} \right)$

$\Rightarrow v = v_m(1 - e^{-gt/Vm})$ $\left[v_m = \frac{Rmg}{B^2\ell^2} \right]$

f) $\frac{ds}{dt} = v \Rightarrow ds = v\ dt$

$\Rightarrow s = vm \int_0^t (1 - e^{-gt/vm})dt$

$= V_m\left(t - \frac{V_m}{g}e^{-gt/vm}\right) = \left(V_m t + \frac{V_m^2}{g}e^{-gt/vm}\right) - \frac{V_m^2}{g}$

$= V_m t - \frac{V_m^2}{g}\left(1 - e^{-gt/vm}\right)$

g) $\frac{d}{dt}mgs = mg\frac{ds}{dt} = mgV_m(1 - e^{-gt/vm)}$

$\frac{d_H}{dt} = i^2R = R\left(\frac{\ell BV}{R}\right)^2 = \frac{\ell^2B^2v^2}{R}$

$\Rightarrow \frac{\ell^2B^2}{R}V_m^2(1 - e^{-gt/vm})^2$

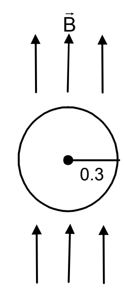
After steady state i.e. $T \to \infty$

$\frac{d}{dt}mgs = mgV_m$

$\frac{d_H}{dt} = \frac{\ell^2B^2}{R}V_m^2 = \frac{\ell^2B^2}{R}V_m\frac{mgR}{\ell^2B^2} = mgV_m$

Hence after steady state $\frac{d_H}{dt} = \frac{d}{dt}mgs$

51. $\ell = 0.3$ m, $\vec{B} = 2.0 \times 10^{-5}$ T, $\omega = 100$ rpm



$v = \frac{100}{60} \times 2\pi = \frac{10}{3}\pi$ rad/s

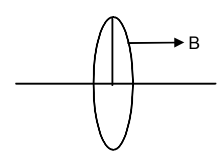
$v = \frac{\ell}{2} \times \omega = \frac{0.3}{2} \times \frac{10}{3}\pi$

Emf = e = $B\ell v$

$= 2.0 \times 10^{-5} \times 0.3 \times \frac{0.33}{2} \times \frac{10}{3}\pi$

$= 3\pi \times 10^{-6}$ V $= 3 \times 3.14 \times 10^{-6}$ V $= 9.42 \times 10^{-6}$ V.

52. V at a distance r/2



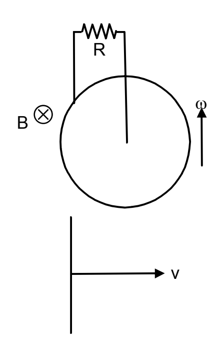
From the centre = $\frac{r\omega}{2}$

$E = B\ell v \Rightarrow E = B \times r \times \frac{r\omega}{2} = \frac{1}{2}Br^2\omega$

53. B = 0.40 T, ω = 10 rad/′, r = 10Ω

r = 5 cm = 0.05 m

Considering a rod of length 0.05 m affixed at the centre and rotating with the same ω.



$v = \frac{\ell}{2} \times \omega = \frac{0.05}{2} \times 10$

$e = B\ell v = 0.40 \times \frac{0.05}{2} \times 10 \times 0.05 = 5 \times 10^{-3}$ V

$I = \frac{e}{R} = \frac{5 \times 10^{-3}}{10} = 0.5$ mA

It leaves from the centre.

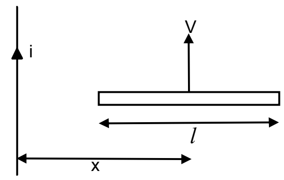
54. $\vec{B} = \frac{B_0}{L} y\hat{K}$

L = Length of rod on y-axis

$V = V_0 \hat{i}$

Considering a small length by of the rod

dE = B V dy



$\Rightarrow dE = \frac{B_0}{L} y \times V_0 \times dy$

$\Rightarrow dE = \frac{B_0 V_0}{L} ydy$

$\Rightarrow E = \frac{B_0 V_0}{L} \int_0^L ydy$

$= \frac{B_0 V_0}{L}\left[\frac{y^2}{2}\right]_0^L = \frac{B_0 V_0}{L}\frac{L^2}{2} = \frac{1}{2} B_0 V_0 L$

55. In this case $\vec{B}$ varies

Hence considering a small element at centre of rod of length dx at a dist x from the wire.

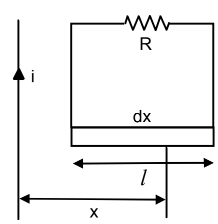
$\vec{B} = \frac{\mu_0 i}{2\pi x}$

So, $de = \frac{\mu_0 i}{2\pi x} \times vxdx$

$e = \int_0^e de = \frac{\mu_0 iv}{2\pi} = \int_{x-t/2}^{x+t/2} \frac{dx}{x} = \frac{\mu_0 iv}{2\pi}$ [ln (x + ℓ/2) – ℓn(x - ℓ/2)]

$= \frac{\mu_0 iv}{2\pi} \ln\left[\frac{x+\ell/2}{x-\ell/2}\right] = \frac{\mu_0 iv}{2x} \ln\left(\frac{2x+\ell}{2x-\ell}\right)$

56. a) emf produced due to the current carrying wire = $\frac{\mu_0 iv}{2\pi} \ln\left(\frac{2x+\ell}{2x-\ell}\right)$



Let current produced in the rod = i′ = $\frac{\mu_0 iv}{2\pi R} \ln\left(\frac{2x+\ell}{2x-\ell}\right)$

Force on the wire considering a small portion dx at a distance x

dF = i′ B ℓ

$\Rightarrow dF = \frac{\mu_0 iv}{2\pi R} \ln\left(\frac{2x+\ell}{2x-\ell}\right) \times \frac{\mu_0 i}{2\pi x} \times dx$

$\Rightarrow dF = \left(\frac{\mu_0 i}{2\pi}\right)^2 \frac{v}{R} \ln\left(\frac{2x+\ell}{2x-\ell}\right) \frac{dx}{x}$

$\Rightarrow F = \left(\frac{\mu_0 i}{2\pi}\right)^2 \frac{v}{R} \ln\left(\frac{2x+\ell}{2x-\ell}\right) \int_{x-t/2}^{x+t/2} \frac{dx}{x}$

$= \left(\frac{\mu_0 i}{2\pi}\right)^2 \frac{v}{R} \ln\left(\frac{2x+\ell}{2x-\ell}\right) \ln\left(\frac{2x+\ell}{2x-\ell}\right)$

$= \frac{v}{R}\left[\frac{\mu_0 i}{2\pi} \ln\left(\frac{2x+\ell}{2x-\ell}\right)\right]^2$

b) Current = $\frac{\mu_0 \ln}{2\pi R} \ln\left(\frac{2x+\ell}{2x-\ell}\right)$

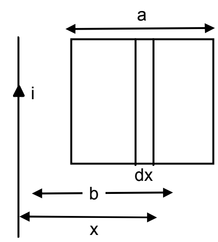
c) Rate of heat developed = i^2R

$$= \left[\frac{\mu_0 iv}{2\pi R}\left(\frac{2x+\ell}{2x-\ell}\right)\right]^2 R = \frac{1}{R}\left[\frac{\mu_0 iv}{2\pi}\ln\left(\frac{2x+\ell}{2x-\ell}\right)^2\right]$$

d) Power developed in rate of heat developed = i^2R

$$= \frac{1}{R}\left[\frac{\mu_0 iv}{2\pi}\ln\left(\frac{2x+\ell}{2x-\ell}\right)\right]^2$$

57. Considering an element dx at a dist x from the wire. We have



a) ϕ = B.A.

$$d\phi = \frac{\mu_0 i \times adx}{2\pi x}$$

$$\phi = \int_0^a d\phi = \frac{\mu_0 ia}{2\pi}\int_b^{a+b}\frac{dx}{x} = \frac{\mu_0 ia}{2\pi}\ln\{1 + a/b\}$$

b) $$e = \frac{d\phi}{dt} = \frac{d}{dt}\frac{\mu_0 ia}{2\pi}\ln[1 + a/b]$$

$$= \frac{\mu_0 a}{2\pi}\ln[1 + a/n]\frac{d}{dt}(i_0 \sin\omega t)$$

$$= \frac{\mu_0 a i_0 \omega \cos\omega t}{2\pi}\ln[1 + a/b]$$

c) $$i = \frac{e}{r} = \frac{\mu_0 a i_0 \omega \cos\omega t}{2\pi r}\ln[1 + a/b]$$

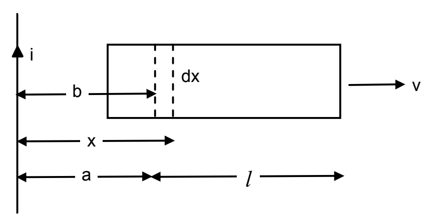
$H = i^2rt$

$$= \left[\frac{\mu_0 a i_0 \omega \cos\omega t}{2\pi r}\ln(1 + a/b)\right]^2 \times r \times t$$

$$= \frac{\mu_0^2 \times a^2 \times i_0^2 \times \omega^2}{4\pi \times r^2}\ln^2[1 + a/b] \times r \times \frac{20\pi}{\omega}$$

$$= \frac{5\mu_0^2 a^2 i_0^2 \omega}{2\pi r}\ln^2[1 + a/b] \quad [\because t = \frac{20\pi}{\omega}]$$

58. a) Using Faraday'' law



Consider a unit length dx at a distance x

$$B = \frac{\mu_0 i}{2\pi x}$$

Area of strip = b dx

$$d\phi = \frac{\mu_0 i}{2\pi x}dx$$

$$\Rightarrow \phi = \int_a^{a+l}\frac{\mu_0 i}{2\pi x}bdx$$

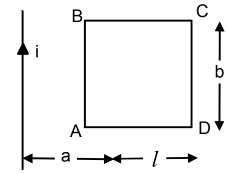
$$= \frac{\mu_0 i}{2\pi}b\int_a^{a+l}\left(\frac{dx}{x}\right) = \frac{\mu_0 ib}{2\pi}\log\left(\frac{a+l}{a}\right)$$

$$\text{Emf} = \frac{d\phi}{dt} = \frac{d}{dt}\left[\frac{\mu_0 ib}{2\pi}\log\left(\frac{a+l}{a}\right)\right]$$

$$= \frac{\mu_0 ib}{2\pi}\frac{a}{a+l}\left(\frac{va - (a+l)v}{a^2}\right) \text{ (where da/dt = V)}$$

$= \frac{\mu_0 ib}{2\pi}\frac{a}{a+l}\frac{vl}{a^2} = \frac{\mu_0 ibvl}{2\pi(a+l)a}$

The velocity of AB and CD creates the emf. since the emf due to AD and BC are equal and opposite to each other.



$B_{AB} = \frac{\mu_0 i}{2\pi a}$ $\Rightarrow$ E.m.f. AB = $\frac{\mu_0 i}{2\pi a}bv$

Length b, velocity v.

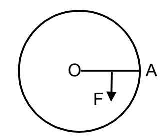
$B_{CD} = \frac{\mu_0 i}{2\pi(a+l)}$

$\Rightarrow$ E.m.f. CD = $\frac{\mu_0 ibv}{2\pi(a+l)}$

Length b, velocity v.

Net emf = $\frac{\mu_0 i}{2\pi a}bv - \frac{\mu_0 ibv}{2\pi(a+l)} = \frac{\mu_0 ibvl}{2\pi a(a+l)}$

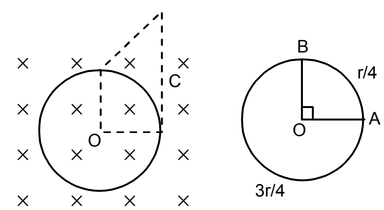
59. e = Bvl = $\frac{B\times a\times\omega\times a}{2}$



$i = \frac{Ba^2\omega}{2R}$

$F = i\ell B = \frac{Ba^2\omega}{2R}\times a\times B = \frac{B^2a^3\omega}{2R}$ towards right of OA.

60. The 2 resistances r/4 and 3r/4 are in parallel.



$R' = \frac{r/4\times 3r/4}{r} = \frac{3r}{16}$

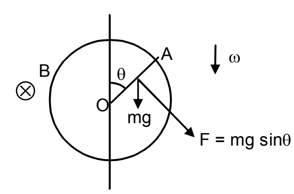
$e = BV\ell$

$= B\times\frac{a}{2}\omega\times a = \frac{Ba^2\omega}{2}$

$i = \frac{e}{R'} = \frac{Ba^2\omega}{2R'} = \frac{Ba^2\omega}{2\times 3r/16}$

$= \frac{Ba^2\omega 16}{2\times 3r} = \frac{8}{3}\frac{Ba^2\omega}{r}$

61. We know

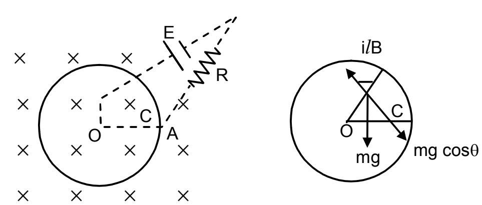


$F = \frac{B^2a^2\omega}{2R} = i\ell B$

Component of mg along F = mg sin θ.

Net force = $\frac{B^2a^3\omega}{2R} - mg\sin\theta$.

62. emf = $\frac{1}{2}B\omega a^2$ [from previous problem]



Current = $\frac{e+E}{R} = \frac{1/2\times B\omega a^2 + E}{R} = \frac{B\omega a^2 + 2E}{2R}$

$\Rightarrow$ mg cos θ = iℓB [Net force acting on the rod is O]

$\Rightarrow$ mg cos θ = $\frac{B\omega a^2 + 2E}{2R}a\times B$

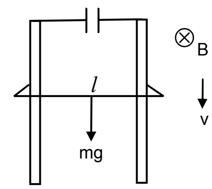
$\Rightarrow R = \frac{(B\omega a^2 + 2E)aB}{2mg\cos\theta}$.

63. Let the rod has a velocity v at any instant,

Then, at the point,

$e = B\ell v$

Now, $q = c \times \text{potential} = ce = CB\ell v$



Current $I = \frac{dq}{dt} = \frac{d}{dt}CBlv$

$= CBl\frac{dv}{dt} = CBla$ (where $a \to$ acceleration)

From figure, force due to magnetic field and gravity are opposite to each other.

So, $mg - I\ell B = ma$

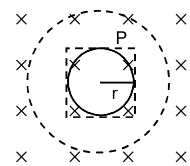
$\Rightarrow mg - CB\ell a \times \ell B = ma$ $\Rightarrow ma + CB^2\ell^2 a = mg$

$\Rightarrow a(m + CB^2\ell^2) = mg$ $\Rightarrow a = \frac{mg}{m + CB^2\ell 2}$

64. a) Work done per unit test charge

$= \phi E.\, dl$ (E = electric field)

$\phi E.\, dl = e$



$\Rightarrow E\phi\, dl = \frac{d\phi}{dt}$ $\Rightarrow E\, 2\pi r = \frac{dB}{dt} \times A$

$\Rightarrow E\, 2\pi r = \pi r^2 \frac{dB}{dt}$

$\Rightarrow E = \frac{\pi r^2}{2\pi}\frac{dB}{dt} = \frac{r}{2}\frac{dB}{dt}$

b) When the square is considered,

$\phi E\, dl = e$

$\Rightarrow E \times 2r \times 4 = \frac{dB}{dt}(2r)^2$

$\Rightarrow E = \frac{dB}{dt}\frac{4r^2}{8r}$ $\Rightarrow E = \frac{r}{2}\frac{dB}{dt}$

$\therefore$ The electric field at the point p has the same value as (a).

65. $\frac{di}{dt} = 0.01$ A/s

For 2s $\frac{di}{dt} = 0.02$ A/s

n = 2000 turn/m, R = 6.0 cm = 0.06 m

r = 1 cm = 0.01 m

a) $\phi = BA$

$\Rightarrow \frac{d\phi}{dt} = \mu_0 nA \frac{di}{dt}$

$= 4\pi \times 10^{-7} \times 2 \times 10^3 \times \pi \times 1 \times 10^{-4} \times 2 \times 10^{-2}$ $[A = \pi \times 1 \times 10^{-4}]$

$= 16\pi^2 \times 10^{-10}\ \omega$

$= 157.91 \times 10^{-10}\ \omega$

$= 1.6 \times 10^{-8}\ \omega$

or, $\frac{d\phi}{dt}$ for 1 s = 0.785 ω.

b) $\int E.dl = \frac{d\phi}{dt}$

$\Rightarrow E\phi dl = \frac{d\phi}{dt} \Rightarrow E = \frac{0.785 \times 10^{-8}}{2\pi \times 10^{-2}} = 1.2 \times 10^{-7}$ V/m

c) $\frac{d\phi}{dt} = \mu_0 n \frac{di}{dt} A = 4\pi \times 10^{-7} \times 2000 \times 0.01 \times \pi \times (0.06)^2$

$E\phi dl = \frac{d\phi}{dt}$

$\Rightarrow E = \frac{d\phi / dt}{2\pi r} = \frac{4\pi \times 10^{-7} \times 2000 \times 0.01 \times \pi \times (0.06)^2}{\pi \times 8 \times 10^{-2}} = 5.64 \times 10^{-7}$ V/m

66. V = 20 V

$dI = I_2 - I_1 = 2.5 - (-2.5) = 5A$

dt = 0.1 s

$V = L\frac{dI}{dt}$

$\Rightarrow 20 = L(5/0.1) \Rightarrow 20 = L \times 50$

$\Rightarrow L = 20 / 50 = 4/10 = 0.4$ Henry.

67. $\frac{d\phi}{dt} = 8 \times 10^{-4}$ weber

n = 200, I = 4A, $E = -nL\ \frac{dI}{dt}$

or, $\frac{-d\phi}{dt} = \frac{-LdI}{dt}$

or, $L = n\frac{d\phi}{dt} = 200 \times 8 \times 10^{-4} = 2 \times 10^{-2}$ H.

68. $E = \frac{\mu_0 N^2 A}{\ell}\frac{dI}{dt}$

$= \frac{4\pi \times 10^{-7} \times (240)^2 \times \pi(2 \times 10^{-2})^2}{12 \times 10^{-2}} \times 0.8$

$= \frac{4\pi \times (24)^2 \times \pi \times 4 \times 8}{12} \times 10^{-8}$

$= 60577.3824 \times 10^{-8} = 6 \times 10^{-4}$ V.

69. We know $i = i_0\ (1 - e^{-t/r})$

a) $\frac{90}{100} i_0 = i_0(1 - e^{-t/r})$

$\Rightarrow 0.9 = 1 - e^{-t/r}$

$\Rightarrow e^{-t/r} = 0.1$

Taking ℓn from both sides

$\ell n\ e^{-t/r} = \ell n\ 0.1 \Rightarrow -t = -2.3 \Rightarrow t/r = 2.3$

b) $\frac{99}{100} i_0 = i_0(1 - e^{-t/r})$

$\Rightarrow e^{-t/r} = 0.01$

$\ell n e^{-t/r} = \ell n\ 0.01$

or, $-t/r = -4.6$ or $t/r = 4.6$

c) $\frac{99.9}{100} i_0 = i_0(1 - e^{-t/r})$

$e^{-t/r} = 0.001$

$\Rightarrow \ln e^{-t/r} = \ln 0.001 \Rightarrow e^{-t/r} = -6.9 \Rightarrow t/r = 6.9.$

70. i = 2A, E = 4V, L = 1H

R = $\frac{E}{i} = \frac{4}{2} = 2$

i = $\frac{L}{R} = \frac{1}{2} = 0.5$

71. L = 2.0 H, R = 20 Ω, emf = 4.0 V, t = 0.20 S

i_0 = $\frac{e}{R} = \frac{4}{20}$, τ = $\frac{L}{R} = \frac{2}{20} = 0.1$

a) i = i_0 (1 – $e^{-t/\tau}$) = $\frac{4}{20}\left(1 - e^{-0.2/0.1}\right)$

= 0.17 A

b) $\frac{1}{2}Li^2 = \frac{1}{2} \times 2 \times (0.17)^2$ = 0.0289 = 0.03 J.

72. R = 40 Ω, E = 4V, t = 0.1, i = 63 mA

i = i_0 – (1 – $e^{tR/2}$)

$\Rightarrow$ 63 × 10^{-3} = 4/40 (1 – $e^{-0.1 \times 40/L}$)

$\Rightarrow$ 63 × 10^{-3} = 10^{-1} (1 – $e^{-4/L}$)

$\Rightarrow$ 63 × 10^{-2} = (1 – $e^{-4/L}$)

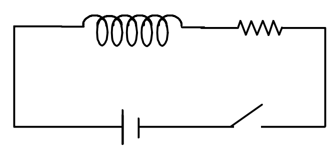
$\Rightarrow$ 1 – 0.63 = $e^{-4/L}$ $\Rightarrow$ $e^{-4/L}$ = 0.37

$\Rightarrow$ –4/L = ln (0.37) = –0.994

$\Rightarrow$ L = $\frac{-4}{-0.994}$ = 4.024 H = 4 H.

73. L = 5.0 H, R = 100 Ω, emf = 2.0 V

t = 20 ms = 20 × 10^{-3} s = 2 × 10^{-2} s



i_0 = $\frac{2}{100}$ now i = i_0 (1 – $e^{-t/\tau}$)

τ = $\frac{L}{R} = \frac{5}{100}$ $\Rightarrow$ i = $\frac{2}{100}\left(1 - e^{\frac{-2\times10^{-2}\times100}{5}}\right)$

$\Rightarrow$ i = $\frac{2}{100}(1 - e^{-2/5})$

$\Rightarrow$ 0.00659 = 0.0066.

V = iR = 0.0066 × 100 = 0.66 V.

74. τ = 40 ms

i_0 = 2 A

a) t = 10 ms

i = i_0 (1 – $e^{-t/\tau}$) = 2(1 – $e^{-10/40}$) = 2(1 – $e^{-1/4}$)

= 2(1 – 0.7788) = 2$(0.2211)^A$ = 0.4422 A = 0.44 A

b) t = 20 ms

i = i_0 (1 – $e^{-t/\tau}$) = 2(1 – $e^{-20/40}$) = 2(1 – $e^{-1/2}$)

= 2(1 – 0.606) = 0.7869 A = 0.79 A

c) t = 100 ms

i = i_0 (1 – $e^{-t/\tau}$) = 2(1 – $e^{-100/40}$) = 2(1 – $e^{-10/4}$)

= 2(1 – 0.082) = 1.835 A =1.8 A

d) t = 1 s

i = i_0 (1 – $e^{-t/\tau}$) = 2(1 – $e^{-1/40\times10^{-3}}$) = 2(1 – $e^{-10/40}$)

= 2(1 – e^{-25}) = 2 × 1 = 2 A

75. L = 1.0 H, R = 20 Ω , emf = 2.0 V

$\tau = \frac{L}{R} = \frac{1}{20}$ = 0.05

$i_0 = \frac{e}{R} = \frac{2}{20}$ = 0.1 A

$i = i_0 (1 - e^{-t}) = i_0 - i_0 e^{-t}$

$\Rightarrow \frac{di}{dt} = \frac{di_0}{dt}(i_0 x - 1/\tau \times e^{-t/\tau}) = i_0 / \tau\ e^{-t/\tau}$.

So,

a) t = 100 ms $\Rightarrow \frac{di}{dt} = \frac{0.1}{0.05} \times e^{-0.1/0.05}$ = 0.27 A

b) t = 200 ms $\Rightarrow \frac{di}{dt} = \frac{0.1}{0.05} \times e^{-0.2/0.05}$ = 0.0366 A

c) t = 1 s $\Rightarrow \frac{di}{dt} = \frac{0.1}{0.05} \times e^{-1/0.05}$ =4 × 10^{-9} A

76. a) For first case at t = 100 ms

$\frac{di}{dt}$ = 0.27

Induced emf = $L\frac{di}{dt}$ = 1 × 0.27 = 0.27 V

b) For the second case at t = 200 ms

$\frac{di}{dt}$ = 0.036

Induced emf = $L\frac{di}{dt}$ = 1 × 0.036 = 0.036 V

c) For the third case at t = 1 s

$\frac{di}{dt}$ = 4.1 × 10^{-9} V

Induced emf = $L\frac{di}{dt}$ = 4.1 × 10^{-9} V

77. L = 20 mH; e = 5.0 V, R = 10 Ω

$\tau = \frac{L}{R} = \frac{20 \times 10^{-3}}{10}$, $i_0 = \frac{5}{10}$

$i = i_0(1 - e^{-t/\tau})^2$

$\Rightarrow i = i_0 - i_0 e^{-t/\tau^2}$

$\Rightarrow iR = i_0 R - i_0 R\, e^{-t/\tau^2}$

a) $10 \times \frac{di}{dt} = \frac{d}{dt} i_0 R + 10 \times \frac{5}{10} \times \frac{10}{20 \times 10^{-3}} \times e^{-0 \times 10/2 \times 10^{-2}}$

$= \frac{5}{2} \times 10^{-3} \times 1 = \frac{5000}{2}$ = 2500 = 2.5 × 10^{-3} V/s.

b) $\frac{Rdi}{dt} = R \times i_0 \times \frac{1}{\tau} \times e^{-t/\tau}$

t = 10 ms = 10 × 10^{-3} s

$\frac{dE}{dt} = 10 \times \frac{5}{10} \times \frac{10}{20 \times 10^{-3}} \times e^{-0.01 \times 10/2 \times 10^{-2}}$

= 16.844 = 17 V/′

c) For t = 1 s

$$\frac{dE}{dt} = \frac{Rdi}{dt} = \frac{5}{2}\,10^3 \times e^{10/2\times10^{-2}} = 0.00 \text{ V/s.}$$

78. L = 500 mH, R = 25 Ω, E = 5 V

a) t = 20 ms

$i = i_0\,(1 - e^{-tR/L}) = \frac{E}{R}(1 - E^{-tR/L})$

$= \frac{5}{25}\left(1 - e^{-20\times10^{-3}\times25/100\times10^{-3}}\right) = \frac{1}{5}(1 - e^{-1})$

$= \frac{1}{5}(1 - 0.3678) = 0.1264$

Potential difference iR = 0.1264 × 25 = 3.1606 V = 3.16 V.

b) t = 100 ms

$i = i_0\,(1 - e^{-tR/L}) = \frac{E}{R}(1 - E^{-tR/L})$

$= \frac{5}{25}\left(1 - e^{-100\times10^{-3}\times25/100\times10^{-3}}\right) = \frac{1}{5}(1 - e^{-5})$

$= \frac{1}{5}(1 - 0.0067) = 0.19864$

Potential difference = iR = 0.19864 × 25 = 4.9665 = 4.97 V.

c) t = 1 sec

$i = i_0\,(1 - e^{-tR/L}) = \frac{E}{R}(1 - E^{-tR/L})$

$= \frac{5}{25}\left(1 - e^{-1\times25/100\times10^{-3}}\right) = \frac{1}{5}(1 - e^{-50})$

$= \frac{1}{5} \times 1 = 1/5$ A

Potential difference = iR = (1/5 × 25) V = 5 V.

79. L = 120 mH = 0.120 H

R = 10 Ω , emf = 6, r = 2

$i = i_0\,(1 - e^{-t/\tau})$

Now, dQ = idt

$= i_0\,(1 - e^{-t/\tau})\,dt$

$Q = \int dQ = \int_0^1 i_0(1 - e^{-t/\tau})dt$

$= i_0\left[\int_0^t dt - \int_0^1 e^{-t/\tau}dt\right] = i_0\left[t - (-\tau)\int_0^t e^{-t/\tau}dt\right]$

$= i_0[t + \tau(e^{-t/\tau - 1})] = i_0[t + \tau e^{-t/\tau}\tau]$

Now, $i_0 = \frac{6}{10+2} = \frac{6}{12} = 0.5$ A

$\tau = \frac{L}{R} = \frac{0.120}{12} = 0.01$

a) t = 0.01 s

So, Q $= 0.5[0.01 + 0.01\,e^{-0.01/0.01} - 0.01]$

$= 0.00183 = 1.8 \times 10^{-3}$ C = 1.8 mC

b) t = 20 ms = 2 × 10^{-2} ′ = 0.02 s

So, Q = 0.5[0.02 + 0.01 $e^{-0.02/0.01}$ – 0.01]

= 0.005676 = 5.6 × 10^{-3} C = 5.6 mC

c) t = 100 ms = 0.1 s

So, Q = 0.5[0.1 + 0.01 $e^{-0.1/0.01}$ – 0.01]

= 0.045 C = 45 mC

80. L = 17 mH, ℓ = 100 m, A = 1 mm^2 = 1 × 10^{-6} m^2, f_{cu} = 1.7 × 10^{-8} Ω-m

$R = \frac{f_{cu}\ell}{A} = \frac{1.7\times10^{-8}\times100}{1\times10^{-6}}$ = 1.7 Ω

$i = \frac{L}{R} = \frac{0.17\times10^{-8}}{1.7}$ = 10^{-2} sec = 10 m sec.

81. τ = L/R = 50 ms = 0.05 ′

a) $\frac{i_0}{2} = i_0(1-e^{-t/0.06})$

$\Rightarrow \frac{1}{2} = 1-e^{-t/0.05} = e^{-t/0.05} = \frac{1}{2}$

$\Rightarrow$ ℓn $e^{-t/0.05}$ = $ℓn^{1/2}$

$\Rightarrow$ t = 0.05 × 0.693 = 0.3465 ′ = 34.6 ms = 35 ms.

b) $P = i^2R = \frac{E^2}{R}(1-E^{-t.R/L})^2$

Maximum power = $\frac{E^2}{R}$

So, $\frac{E^2}{2R} = \frac{E^2}{R}(1-e^{-tR/L})^2$

$\Rightarrow 1 - e^{-tR/L} = \frac{1}{\sqrt{2}}$ = 0.707

$\Rightarrow e^{-tR/L} = 0.293$

$\Rightarrow \frac{tR}{L} = -\ln 0.293 = 1.2275$

$\Rightarrow$ t = 50 × 1.2275 ms = 61.2 ms.

82. Maximum current = $\frac{E}{R}$

In steady state magnetic field energy stored = $\frac{1}{2}L\frac{E^2}{R^2}$

The fourth of steady state energy = $\frac{1}{8}L\frac{E^2}{R^2}$

One half of steady energy = $\frac{1}{4}L\frac{E^2}{R^2}$

$\frac{1}{8}L\frac{E^2}{R^2} = \frac{1}{2}L\frac{E^2}{R^2}(1-e^{-t_1R/L})^2$

$\Rightarrow 1 - e^{t_1R/L} = \frac{1}{2}$

$\Rightarrow e^{t_1R/L} = \frac{1}{2} \Rightarrow t_1 \frac{R}{L}$ = ℓn 2 ⇒ t_1 = τℓn2

Again $\frac{1}{4}L\frac{E^2}{R^2} = \frac{1}{2}L\frac{E^2}{R^2}(1-e^{-t_2R/L})^2$

$\Rightarrow\ e^{t_2R/L} = \frac{\sqrt{2}-1}{\sqrt{2}} = \frac{2-\sqrt{2}}{2}$

$\Rightarrow\ t_2 = \tau\left[\ell n\left(\frac{1}{2-\sqrt{2}}\right) + \ell n2\right]$

So, $t_2 - t_1 = \tau\ell n\frac{1}{2-\sqrt{2}}$

83. L = 4.0 H, R = 10 Ω, E = 4 V

a) Time constant = $\tau = \frac{L}{R} = \frac{4}{10}$ = 0.4 s.

b) i = 0.63 i_0

Now, 0.63 i_0 = i_0 $(1 - e^{-t/\tau})$

$\Rightarrow e^{-t/\tau} = 1 - 0.63 = 0.37$

$\Rightarrow\ \ell n e^{-t/\tau}$ = In 0.37

$\Rightarrow -t/\tau = -0.9942$

$\Rightarrow$ t = 0.9942 $\times$ 0.4 = 0.3977 = 0.40 s.

c) $i = i_0\ (1 - e^{-t/\tau})$

$\Rightarrow\ \frac{4}{10}(1 - e^{-0.4/0.4})$ = 0.4 $\times$ 0.6321 = 0.2528 A.

Power delivered = VI

= 4 $\times$ 0.2528 = 1.01 = 1 ω.

d) Power dissipated in Joule heating $= I^2R$

$= (0.2528)^2 \times 10 = 0.639 = 0.64\ \omega$.

84. $i = i_0(1 - e^{-t/\tau})$

$\Rightarrow\ \mu_0 ni = \mu_0 n\ i_0(1 - e^{-t/\tau})$ $\Rightarrow$ $B = B_0\ (1 - e^{-IR/L})$

$\Rightarrow\ 0.8\ B_0 = B_0\ (1 - e^{-20\times10^{-5}\times R/2\times10^{-3}})$ $\Rightarrow$ $0.8 = (1 - e^{-R/100})$

$\Rightarrow\ e^{-R/100} = 0.2$ $\Rightarrow$ $\ell n(e^{-R/100}) = \ell n(0.2)$

$\Rightarrow\ -R/100 = -1.609$ $\Rightarrow$ R = 16.9 = 160 Ω.

85. Emf = E LR circuit

a) dq = idt

$= i_0\ (1 - e^{-t/\tau})dt$

$= i_0\ (1 - e^{-IR.L})dt$ [$\therefore\ \tau$ = L/R]

$$Q = \int_0^t dq = i_0\left[\int_0^t dt - \int_0^t e^{-tR/L}dt\right]$$

$= i_0\ [t - (-L/R)\ (e^{-IR/L})\ t_0]$

$= i_0\ [t - L/R\ (1 - e^{-IR/L})]$

$Q = E/R\ [t - L/R\ (1 - e^{-IR/L})]$

b) Similarly as we know work done = VI = EI

$= E\ i_0\ [t - L/R\ (1 - e^{-IR/L})]$

$= \frac{E^2}{R}\ [t - L/R\ (1 - e^{-IR/L})]$

c) $H = \int_0^t i^2R\cdot dt = \frac{E^2}{R^2}\cdot R\cdot\int_0^t (1 - e^{-tR/L})^2\cdot dt$

$= \frac{E^2}{R}\int_0^t (1 + e^{(-2+B)/L} - 2e^{-tR/L})\cdot dt$

$= \frac{E^2}{R}\left(t - \frac{L}{2R}e^{-2tR/L} + \frac{L}{R}2 \cdot e^{-tR/L}\right)_0^t$

$= \frac{E^2}{R}\left(t - \frac{L}{2R}e^{-2tR/L} + \frac{2L}{R} \cdot e^{-tR/L}\right) - \left(-\frac{L}{2R} + \frac{2L}{R}\right)$

$= \frac{E^2}{R}\left[\left(t - \frac{L}{2R}x^2 + \frac{2L}{R} \cdot x\right) - \frac{3}{2}\frac{L}{R}\right]$

$= \frac{E^2}{2}\left(t - \frac{L}{2R}(x^2 - 4x + 3)\right)$

d) $E = \frac{1}{2}Li^2$

$= \frac{1}{2}L \cdot \frac{E^2}{R^2} \cdot (1 - e^{-tR/L})^2$ [$x = e^{-tR/L}$]

$= \frac{LE^2}{2R^2}(1 - x)^2$

e) Total energy used as heat as stored in magnetic field

$= \frac{E^2}{R}T - \frac{E^2}{R} \cdot \frac{L}{2R}x^2 + \frac{E^2}{R}\frac{L}{r} \cdot 4x^2 - \frac{3L}{2R} \cdot \frac{E^2}{R} + \frac{LE^2}{2R^2} + \frac{LE^2}{2R^2}x^2 - \frac{LE^2}{R^2}x$

$= \frac{E^2}{R}t + \frac{E^2L}{R^2}x - \frac{LE^2}{R^2}$

$= \frac{E^2}{R}\left(t - \frac{L}{R}(1 - x)\right)$

= Energy drawn from battery.

(Hence conservation of energy holds good).

86. $L = 2H$, $R = 200\ \Omega$, $E = 2$ V, $t = 10$ ms

a) $\ell = \ell_0 (1 - e^{-t/\tau})$

$= \frac{2}{200}\left(1 - e^{-10 \times 10^{-3} \times 200/2}\right)$

$= 0.01 (1 - e^{-1}) = 0.01 (1 - 0.3678)$

$= 0.01 \times 0.632 = 6.3$ A.

b) Power delivered by the battery

= VI

$= EI_0 (1 - e^{-t/\tau}) = \frac{E^2}{R}(1 - e^{-t/\tau})$

$= \frac{2 \times 2}{200}(1 - e^{-10 \times 10^{-3} \times 200/2}) = 0.02 (1 - e^{-1}) = 0.1264 = 12$ mw.

c) Power dissepited in heating the resistor = I^2R

$= [i_0(1 - e^{-t/\tau})]^2R$

$= (6.3 \text{ mA})^2 \times 200 = 6.3 \times 6.3 \times 200 \times 10^{-6}$

$= 79.38 \times 10^{-4} = 7.938 \times 10^{-3} = 8$ mA.

d) Rate at which energy is stored in the magnetic field

d/dt (1/2 LI^2]

$= \frac{LI_0^2}{\tau}(e^{-t/\tau} - e^{-2t/\tau}) = \frac{2 \times 10^{-4}}{10^{-2}}(e^{-1} - e^{-2})$

$= 2 \times 10^{-2} (0.2325) = 0.465 \times 10^{-2}$

$= 4.6 \times 10^{-3} = 4.6$ mW.

87. $L_A = 1.0$ H ; $L_B = 2.0$ H ; R = 10 Ω

a) t = 0.1 s, $\tau_A = 0.1$, $\tau_B = L/R = 0.2$

$i_A = i_0(1 - e^{-t/\tau})$

$= \frac{2}{10}\left(1 - e^{\frac{-0.1\times10}{1}}\right) = 0.2\,(1 - e^{-1}) = 0.126424111$

$i_B = i_0(1 - e^{-t/\tau})$

$= \frac{2}{10}\left(1 - e^{\frac{-0.1\times10}{2}}\right) = 0.2\,(1 - e^{-1/2}) = 0.078693$

$\frac{i_A}{i_B} = \frac{0.12642411}{0.78693} = 1.6$

b) t = 200 ms = 0.2 s

$i_A = i_0(1 - e^{-t/\tau})$

$= 0.2(1 - e^{-0.2\times10/1}) = 0.2 \times 0.864664716 = 0.172932943$

$i_B = 0.2(1 - e^{-0.2\times10/2}) = 0.2 \times 0.632120 = 0.126424111$

$\frac{i_A}{i_B} = \frac{0.172932943}{0.126424111} = 1.36 = 1.4$

c) t = 1 s

$i_A = 0.2(1 - e^{-1\times10/1}) = 0.2 \times 0.9999546 = 0.19999092$

$i_B = 0.2(1 - e^{-1\times10/2}) = 0.2 \times 0.99326 = 0.19865241$

$\frac{i_A}{i_B} = \frac{0.19999092}{0.19865241} = 1.0$

88. a) For discharging circuit

$i = i_0\, e^{-t/\tau}$

$\Rightarrow 1 = 2\, e^{-0.1/\tau}$

$\Rightarrow (1/2) = e^{-0.1/\tau}$

$\Rightarrow \ell n\,(1/2) = \ell n\,(e^{-0.1/\tau})$

$\Rightarrow -0.693 = -0.1/\tau$

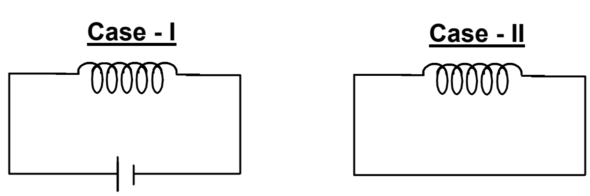
$\Rightarrow \tau = 0.1/0.693 = 0.144 = 0.14.$

b) L = 4 H, i = L/R

$\Rightarrow 0.14 = 4/R$

$\Rightarrow R = 4 / 0.14 = 28.57 = 28\ \Omega.$

89.



In this case there is no resistor in the circuit.

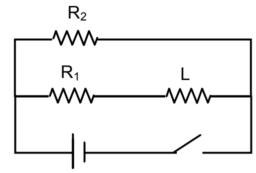
So, the energy stored due to the inductor before and after removal of battery remains same. i.e.

$V_1 = V_2 = \frac{1}{2}Li^2$

So, the current will also remain same.

Thus charge flowing through the conductor is the same.

90. a) The inductor does not work in DC. When the switch is closed the current charges so at first inductor works. But after a long time the current flowing is constant.



Thus effect of inductance vanishes.

$$i = \frac{E}{R_{net}} = \frac{E}{\frac{R_1R_2}{R_1+R_2}} = \frac{E(R_1+R_2)}{R_1R_2}$$

b) When the switch is opened the resistors are in series.

$$\tau = \frac{L}{R_{net}} = \frac{L}{R_1+R_2}.$$

91. i = 1.0 A, r = 2 cm, n = 1000 turn/m

Magnetic energy stored = $\frac{B^2V}{2\mu_0}$

Where B $\rightarrow$ Magnetic field, V $\rightarrow$ Volume of Solenoid.

$= \frac{\mu_0 n^2 i^2}{2\mu_0} \times \pi r^2 h$

$= \frac{4\pi\times10^{-7}\times10^{6}\times1\times\pi\times4\times10^{-4}\times1}{2}$ [h = 1 m]

$= 8\pi^2 \times 10^{-5}$

$= 78.956 \times 10^{-5} = 7.9 \times 10^{-4}$ J.

92. Energy density = $\frac{B^2}{2\mu_0}$

Total energy stored = $\frac{B^2V}{2\mu_0} = \frac{(\mu_0 i/2r)^2}{2\mu_0}V = \frac{\mu_0 i^2}{4r^2\times2}V$

$= \frac{4\pi\times10^{-7}\times4^2\times1\times10^{-9}}{4\times(10^{-1})^2\times2} = 8\pi \times 10^{-14}$ J.

93. I = 4.00 A, V = 1 mm^3,

d = 10 cm = 0.1 m

$\vec{B} = \frac{\mu_0 i}{2\pi r}$

Now magnetic energy stored = $\frac{B^2}{2\mu_0}V$

$= \frac{\mu_0^2 i^2}{4\pi r^2}\times\frac{1}{2\mu_0}\times V = \frac{4\pi\times10^{-7}\times16\times1\times1\times10^{-9}}{4\times1\times10^{-2}\times2}$

$= \frac{8}{\pi}\times10^{-14}$ J

$= 2.55 \times 10^{-14}$ J

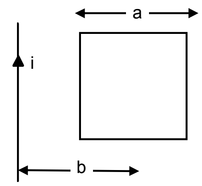
94. M = 2.5 H

$\frac{dI}{dt} = \frac{\ell A}{s}$

$E = -\mu\frac{dI}{dt}$

$\Rightarrow$ E = 2.5 $\times$ 1 = 2.5 V

95. We know



$$\frac{d\phi}{dt} = E = M \times \frac{di}{dt}$$

From the question,

$$\frac{di}{dt} = \frac{d}{dt}(i_0 \sin\omega t) = i_0\omega\cos\omega t$$

$$\frac{d\phi}{dt} = E = \frac{\mu_0 a i_0 \omega\cos\omega t}{2\pi}\ell n[1 + a/b]$$

Now, E = $M \times \frac{di}{dt}$

or, $\frac{\mu_0 a i_0 \omega\cos\omega t}{2\pi}\ell n[1 + a/b] = M \times i_0\omega\cos\omega t$

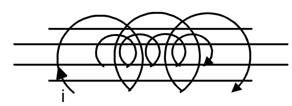
$\Rightarrow$ $M = \frac{\mu_0 a}{2\pi}\ell n[1 + a/b]$

96. emf induced = $\frac{\pi\mu_0 N a^2 a'^2 ERV}{2L(a^2 + x^2)^{3/2}(R/Lx + r)^2}$

$\frac{dI}{dt} = \frac{ERV}{L\left(\frac{Rx}{L} + r\right)^2}$ (from question 20)

μ = $\frac{E}{di/dt} = \frac{N\mu_0\pi a^2 a'^2}{2(a^2 + x^2)^{3/2}}$.

97. **Solenoid I :**



a_1 = 4 cm^2 ; n_1 = 4000/0.2 m ; ℓ_1 = 20 cm = 0.20 m

Solenoid II :

a_2 = 8 cm^2 ; n_2 = 2000/0.1 m ; ℓ_2 = 10 cm = 0.10 m

B = $\mu_0 n_2 i$ let the current through outer solenoid be i.

ϕ = n_1B.A = $n_1\, n_2\, \mu_0$ i × a_1

= $2000 \times \frac{2000}{0.1} \times 4\pi \times 10^{-7} \times i \times 4 \times 10^{-4}$

E = $\frac{d\phi}{dt} = 64\pi \times 10^{-4} \times \frac{di}{dt}$

Now M = $\frac{E}{di/dt}$ = $64\pi \times 10^{-4}$ H = 2×10^{-2} H. [As E = Mdi/dt]

98. a) B = Flux produced due to first coil

= μ_0 n i

Flux ϕ linked with the second

= μ_0 n i × NA = μ_0 n i N π R^2

Emf developed

= $\frac{dI}{dt} = \frac{dt}{dt}(\mu_0 n i N\pi R^2)$

= $\mu_0 n N\pi R^2\frac{di}{dt} = \mu_0 n N\pi R^2 i_0\omega\cos\omega t$.

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CHAPTER – 39
ALTERNATING CURRENT

1. $f = 50$ Hz

   $I = I_0 \sin Wt$

   Peak value $I = \frac{I_0}{\sqrt{2}}$

   $\frac{I_0}{\sqrt{2}} = I_0 \sin Wt$

   $\Rightarrow \frac{1}{\sqrt{2}} = \sin Wt = \sin \frac{\pi}{4}$

   $\Rightarrow \frac{\pi}{4} = Wt.$ or, $t = \frac{\pi}{400} = \frac{\pi}{4 \times 2\pi f} = \frac{1}{8f} = \frac{1}{8 \times 50} = 0.0025$ s = 2.5 ms

2. $E_{rms} = 220$ V

   Frequency = 50 Hz

   (a) $E_{rms} = \frac{E_0}{\sqrt{2}}$

   $\Rightarrow E_0 = E_{rms}\sqrt{2} = \sqrt{2} \times 220 = 1.414 \times 220 = 311.08$ V = 311 V

   (b) Time taken for the current to reach the peak value = Time taken to reach the 0 value from r.m.s

   $I = \frac{I_0}{\sqrt{2}} \Rightarrow \frac{I_0}{\sqrt{2}} = I_0 \sin \omega t$

   $\Rightarrow \omega t = \frac{\pi}{4}$

   $\Rightarrow t = \frac{\pi}{4\omega} = \frac{\pi}{4 \times 2\pi f} = \frac{\pi}{8\pi 50} = \frac{1}{400} = 2.5$ ms

3. P = 60 W V = 220 V = E

   $R = \frac{v^2}{P} = \frac{220 \times 220}{60} = 806.67$

   $\varepsilon_0 = \sqrt{2}\,E = 1.414 \times 220 = 311.08$

   $I_0 = \frac{\varepsilon_0}{R} = \frac{806.67}{311.08} = 0.385 \approx 0.39$ A

4. E = 12 volts

   $i^2 Rt = i^2_{rms} RT$

   $\Rightarrow \frac{E^2}{R^2} = \frac{E^2{}_{rms}}{R^2} \Rightarrow E^2 = \frac{{E_0}^2}{2}$

   $\Rightarrow {E_0}^2 = 2E^2 \Rightarrow {E_0}^2 = 2 \times 12^2 = 2 \times 144$

   $\Rightarrow E_0 = \sqrt{2 \times 144} = 16.97 \approx 17$ V

5. $P_0 = 80$ W (given)

   $P_{rms} = \frac{P_0}{2} = 40$ W

   Energy consumed = P × t = 40 × 100 = 4000 J = 4.0 KJ

6. $E = 3 \times 10^6$ V/m, $A = 20$ cm^2, d = 0.1 mm

   Potential diff. across the capacitor = $Ed = 3 \times 10^6 \times 0.1 \times 10^{-3} = 300$ V

   Max. rms Voltage = $\frac{V}{\sqrt{2}} = \frac{300}{\sqrt{2}} = 212$ V

7. $i = i_0 e^{-ur}$

$$\overline{i^2} = \frac{1}{\tau}\int_0^{\tau} i_0^{\,2} e^{-2t/\tau} dt = \frac{i_0^{\,2}}{\tau}\int_0^{\tau} e^{-2t/\tau} dt = \frac{i_0^{\,2}}{\tau} \times \left[\frac{\tau}{2} e^{-2t/\tau}\right]_0^{\tau} = -\frac{i_0^{\,2}}{\tau} \times \frac{\tau}{2} \times \left[e^{-2} - 1\right]$$

$$\sqrt{\overline{i^2}} = \sqrt{-\frac{i_0^{\,2}}{2}\left(\frac{1}{e^2} - 1\right)} = \frac{i_0}{e}\sqrt{\left(\frac{e^2 - 1}{2}\right)}$$

8. $C = 10\ \mu F = 10 \times 10^{-6}\ F = 10^{-5}\ F$

$E = (10\ V)\ \text{Sin}\ \omega t$

a) $I = \frac{E_0}{X_C} = \frac{E_0}{\left(\frac{1}{\omega C}\right)} = \frac{10}{\left(\frac{1}{10 \times 10^{-5}}\right)} = 1 \times 10^{-3}\ A$

b) $\omega = 100\ s^{-1}$

$$I = \frac{E_0}{\left(\frac{1}{\omega C}\right)} = \frac{10}{\left(\frac{1}{100 \times 10^{-5}}\right)} = 1 \times 10^{-2}\ A = 0.01\ A$$

c) $\omega = 500\ s^{-1}$

$$I = \frac{E_0}{\left(\frac{1}{\omega C}\right)} = \frac{10}{\left(\frac{1}{500 \times 10^{-5}}\right)} = 5 \times 10^{-2}\ A = 0.05\ A$$

d) $\omega = 1000\ s^{-1}$

$$I = \frac{E_0}{\left(\frac{1}{\omega C}\right)} = \frac{10}{\left(\frac{1}{1000 \times 10^{-5}}\right)} = 1 \times 10^{-1}\ A = 0.1\ A$$

9. Inductance = 5.0 mH = 0.005 H

a) $\omega = 100\ s^{-1}$

$$X_L = \omega L = 100 \times \frac{5}{1000} = 0.5\ \Omega$$

$$i = \frac{\varepsilon_0}{X_L} = \frac{10}{0.5} = 20\ A$$

b) $\omega = 500\ s^{-1}$

$$X_L = \omega L = 500 \times \frac{5}{1000} = 2.5\ \Omega$$

$$i = \frac{\varepsilon_0}{X_L} = \frac{10}{2.5} = 4\ A$$

c) $\omega = 1000\ s^{-1}$

$$X_L = \omega L = 1000 \times \frac{5}{1000} = 5\ \Omega$$

$$i = \frac{\varepsilon_0}{X_L} = \frac{10}{5} = 2\ A$$

10. $R = 10\ \Omega$, $L = 0.4$ Henry

$E = 6.5\ V$, $f = \frac{30}{\pi}$ Hz

$$Z = \sqrt{R^2 + X_L^{\,2}} = \sqrt{R^2 + (2\pi f L)^2}$$

Power = $V_{rms}\ I_{rms} \cos \phi$

$$= 6.5 \times \frac{6.5}{Z} \times \frac{R}{Z} = \frac{6.5 \times 6.5 \times 10}{\left[\sqrt{R^2 + (2\pi f L)^2}\right]^2} = \frac{6.5 \times 6.5 \times 10}{10^2 + \left(2\pi \times \frac{30}{\pi} \times 0.4\right)^2} = \frac{6.5 \times 6.5 \times 10}{100 + 576} = 0.625 = \frac{5}{8}\ \omega$$

11. $H = \frac{V^2}{R} T$, $E_0 = 12$ V, $\omega = 250\pi$, $R = 100\ \Omega$

$$H = \int_0^H dH = \int \frac{E_0{}^2 \text{Sin}^2 \omega t}{R} dt = \frac{144}{100} \int \sin^2 \omega t \; dt = 1.44 \int \left(\frac{1 - \cos 2\omega t}{2} \right) dt$$

$$= \frac{1.44}{2} \left[\int_0^{10^{-3}} dt - \int_0^{10^{-3}} \text{Cos} 2\omega t \; dt \right] = 0.72 \left[10^{-3} - \left(\frac{\text{Sin} 2\omega t}{2\omega} \right)_0^{10^{-3}} \right]$$

$$= 0.72 \left[\frac{1}{1000} - \frac{1}{500\pi} \right] = \frac{(\pi - 2)}{1000\pi} \times 0.72 = 0.0002614 = 2.61 \times 10^{-4} \text{ J}$$

12. $R = 300\Omega$, $C = 25\ \mu F = 25 \times 10^{-6}$ F, $\varepsilon_0 = 50$ V, $f = 50$ Hz

$$X_c = \frac{1}{\omega c} = \frac{1}{\frac{50}{\pi} \times 2\pi \times 25 \times 10^{-6}} = \frac{10^4}{25}$$

$$Z = \sqrt{R^2 + X_c{}^2} = \sqrt{(300)^2 + \left(\frac{10^4}{25} \right)^2} = \sqrt{(300)^2 + (400)^2} = 500$$

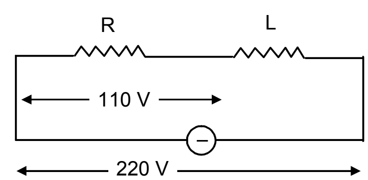
(a) Peak current = $\frac{E_0}{Z} = \frac{50}{500} = 0.1$ A

(b) Average Power dissipitated, = $E_{rms} I_{rms} \text{Cos}\, \phi$

$$= \frac{E_0}{\sqrt{2}} \times \frac{E_0}{\sqrt{2} Z} \times \frac{R}{Z} = \frac{E_0{}^2}{2Z^2} = \frac{50 \times 50 \times 300}{2 \times 500 \times 500} = \frac{3}{2} = 1.5\ \omega.$$

13. Power = 55 W, Voltage = 110 V, Resistance = $\frac{V^2}{P} = \frac{110 \times 110}{55} = 220\ \Omega$

frequency (f) = 50 Hz, $\omega = 2\pi f = 2\pi \times 50 = 100\pi$



Current in the circuit = $\frac{V}{Z} = \frac{V}{\sqrt{R^2 + (\omega L)^2}}$

Voltage drop across the resistor = ir = $\frac{VR}{\sqrt{R^2 + (\omega L)^2}}$

$$= \frac{220 \times 220}{\sqrt{(220)^2 + (100\pi L)^2}} = 110$$

$\Rightarrow 220 \times 2 = \sqrt{(220)^2 + (100\pi L)^2}$ $\Rightarrow (220)^2 + (100\pi L)^2 = (440)^2$

$\Rightarrow 48400 + 10^4 \pi^2 L^2 = 193600$ $\Rightarrow 10^4 \pi^2 L^2 = 193600 - 48400$

$\Rightarrow L^2 = \frac{142500}{\pi^2 \times 10^4} = 1.4726$ $\Rightarrow L = 1.2135 \approx 1.2$ Hz

14. $R = 300\ \Omega$, $C = 20\ \mu F = 20 \times 10^{-6}$ F

L = 1 Henry, E = 50 V $V = \frac{50}{\pi}$ Hz

(a) $I_0 = \frac{E_0}{Z}$,

$$Z = \sqrt{R^2 + (X_c - X_L)^2} = \sqrt{(300)^2 + \left(\frac{1}{2\pi f C} - 2\pi f L \right)^2}$$

$$= \sqrt{(300)^2 + \left(\frac{1}{2\pi \times \frac{50}{\pi} \times 20 \times 10^{-6}} - 2\pi \times \frac{50}{\pi} \times 1 \right)^2} = \sqrt{(300)^2 + \left(\frac{10^4}{20} - 100 \right)^2} = 500$$

$$I_0 = \frac{E_0}{Z} = \frac{50}{500} = 0.1 \text{ A}$$

(b) Potential across the capacitor = $i_0 \times X_c$ = 0.1 × 500 = 50 V
Potential difference across the resistor = $i_0 \times R$ = 0.1 × 300 = 30 V
Potential difference across the inductor = $i_0 \times X_L$ = 0.1 × 100 = 10 V
Rms. potential = 50 V
Net sum of all potential drops = 50 V + 30 V + 10 V = 90 V
Sum or potential drops > R.M.S potential applied.

15. R = 300 Ω
C = 20 μF = 20 × 10^{-6} F
L = 1H, Z = 500 (from 14)
$\varepsilon_0 = 50$ V, $I_0 = \frac{E_0}{Z} = \frac{50}{500} = 0.1$ A
Electric Energy stored in Capacitor = (1/2) CV^2 = (1/2) × 20 × 10^{-6} × 50 × 50 = 25 × 10^{-3} J = 25 mJ
Magnetic field energy stored in the coil = (1/2) L I_0^2 = (1/2) × 1 × $(0.1)^2$ = 5 × 10^{-3} J = 5 mJ

16. (a)For current to be maximum in a circuit
$X_l = X_c$ (Resonant Condition)

$$\Rightarrow WL = \frac{1}{WC}$$

$$\Rightarrow W^2 = \frac{1}{LC} = \frac{1}{2 \times 18 \times 10^{-6}} = \frac{10^6}{36}$$

$$\Rightarrow W = \frac{10^3}{6} \Rightarrow 2\pi f = \frac{10^3}{6}$$

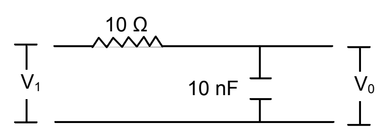
$$\Rightarrow f = \frac{1000}{6 \times 2\pi} = 26.537 \text{ Hz} \approx 27 \text{ Hz}$$

(b) Maximum Current = $\frac{E}{R}$ (in resonance and)

$$= \frac{20}{10 \times 10^3} = \frac{2}{10^3} \text{ A} = 2 \text{ mA}$$

17. E_{rms} = 24 V
r = 4 Ω, I_{rms} = 6 A
$R = \frac{E}{I} = \frac{24}{6} = 4\ \Omega$
Internal Resistance = 4 Ω
Hence net resistance = 4 + 4 = 8 Ω
$\therefore$ Current = $\frac{12}{8}$ = 1.5 A

18. $V_1 = 10 \times 10^{-3}$ V
R = 1 × 10^3 Ω
C = 10 × 10^{-9} F



(a) $X_c = \frac{1}{WC} = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 10 \times 10^3 \times 10 \times 10^{-9}} = \frac{1}{2\pi \times 10^{-4}} = \frac{10^4}{2\pi} = \frac{5000}{\pi}$

$$Z = \sqrt{R^2 + X_c^2} = \sqrt{(1 \times 10^3)^2 + \left(\frac{5000}{\pi}\right)^2} = \sqrt{10^6 + \left(\frac{5000}{\pi}\right)^2}$$

$$I_0 = \frac{E_0}{Z} = \frac{V_1}{Z} = \frac{10 \times 10^{-3}}{\sqrt{10^6 + \left(\frac{5000}{\pi}\right)^2}}$$

(b) $X_c = \frac{1}{WC} = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 10^5 \times 10 \times 10^{-9}} = \frac{1}{2\pi \times 10^{-3}} = \frac{10^3}{2\pi} = \frac{500}{\pi}$

$Z = \sqrt{R^2 + X_c^2} = \sqrt{(10^3)^2 + \left(\frac{500}{\pi}\right)^2} = \sqrt{10^6 + \left(\frac{500}{\pi}\right)^2}$

$I_0 = \frac{E_0}{Z} = \frac{V_1}{Z} = \frac{10 \times 10^{-3}}{\sqrt{10^6 + \left(\frac{500}{\pi}\right)^2}}$

$V_0 = I_0 X_c = \frac{10 \times 10^{-3}}{\sqrt{10^6 + \left(\frac{500}{\pi}\right)^2}} \times \frac{500}{\pi} = 1.6124 \text{ V} \approx 1.6 \text{ mV}$

(c) $f = 1 \text{ MHz} = 10^6 \text{ Hz}$

$X_c = \frac{1}{WC} = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 10^6 \times 10 \times 10^{-9}} = \frac{1}{2\pi \times 10^{-2}} = \frac{10^2}{2\pi} = \frac{50}{\pi}$

$Z = \sqrt{R^2 + X_c^2} = \sqrt{(10^3)^2 + \left(\frac{50}{\pi}\right)^2} = \sqrt{10^6 + \left(\frac{50}{\pi}\right)^2}$

$I_0 = \frac{E_0}{Z} = \frac{V_1}{Z} = \frac{10 \times 10^{-3}}{\sqrt{10^6 + \left(\frac{50}{\pi}\right)^2}}$

$V_0 = I_0 X_c = \frac{10 \times 10^{-3}}{\sqrt{10^6 + \left(\frac{50}{\pi}\right)^2}} \times \frac{50}{\pi} \approx 0.16 \text{ mV}$

(d) $f = 10 \text{ MHz} = 10^7 \text{ Hz}$

$X_c = \frac{1}{WC} = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 10^7 \times 10 \times 10^{-9}} = \frac{1}{2\pi \times 10^{-1}} = \frac{10}{2\pi} = \frac{5}{\pi}$

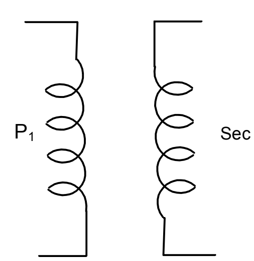
$Z = \sqrt{R^2 + X_c^2} = \sqrt{(10^3)^2 + \left(\frac{5}{\pi}\right)^2} = \sqrt{10^6 + \left(\frac{5}{\pi}\right)^2}$

$I_0 = \frac{E_0}{Z} = \frac{V_1}{Z} = \frac{10 \times 10^{-3}}{\sqrt{10^6 + \left(\frac{5}{\pi}\right)^2}}$

$V_0 = I_0 X_c = \frac{10 \times 10^{-3}}{\sqrt{10^6 + \left(\frac{5}{\pi}\right)^2}} \times \frac{5}{\pi} \approx 16 \ \mu\text{V}$

19. Transformer works upon the principle of induction which is only possible in case of AC.

Hence when DC is supplied to it, the primary coil blocks the Current supplied to it and hence induced current supplied to it and hence induced Current in the secondary coil is zero.



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ELECTROMAGNETIC WAVES
CHAPTER - 40

1. $\frac{\epsilon_0\, d\phi_E}{dt} = \frac{\epsilon_0\, EA}{dt\; 4\pi\, \varepsilon_0 r^2}$

$= \frac{M^{-1}L^{-3}T^4A^2}{M^{-1}L^{-3}A^2} \times \frac{A^1T^1}{L^2} \times \frac{L^2}{T} = A^1$

= (Current) (proved).

2. $E = \frac{Kq}{x^2}$, [from coulomb's law]

$\phi_E = EA = \frac{KqA}{x^2}$

$I_d = \epsilon_0 \frac{d\phi E}{dt} = \epsilon_0 \frac{d}{dt}\frac{kqA}{x^2} = \epsilon_0\, KqA = \frac{d}{dt} x^{-2}$

$= \epsilon_0 \times \frac{1}{4\pi\, \epsilon_0} \times q \times A \times -2 \times x^{-3} \times \frac{dx}{dt} = \frac{qAv}{2\pi x^3}$.

3. $E = \frac{Q}{\epsilon_0\, A}$ (Electric field)

$\phi = E.A. = \frac{Q}{\epsilon_0\, A}\frac{A}{2} = \frac{Q}{\epsilon_0\, 2}$

$i_0 = \epsilon_0 \frac{d\phi_E}{dt} = \epsilon_0 \frac{d}{dt}\left(\frac{Q}{\epsilon_0\, 2}\right) = \frac{1}{2}\left(\frac{dQ}{dt}\right)$

$= \frac{1}{2}\frac{d}{dt}(EC\, e^{-t/RC}) = \frac{1}{2}EC - \frac{1}{RC}e^{-t/RC} = \frac{-E}{2R}e^{\frac{-td}{RE_0\lambda}}$

4. $E = \frac{Q}{\epsilon_0\, A}$ (Electric field)

$\phi = E.A. = \frac{Q}{\epsilon_0\, A}\frac{A}{2} = \frac{Q}{\epsilon_0\, 2}$

$i_0 = \epsilon_0 \frac{d\phi_E}{dt} = \epsilon_0 \frac{d}{dt}\left(\frac{Q}{\epsilon_0\, 2}\right) = \frac{1}{2}\left(\frac{dQ}{dt}\right)$

5. $B = \mu_0 H$

$\Rightarrow H = \frac{B}{\mu_0}$

$\frac{E_0}{H_0} = \frac{B_0/(\mu_0\, \epsilon_0\, C)}{B_0/\mu_0} = \frac{1}{\epsilon_0\, C}$

$= \frac{1}{8.85 \times 10^{-12} \times 3 \times 10^8} = 376.6\ \Omega = 377\ \Omega.$

Dimension $\frac{1}{\epsilon_0\, C} = \frac{1}{[LT^{-1}][M^{-1}L^{-3}T^4A^2]} = \frac{1}{M^{-1}L^{-2}T^3A^2} = M^1L^2T^{-3}A^{-2} = [R].$

6. $E_0 = 810$ V/m, $B_0 = ?$

We know, $B_0 = \mu_0\, \epsilon_0\, C\, E_0$

Putting the values,

$B_0 = 4\pi \times 10^{-7} \times 8.85 \times 10^{-12} \times 3 \times 10^8 \times 810$

$= 27010.9 \times 10^{-10} = 2.7 \times 10^{-6}\ T = 2.7\ \mu T.$

7. B = (200 μT) Sin [(4 × 10^{15} 5^{-1}) (t – x/C)]

   a) B_0 = 200 μT

      $E_0 = C \times B_0 = 200 \times 10^{-6} \times 3 \times 10^8 = 6 \times 10^4$

   b) Average energy density = $\frac{1}{2\mu_0}B_0^2 = \frac{(200\times10^{-6})^2}{2\times4\pi\times10^{-7}} = \frac{4\times10^{-8}}{8\pi\times10^{-7}} = \frac{1}{20\pi}$ = 0.0159 = 0.016.

8. $I = 2.5 \times 10^{14}$ W/m^2

   We know, $I = \frac{1}{2}\epsilon_0 E_0^2 C$

   $\Rightarrow E_0^2 = \frac{2I}{\epsilon_0 C}$ or $E_0 = \sqrt{\frac{2I}{\epsilon_0 C}}$

   $E_0 = \sqrt{\frac{2\times2.5\times10^{14}}{8.85\times10^{-12}\times3\times10^8}} = 0.4339 \times 10^9 = 4.33 \times 10^8$ N/c.

   $B_0 = \mu_0 \epsilon_0 C E_0$

   $= 4 \times 3.14 \times 10^{-7} \times 8.854 \times 10^{-12} \times 3 \times 10^8 \times 4.33 \times 10^8 = 1.44$ T.

9. Intensity of wave = $\frac{1}{2}\epsilon_0 E_0^2 C$

   $\epsilon_0 = 8.85 \times 10^{-12}$; E_0 = ? ; $C = 3 \times 10^8$, $I = 1380$ W/m^2

   $1380 = 1/2 \times 8.85 \times 10^{-12} \times E_0^2 \times 3 \times 10^8$

   $\Rightarrow E_0^2 = \frac{2\times1380}{8.85\times3\times10^{-4}} = 103.95 \times 10^4$

   $\Rightarrow E_0 = 10.195 \times 10^2 = 1.02 \times 10^3$

   $E_0 = B_0 C$

   $\Rightarrow B_0 = E_0/C = \frac{1.02\times10^3}{3\times10^8} = 3.398 \times 10^{-5} = 3.4 \times 10^{-5}$ T.

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ELECTRIC CURRENT THROUGH GASES
CHAPTER 41

1. Let the two particles have charge 'q'

Mass of electron $m_a = 9.1 \times 10^{-31}$ kg

Mass of proton $m_p = 1.67 \times 10^{-27}$ kg

Electric field be E

Force experienced by Electron = qE

accln. = qE/m_e

For time dt

$$S_e = \frac{1}{2} \times \frac{qE}{m_e} \times dt^2 \quad \ldots(1)$$

For the positive ion,

$$\text{accln.} = \frac{qE}{4 \times m_p}$$

$$S_p = \frac{1}{2} \times \frac{qE}{4 \times m_p} \times dt^2 \quad \ldots(2)$$

$$\frac{S_e}{S_p} = \frac{4m_p}{m_e} = 7340.6$$

2. $E = 5$ Kv/m $= 5 \times 10^3$ v/m ; $t = 1\ \mu s = 1 \times 10^{-6}$ s

$F = qE = 1.6 \times 10^{-9} \times 5 \times 10^3$

$$a = \frac{qE}{m} = \frac{1.6 \times 5 \times 10^{-16}}{9.1 \times 10^{-31}}$$

a) S = distance travelled

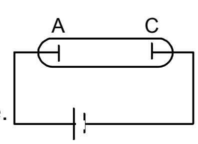
$$= \frac{1}{2}at^2 = 439.56 \text{ m} = 440 \text{ m}$$

b) $d = 1$ mm $= 1 \times 10^{-3}$ m

$$1 \times 10^{-3} = \frac{1}{2} \times \frac{1.6 \times 5}{9.1} 10^5 \times t^2$$

$$\Rightarrow t^2 = \frac{9.1}{0.8 \times 5} \times 10^{-18} \Rightarrow t = 1.508 \times 10^{-9} \text{ sec} \Rightarrow 1.5 \text{ ns.}$$

3. Let the mean free path be 'L' and pressure be 'P'



$L \propto 1/p$ for L = half of the tube length, P = 0.02 mm of Hg

As 'P' becomes half, 'L' doubles, that is the whole tube is filled with Crook's dark space.

Hence the required pressure = 0.02/2 = 0.01 m of Hg.

4. V = f(Pd)

$v_s = P_s d_s$

$v_L = P_l d_l$

$$\Rightarrow \frac{V_s}{V_l} = \frac{P_s}{P_l} \times \frac{d_s}{d_l} \Rightarrow \frac{100}{100} = \frac{10}{20} \times \frac{1\text{mm}}{x}$$

$\Rightarrow$ x = 1 mm / 2 = 0.5 mm

5. i = ne or n = i/e

'e' is same in all cases.

We know,

$i = AST^2\ e^{-\phi/RT}$ ϕ = 4.52 eV, K = 1.38 × 10^{-23} J/k

$n(1000) = As \times (1000)^2 \times\ e^{-4.52\times1.6\times10^{-19}/1.38\times10^{-23}\times1000}$

$\Rightarrow 1.7396 \times 10^{-17}$

a) T = 300 K

$$\frac{n(T)}{n(1000K)} = \frac{AS\times(300)^2\times e^{-4.52\times1.6\times10^{-19}/1.38\times10^{-23}\times300}}{AS\times1.7396\times10^{-17}} = 7.05\times10^{-55}$$

b) T = 2000 K

$$\frac{n(T)}{n(1000K)} = \frac{AS\times(2000)^2\times e^{-4.52\times1.6\times10^{-19}/1.38\times10^{-23}\times2000}}{AS\times1.7396\times10^{-17}} = 9.59\times10^{11}$$

c) T = 3000 K

$$\frac{n(T)}{n(1000K)} = \frac{AS\times(3000)^2\times e^{-4.52\times1.6\times10^{-19}/1.38\times10^{-23}\times3000}}{AS\times1.7396\times10^{-17}} = 1.340\times10^{16}$$

6. $i = AST^2\ e^{-\phi/KT}$

$i_1 = i$	i_2 = 100 mA
$A_1 = 60 \times 10^4$	$A_2 = 3 \times 10^4$
$S_1 = S$	$S_2 = S$
$T_1 = 2000$	$T_2 = 2000$
ϕ_1 = 4.5 eV	ϕ_2 = 2.6 eV

K = 1.38 × 10^{-23} J/k

$$i = (60\times10^4)\,(S)\times(2000)^2\ e^{\frac{-4.5\times1.6\times10^{-19}}{1.38\times10^{-23}\times2000}}$$

$$100 = (3\times10^4)\,(S)\times(2000)^2\ e^{\frac{-2.6\times1.6\times10^{-19}}{1.38\times10^{-23}\times2000}}$$

Dividing the equation

$$\frac{i}{100} = e^{\left[\frac{-4.5\times1.6\times10}{1.38\times2}\left(\frac{-2.6\times1.6\times10}{1.38\times20}\right)\right]}$$

$$\Rightarrow \frac{i}{100} = 20\times e^{-11.014} \Rightarrow \frac{i}{100} = 20\times0.000016$$

$\Rightarrow$ i = 20 × 0.0016 = 0.0329 mA = 33 μA

7. Pure tungsten — Thoriated tungsten

Pure tungsten	Thoriated tungsten
ϕ = 4.5 eV	ϕ = 2.6 eV
A = 60 × 10^4 A/m^2 – k^2	A = 3 × 10^4 A/m^2 – k^2

$i = AST^2 e^{-\phi/KT}$

$i_{Thoriated\ Tungsten} = 5000\ i_{Tungsten}$

$$\text{So, } 5000\times S\times60\times10^4\times T^2\times\ e^{\frac{-4.5\times1.6\times10^{-19}}{1.38\times T\times10^{-23}}}$$

$$\Rightarrow S\times3\times10^4\times T^2\times\ e^{\frac{-2.65\times1.6\times10^{-19}}{1.38\times T\times10^{-23}}}$$

$$\Rightarrow 3\times10^8\times\ e^{\frac{-4.5\times1.6\times10^{-19}}{1.38\times T\times10^{-23}}} = e^{\frac{-2.65\times1.6\times10^{-19}}{1.38\times T\times10^{-23}}}\times3\times10^4$$

Taking 'ln'

$\Rightarrow$ 9.21 T = 220.29

$\Rightarrow$ T = 22029 / 9.21 = 2391.856 K

8. $i = AST^2\ e^{-\phi/KT}$

$i' = AST^{12}\ e^{-\phi/KT'}$

$$\frac{i}{i'} = \frac{T^2}{T^{12}}\frac{e^{-\phi/KT}}{e^{-\phi/KT'}}$$

$$\Rightarrow \frac{i}{i'} = \left(\frac{T}{T'}\right)^2 e^{-\phi/KT+\phi KT'} = \left(\frac{T}{T'}\right)^2 e^{\phi KT'-\phi/KT}$$

$$= \frac{i}{i'} = \left(\frac{2000}{2010}\right)^2 e^{\frac{4.5\times1.6\times10^{-19}}{1.38\times10^{-23}}}\left(\frac{1}{2010}-\frac{1}{2000}\right) = 0.8690$$

$$\Rightarrow \frac{i}{i'} = \frac{1}{0.8699} = 1.1495\ = 1.14$$

9. $A = 60 \times 10^4\ A/m^2 - k^2$

$\phi = 4.5$ eV $\qquad \sigma = 6 \times 10^{-8}\ \omega/m^2 - k^4$

$S = 2 \times 10^{-5}\ m^2 \qquad K = 1.38 \times 10^{-23}$ J/K

$H = 24\ \omega'$

The Cathode acts as a black body, i.e. emissivity = 1

$\therefore E = \sigma A T^4$ (A is area)

$$\Rightarrow T^4 = \frac{E}{\sigma A} = \frac{24}{6\times10^{-8}\times2\times10^{-5}} = 2\times10^{13}K = 20\times10^{12}K$$

$\Rightarrow T = 2.1147 \times 10^3 = 2114.7$ K

Now, $i\ = AST^2 e^{-\phi/KT}$

$$= 6 \times 10^5 \times 2 \times 10^{-5} \times (2114.7)^2 \times\ e^{\frac{-4.5\times1.6\times10^{-19}}{1.38\times T\times10^{-23}}}$$

$= 1.03456 \times 10^{-3}$ A = 1 mA

10. $i_p = CV_p^{3/2}$ …(1)

$\Rightarrow di_p = C\ 3/2\ V_p^{(3/2)-1}dv_p$

$$\Rightarrow \frac{di_p}{dv_p} = \frac{3}{2}CV_p^{1/2} \quad …(2)$$

Dividing (2) and (1)

$$\frac{i}{i_p}\frac{di_p}{dv_p} = \frac{3/2CV_p^{1/2}}{CVp^{3/2}}$$

$$\Rightarrow \frac{1}{i_p}\frac{di_p}{dv_p} = \frac{3}{2V}$$

$$\Rightarrow \frac{dv_p}{di_p} = \frac{2V}{3i_p}$$

$$\Rightarrow R = \frac{2V}{3i_p} = \frac{2\times60}{3\times10\times10^{-3}} = 4\times10^3 = 4k\Omega$$

11. For plate current 20 mA, we find the voltage 50 V or 60 V.

Hence it acts as the saturation current. Therefore for the same temperature, the plate current is 20 mA for all other values of voltage.

Hence the required answer is 20 mA.

12. P = 1 W, p = ?

$V_p = 36$ V, $V_p = 49$ V, $P = I_pV_p$

$\Rightarrow I_p = \frac{P}{V_p} = \frac{1}{36}$

$I_p \propto (V_p)^{3/2}$

$I'_p \propto (V'_p)^{3/2}$

$\Rightarrow \frac{I_p}{I'_p} = \frac{(V_p)^{3/2}}{V'_p}$

$\Rightarrow \frac{1/36}{I'_p} = \left(\frac{36}{49}\right)^{3/2}$

$\Rightarrow \frac{1}{36\, I'_p} = \frac{36}{49} \times \frac{6}{7} \Rightarrow I'_p = 0.4411$

$P' = V'_p\, I'_p = 49 \times 0.4411 = 2.1613\ W = 2.2\ W$

13. Amplification factor for triode value

$= \mu = \frac{\text{Charge in Plate Voltage}}{\text{Charge in Grid Voltage}} = \frac{\delta V_p}{\delta V_g}$

$= \frac{250 - 225}{2.5 - 0.5} = \frac{25}{2} = 12.5$ $[\therefore\ \delta Vp = 250 - 225,\ \delta Vg = 2.5 - 0.5]$

14. $r_p = 2\ K\Omega = 2 \times 10^3\ \Omega$

$g_m = 2$ milli mho $= 2 \times 10^{-3}$ mho

$\mu = r_p \times g_m = 2 \times 10^3 \times 2 \times 10^{-3} = 4$ Amplification factor is 4.

15. Dynamic Plate Resistance $r_p = 10\ K\Omega = 10^4\ \Omega$

$\delta I_p = ?$

$\delta V_p = 220 - 220 = 20\ V$

$\delta I_p = (\delta V_p / r_p) / V_g$ = constant.

$= 20/10^4 = 0.002\ A = 2\ mA$

16. $r_p = \left(\frac{\delta V_p}{\delta I_p}\right)$ at constant V_g

Consider the two points on $V_g = -6$ line

$r_p = \frac{(240 - 160)V}{(13 - 3) \times 10^{-3} A} = \frac{80}{10} \times 10^3 \Omega = 8K\Omega$

$g_m = \left(\frac{\delta I_p}{\delta V_g}\right) v_p$ = constant

Considering the points on 200 V line,

$g_m = \frac{(13 - 3) \times 10^{-3}}{[(-4) + (-8)]} A = \frac{10 \times 10^{-3}}{4} = 2.5$ milli mho

$\mu = r_p \times gm$

$= 8 \times 10^3\ \Omega \times 2.5 \times 10^{-3}\ \Omega^{-1} = 8 \times 1.5 = 20$

17. a) $r_p = 8\ K\Omega = 8000\ \Omega$

$\delta V_p = 48\ V$ $\delta I_p = ?$

$\delta I_p = (\delta V_p / r_p) / V_g$ = constant.

So, $\delta I_p = 48 / 8000 = 0.006\ A = 6\ mA$

b) Now, V_p is constant.

$\delta I_p = 6\ mA = 0.006\ A$

$g_m = 0.0025$ mho

$\delta V_g = (\delta I_p / g_m) / V_p$ = constant.

$= \dfrac{0.006}{0.0025} = 2.4$ V

18. $r_p = 10\ K\Omega = 10 \times 10^3\ \Omega$

$\mu = 20$ $\quad$ $V_p = 250$ V

$V_g = -7.5$ V $\quad$ $I_p = 10$ mA

a) $g_m = \left(\dfrac{\delta I_p}{\delta V_g}\right) V_p$ = constant

$\Rightarrow \delta V_g = \dfrac{\delta I_p}{g_m} = \dfrac{15 \times 10^{-3} - 10 \times 10^{-3}}{\mu / r_p}$

$= \dfrac{5 \times 10^{-3}}{20/10 \times 10^3} = \dfrac{5}{2} = 2.5$

$r'_g = +2.5 - 7.5 = -5$ V

b) $r_p = \left(\dfrac{\delta V_p}{\delta I_p}\right) V_g$ = constnant

$\Rightarrow 10^4 = \dfrac{\delta V_p}{(15 \times 10^{-3} - 10 \times 10^{-3})}$

$\Rightarrow \delta V_p = 10^4 \times 5 \times 10^{-3} = 50$ V

$V'_p - V_p = 50 \Rightarrow V'_p = -50 + V_p = 200$ V

19. $V_p = 250$ V, $V_g = -20$ V

a) $i_p = 41(V_p + 7V_g)^{1.41}$

$\Rightarrow 41(250 - 140)^{1.41} = 41 \times (110)^{1.41} = 30984\ \mu A = 30$ mA

b) $i_p = 41(V_p + 7V_g)^{1.41}$

Differentiating,

$di_p = 41 \times 1.41 \times (V_p + 7V_g)^{0.41} \times (dV_p + 7dV_g)$

Now $r_p = \dfrac{dV_p}{di_p} V_g$ = constant.

or $\dfrac{dV_p}{di_p} = \dfrac{1 \times 10^6}{41 \times 1.41 \times 110^{0.41}} = 10^6 \times 2.51 \times 10^{-3} \Rightarrow 2.5 \times 10^3\ \Omega = 2.5\ K\Omega$

c) From above,

$dI_p = 41 \times 1.41 \times 6.87 \times 7\ d\ V_g$

$g_m = \dfrac{dI_p}{dV_g} = 41 \times 1.41 \times 6.87 \times 7\ \mu$ mho

$= 2780\ \mu$ mho = 2.78 milli mho.

d) Amplification factor

$\mu = r_p \times g_m = 2.5 \times 10^3 \times 2.78 \times 10^{-3} = 6.95 = 7$

20. $i_p = K(V_g + V_p/\mu)^{3/2}$ …(1)

Diff. the equation :

$di_p = K\ 3/2\ (V_g + V_p/\mu)^{1/2}\ d\ V_g$

$\Rightarrow \dfrac{di_p}{dV_g} = \dfrac{3}{2} K \left(V_g + \dfrac{V_0}{\mu}\right)^{1/2}$

$\Rightarrow$ $g_m = 3/2\ K\ (V_g + V_p/\mu)^{1/2}$ …(2)

From (1) $i_p = [3/2\ K\ (V_g + V_p/\mu)^{1/2}]^3 \times 8/K^2\ 27$

$\Rightarrow i_p = k'\ (g_m)^3 \Rightarrow g_m \propto 3\sqrt{i_p}$

21. $r_p = 20\ K\Omega$ = Plate Resistance

Mutual conductance = g_m = 2.0 milli mho = 2×10^{-3} mho

Amplification factor $\mu = 30$

Load Resistance = R_L = ?

We know

$A = \dfrac{\mu}{1+\dfrac{r_p}{R_L}}$ where A = voltage amplification factor

$\Rightarrow A = \dfrac{r_p \times g_m}{1+\dfrac{r_p}{R_L}}$ where $\mu = r_p \times g_m$

$\Rightarrow 30 = \dfrac{20\times10^3 \times 2\times10^{-3}}{1+\dfrac{20000}{R_L}} \Rightarrow 3 = \dfrac{4R_L}{R_L + 20000}$

$\Rightarrow$ $3R_L + 60000 = 4\ R_L$

$\Rightarrow$ $R_L = 60000\ \Omega = 60\ K\Omega$

22. Voltage gain = $\dfrac{\mu}{1+\dfrac{r_p}{R_L}}$

When A = 10, $R_L = 4\ K\Omega$

$10 = \dfrac{\mu}{1+\dfrac{r_p}{4\times10^3}} \Rightarrow 10 = \dfrac{\mu\times4\times10^3}{4\times10^3 + r_p}$

$\Rightarrow$ $40 \times 10^3 \times 10r_p = 4 \times 10^3\ \mu$ …(1)

when A = 12, $R_L = 8\ K\Omega$

$12 = \dfrac{\mu}{1+\dfrac{r_p}{8\times10^3}} \Rightarrow 12 = \dfrac{\mu\times8\times10^3}{8\times10^3 + r_p}$

$\Rightarrow$ $96 \times 10^3 + 12\ r_p = 8 \times 10^3\ \mu$ …(2)

Multiplying (2) in equation (1) and equating with equation (2)

$2(40 \times 10^3 + 10\ r_p) = 96 \times 10+3 + 12r_p$

$\Rightarrow$ $r_p = 2 \times 10^3\ \Omega = 2\ K\Omega$

Putting the value in equation (1)

$40 \times 10^3 + 10(2 \times 10^3) = 4 \times 10^3\ \mu$

$\Rightarrow$ $40 \times 10^3 + 20 \times 10^3) = 4 \times 10^3\ \mu$

$\Rightarrow$ $\mu = 60/4 = 15$

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PHOTO ELECTRIC EFFECT AND WAVE PARTICLE QUALITY
CHAPTER 42

1. λ_1 = 400 nm to λ_2 = 780 nm

$E = h\nu = \frac{hc}{\lambda}$ h = 6.63×10^{-34} j - s, c = 3×10^8 m/s, λ_1 = 400 nm, λ_2 = 780 nm

$E_1 = \frac{6.63\times10^{-34}\times3\times10^8}{400\times10^{-9}} = \frac{6.63\times3}{4}\times10^{-19} = 5 \times 10^{-19}$ J

$E_2 = \frac{6.63\times3}{7.8}\times10^{-19} = 2.55 \times 10^{-19}$ J

So, the range is 5×10^{-19} J to 2.55×10^{-19} J.

2. λ = h/p

$\Rightarrow$ P = h/λ = $\frac{6.63\times10^{-34}}{500\times10^{-9}}$ J-S = 1.326×10^{-27} = 1.33×10^{-27} kg – m/s.

3. λ_1 = 500 nm = 500×10^{-9}m, λ_2 = 700 nm = 700×10^{-9} m

$E_1 - E_2$ = Energy absorbed by the atom in the process. = hc $[1/\lambda_1 - 1/\lambda_2]$

$\Rightarrow 6.63 \times 3[1/5 - 1/7] \times 10^{-19} = 1.136 \times 10^{-19}$ J

4. P = 10 W $\therefore$ E in 1 sec = 10 J % used to convert into photon = 60%

$\therefore$Energy used = 6 J

Energy used to take out 1 photon = hc/λ = $\frac{6.63\times10^{-34}\times3\times10^8}{590\times10^{-9}} = \frac{6.633}{590}\times10^{-17}$

No. of photons used = $\frac{6}{\frac{6.63\times3}{590}\times10^{-17}} = \frac{6\times590}{6.63\times3}\times10^{17} = 176.9\times10^{17}$ = 1.77×10^{19}

5. a) Here intensity = I = 1.4×10^3 ω/m^2 Intensity, I = $\frac{\text{power}}{\text{area}}$ = 1.4×10^3 ω/m^2

Let no.of photons/sec emitted = n $\therefore$ Power = Energy emitted/sec = nhc/λ = P

No.of photons/m^2 = nhc/λ = intensity

$n = \frac{\text{int ensity}\times\lambda}{hc} = \frac{1.9\times10^3\times5\times10^{-9}}{6.63\times10^{-34}\times3\times10^8} = 3.5\times10^{21}$

b) Consider no.of two parts at a distance r and r + dr from the source.

The time interval 'dt' in which the photon travel from one point to another = dv/e = dt.

In this time the total no.of photons emitted = N = n dt = $\left(\frac{p\lambda}{hc}\right)\frac{dr}{C}$

These points will be present between two spherical shells of radii 'r' and r+dr. It is the distance of the 1st point from the sources. No.of photons per volume in the shell

(r + r + dr) = $\frac{N}{2\pi r2dr} = \frac{P\lambda dr}{hc^2} = \frac{1}{4\pi r^2 ch} = \frac{p\lambda}{4\pi hc^2 r^2}$

In the case = 1.5×10^{11} m, λ = 500 nm, = 500×10^{-9} m

$\frac{P}{4\pi r^2} = 1.4\times10^3$, $\therefore$ No.of photons/m^3 = $\frac{P}{4\pi r^2}\frac{\lambda}{hc^2}$

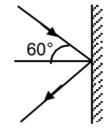
= $1.4 \times 10^3 \times \frac{500\times10^{-9}}{6.63\times10^{-34}\times3\times10^8} = 1.2\times10^{13}$

c) No.of photons = (No.of photons/sec/m^2) $\times$ Area

= $(3.5 \times 10^{21}) \times 4\pi r^2$

= $3.5 \times 10^{21} \times 4(3.14)(1.5 \times 10^{11})^2 = 9.9 \times 10^{44}$.

6. $\lambda = 663 \times 10^{-9}$ m, $\theta = 60°$, $n = 1 \times 10^{19}$, $\lambda = h/p$



$\Rightarrow P = p/\lambda = 10^{-27}$

Force exerted on the wall = $n(mv \cos\theta - (-mv \cos\theta)) = 2n\, mv \cos\theta$.

$= 2 \times 1 \times 10^{19} \times 10^{-27} \times ½ = 1 \times 10^{-8}$ N.

7. Power = 10 W P → Momentum

$\lambda = \dfrac{h}{p}$ or, $P = \dfrac{h}{\lambda}$ or, $\dfrac{P}{t} = \dfrac{h}{\lambda t}$

$E = \dfrac{hc}{\lambda}$ or, $\dfrac{E}{t} = \dfrac{hc}{\lambda t}$ = Power (W)

W = Pc/t or, P/t = W/c = force.

or Force = 7/10 (absorbed) + 2 × 3/10 (reflected)

$= \dfrac{7}{10} \times \dfrac{W}{C} + 2 \times \dfrac{3}{10} \times \dfrac{W}{C} \Rightarrow \dfrac{7}{10} \times \dfrac{10}{3 \times 10^8} + 2 \times \dfrac{3}{10} \times \dfrac{10}{3 \times 10^8}$

$= 13/3 \times 10^{-8} = 4.33 \times 10^{-8}$ N.

8. m = 20 g

The weight of the mirror is balanced. Thus force exerted by the photons is equal to weight

$P = \dfrac{h}{\lambda}$ $E = \dfrac{hc}{\lambda} = PC$

$\Rightarrow \dfrac{E}{t} = \dfrac{P}{t} C$

⇒ Rate of change of momentum = Power/C

30% of light passes through the lens.

Thus it exerts force. 70% is reflected.

∴ Force exerted = 2(rate of change of momentum)

= 2 × Power/C

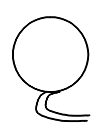
$30\% \left(\dfrac{2 \times \text{Power}}{C}\right) = mg$

$\Rightarrow$ Power = $\dfrac{20 \times 10^{-3} \times 10 \times 3 \times 10^8 \times 10}{2 \times 3}$ = 10 w = 100 MW.

9. Power = 100 W

Radius = 20 cm

60% is converted to light = 60 w



Now, Force = $\dfrac{\text{power}}{\text{velocity}} = \dfrac{60}{3 \times 10^8} = 2 \times 10^{-7}$ N.

Pressure = $\dfrac{\text{force}}{\text{area}} = \dfrac{2 \times 10^{-7}}{4 \times 3.14 \times (0.2)^2} = \dfrac{1}{8 \times 3.14} \times 10^{-5}$

$= 0.039 \times 10^{-5} = 3.9 \times 10^{-7} = 4 \times 10^{-7}$ N/m^2.

10. We know,

If a perfectly reflecting solid sphere of radius ‘r’ is kept in the path of a parallel beam of light of large aperture if intensity is I,

Force = $\dfrac{\pi r^2 I}{C}$

I = 0.5 W/m^2, r = 1 cm, C = 3×10^8 m/s

Force = $\dfrac{\pi \times (1)^2 \times 0.5}{3 \times 10^8} = \dfrac{3.14 \times 0.5}{3 \times 10^8}$

$= 0.523 \times 10^{-8} = 5.2 \times 10^{-9}$ N.

11. For a perfectly reflecting solid sphere of radius 'r' kept in the path of a parallel beam of light of large aperture with intensity 'I', force exerted = $\frac{\pi r^2 I}{C}$

12. If the i undergoes an elastic collision with a photon. Then applying energy conservation to this collision.
We get, $hC/\lambda + m_0c^2 = mc^2$
and applying conservation of momentum $h/\lambda = mv$
Mass of e = m = $\frac{m_0}{\sqrt{1-v^2/c^2}}$
from above equation it can be easily shown that
V = C or V = 0
both of these results have no physical meaning hence it is not possible for a photon to be completely absorbed by a free electron.

13. r = 1 m
Energy = $\frac{kq^2}{R} = \frac{kq^2}{1}$
Now, $\frac{kq^2}{1} = \frac{hc}{\lambda}$ or $\lambda = \frac{hc}{kq^2}$
For max 'λ', 'q' should be min,
For minimum 'e' = 1.6×10^{-19} C
Max $\lambda = \frac{hc}{kq^2} = 0.863 \times 10^3 = 863$ m.
For next smaller wavelength = $\frac{6.63 \times 3 \times 10^{-34} \times 10^8}{9 \times 10^9 \times (1.6 \times 2)^2 \times 10^{-38}} = \frac{863}{4}$ = 215.74 m

14. λ = 350 nn = 350×10^{-9} m
$\phi = 1.9$ eV
Max KE of electrons = $\frac{hC}{\lambda} - \phi = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{350 \times 10^{-9} \times 1.6 \times 10^{-19}} - 1.9$
= 1.65 ev = 1.6 ev.

15. $W_0 = 2.5 \times 10^{-19}$ J
a) We know $W_0 = h\nu_0$
$\nu_0 = \frac{W_0}{h} = \frac{2.5 \times 10^{-19}}{6.63 \times 10^{-34}} = 3.77 \times 10^{14}$ Hz = 3.8×10^{14} Hz
b) $eV_0 = h\nu - W_0$
or, $V_0 = \frac{h\nu - W_0}{e} = \frac{6.63 \times 10^{-34} \times 6 \times 10^{14} - 2.5 \times 10^{-19}}{1.6 \times 10^{-19}}$ = 0.91 V

16. $\phi = 4$ eV = $4 \times 1.6 \times 10^{-19}$ J
a) Threshold wavelength = λ
$\phi = hc/\lambda$
$\Rightarrow \lambda = \frac{hC}{\phi} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{4 \times 1.6 \times 10^{-19}} = \frac{6.63 \times 3}{6.4} \times \frac{10^{-27}}{10^{-9}} = 3.1 \times 10^{-7}\text{m}$ = 310 nm.
b) Stopping potential is 2.5 V
$E = \phi + eV$
$\Rightarrow hc/\lambda = 4 \times 1.6 \times 10^{-19} + 1.6 \times 10^{-19} \times 2.5$
$\Rightarrow \lambda = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{\lambda \times 1.6 \times 10^{-19}} = 4 + 2.5$
$\Rightarrow \frac{6.63 \times 3 \times 10^{-26}}{1.6 \times 10^{-19} \times 6.5} = 1.9125 \times 10^{-7}$ = 190 nm.

17. Energy of photoelectron

$\Rightarrow$ ½ mv^2 = $\frac{hc}{\lambda} - hv_0$ = $\frac{4.14 \times 10^{-15} \times 3 \times 10^8}{4 \times 10^{-7}} - 2.5ev$ = 0.605 ev.

We know KE = $\frac{P^2}{2m}$ $\Rightarrow$ P^2 = 2m × KE.

$P^2 = 2 \times 9.1 \times 10^{-31} \times 0.605 \times 1.6 \times 10^{-19}$

$P = 4.197 \times 10^{-25}$ kg – m/s

18. λ = 400 nm = 400×10^{-9} m

V_0 = 1.1 V

$$\frac{hc}{\lambda} = \frac{hc}{\lambda_0} + ev_0$$

$$\Rightarrow \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{400 \times 10^{-9}} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{\lambda_0} + 1.6 \times 10^{-19} \times 1.1$$

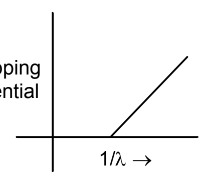
$$\Rightarrow 4.97 = \frac{19.89 \times 10^{-26}}{\lambda_0} + 1.76$$

$$\Rightarrow \frac{19.89 \times 10^{-26}}{\lambda_0} = 4.97 - 17.6 = 3.21$$

$$\Rightarrow \lambda_0 = \frac{19.89 \times 10^{-26}}{3.21} = 6.196 \times 10^{-7} \text{ m} = 620 \text{ nm.}$$

19. a) When λ = 350, V_s = 1.45

and when λ = 400, V_s = 1



$\therefore$ $\frac{hc}{350}$ = W + 1.45 …(1)

and $\frac{hc}{400}$ = W + 1 …(2)

Subtracting (2) from (1) and solving to get the value of h we get

$h = 4.2 \times 10^{-15}$ ev-sec

b) Now work function = w = $\frac{hc}{\lambda}$ = ev - s

$= \frac{1240}{350} - 1.45$ = 2.15 ev.

c) w = $\frac{hc}{\lambda} = \lambda_{\text{there cathod}} = \frac{hc}{w}$

$= \frac{1240}{2.15}$ = 576.8 nm.

20. The electric field becomes 0 1.2×10^{45} times per second.

$\therefore$ Frequency = $\frac{1.2 \times 10^{15}}{2}$ = 0.6×10^{15}

$h\nu = \phi_0$ + kE

$\Rightarrow$ $h\nu - \phi_0$ = KE

$\Rightarrow$ KE = $\frac{6.63 \times 10^{-34} \times 0.6 \times 10^{15}}{1.6 \times 10^{-19}} - 2$

= 0.482 ev = 0.48 ev.

21. $E = E_0 \sin[(1.57 \times 10^7 \text{ m}^{-1})(x - ct)]$

$W = 1.57 \times 10^7 \times C$

$\Rightarrow$ f = $\dfrac{1.57 \times 10^7 \times 3 \times 10^8}{2\pi}$ Hz W_0 = 1.9 ev

Now $eV_0 = h\nu - W_0$

$= 4.14 \times 10^{-15} \times \dfrac{1.57 \times 3 \times 10^{15}}{2\pi} - 1.9$ ev

= 3.105 – 1.9 = 1.205 ev

So, $V_0 = \dfrac{1.205 \times 1.6 \times 10^{-19}}{1.6 \times 10^{-19}}$ = 1.205 V.

22. E $= 100 \sin[(3 \times 10^{15} s^{-1})t] \sin [6 \times 10^{15} s^{-1})t]$

$= 100$ ½ $[\cos[(9 \times 10^{15} s^{-1})t] - \cos [3 \times 10^{15} s^{-1})t]$

The w are 9×10^{15} and 3×10^{15}

for largest K.E.

$f_{max} = \dfrac{w_{max}}{2\pi} = \dfrac{9 \times 10^{15}}{2\pi}$

$E - \phi_0$ = K.E.

$\Rightarrow hf - \phi_0$ = K.E.

$\Rightarrow \dfrac{6.63 \times 10^{-34} \times 9 \times 10^{15}}{2\pi \times 1.6 \times 10^{-19}} - 2 = KE$

$\Rightarrow$ KE = 3.938 ev = 3.93 ev.

23. $W_0 = hv - ev_0$

$= \dfrac{5 \times 10^{-3}}{8 \times 10^{15}} - 1.6 \times 10^{-19} \times 2$ (Given V_0 = 2V, No. of photons = 8×10^{15}, Power = 5 mW)

$= 6.25 \times 10^{-19} - 3.2 \times 10^{-19} = 3.05 \times 10^{-19}$ J

$= \dfrac{3.05 \times 10^{-19}}{1.6 \times 10^{-19}}$ = 1.906 eV.

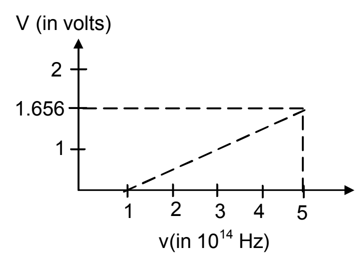
24. We have to take two cases :

Case I … v_0 = 1.656

$\nu = 5 \times 10^{14}$ Hz

Case II… v_0 = 0

$\nu = 1 \times 10^{14}$ Hz



We know ;

a) $ev_0 = h\nu - w_0$

$1.656e = h \times 5 \times 10^{14} - w_0$ …(1)

$0 = 5h \times 10^{14} - 5w_0$ …(2)

$1.656e = 4w_0$

$\Rightarrow w_0 = \dfrac{1.656}{4}$ ev = 0.414 ev

b) Putting value of w_0 in equation (2)

$\Rightarrow 5w_0 = 5h \times 10^{14}$

$\Rightarrow 5 \times 0.414 = 5 \times h \times 10^{14}$

$\Rightarrow h = 4.414 \times 10^{-15}$ ev-s

25. w_0 = 0.6 ev

For w_0 to be min ‘λ’ becomes maximum.

$w_0 = \dfrac{hc}{\lambda}$ or $\lambda = \dfrac{hc}{w_0} = \dfrac{6.63 \times 10^{-34} \times 3 \times 10^8}{0.6 \times 1.6 \times 10^{-19}}$

$= 20.71 \times 10^{-7}$ m = 2071 nm

26. λ = 400 nm, P = 5 w

E of 1 photon = $\dfrac{hc}{\lambda} = \left(\dfrac{1242}{400}\right)$ ev

No.of electrons = $\dfrac{5}{\text{Energy of 1 photon}} = \dfrac{5 \times 400}{1.6 \times 10^{-19} \times 1242}$

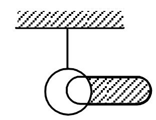
No.of electrons = 1 per 10^6 photon.

No.of photoelectrons emitted = $\dfrac{5 \times 400}{1.6 \times 1242 \times 10^{-19} \times 10^{6}}$

Photo electric current = $\dfrac{5 \times 400}{1.6 \times 1242 \times 10^{6} \times 10^{-19}} \times 1.6 \times 10^{-19}$ = 1.6×10^{-6} A = 1.6 μA.

27. λ = 200 nm = 2×10^{-7} m

E of one photon = $\dfrac{hc}{\lambda} = \dfrac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{2 \times 10^{-7}}$ = 9.945×10^{-19}



No.of photons = $\dfrac{1 \times 10^{-7}}{9.945 \times 10^{-19}}$ = 1×10^{11} no.s

Hence, No.of photo electrons = $\dfrac{1 \times 10^{11}}{10^{4}}$ = 1×10^{7}

Net amount of positive charge 'q' developed due to the outgoing electrons

= $1 \times 10^{7} \times 1.6 \times 10^{-19} = 1.6 \times 10^{-12}$ C.

Now potential developed at the centre as well as at the surface due to these charger

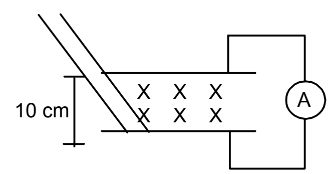
= $\dfrac{Kq}{r} = \dfrac{9 \times 10^{9} \times 1.6 \times 10^{-12}}{4.8 \times 10^{-2}}$ = 3×10^{-1} V = 0.3 V.

28. ϕ_0 = 2.39 eV

λ_1 = 400 nm, λ_2 = 600 nm

for B to the minimum energy should be maximum

$\therefore$ λ should be minimum.



E = $\dfrac{hc}{\lambda} - \phi_0$ = 3.105 – 2.39 = 0.715 eV.

The presence of magnetic field will bend the beam there will be no current if the electron does not reach the other plates.

r = $\dfrac{mv}{qB}$

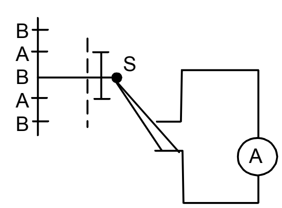
$\Rightarrow$ r = $\dfrac{\sqrt{2mE}}{qB}$

$\Rightarrow$ 0.1 = $\dfrac{\sqrt{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19} \times 0.715}}{1.6 \times 10^{-19} \times B}$

$\Rightarrow$ B = 2.85×10^{-5} T

29. Given : fringe width,

y = 1.0 mm $\times$ 2 = 2.0 mm, D = 0.24 mm, W_0 = 2.2 ev, D = 1.2 m



y = $\dfrac{\lambda D}{d}$

or, λ = $\dfrac{yd}{D} = \dfrac{2 \times 10^{-3} \times 0.24 \times 10^{-3}}{1.2}$ = 4×10^{-7} m

E = $\dfrac{hc}{\lambda} = \dfrac{4.14 \times 10^{-15} \times 3 \times 10^{8}}{4 \times 10}$ = 3.105 ev

Stopping potential eV_0 = 3.105 – 2.2 = 0.905 V

30. ϕ = 4.5 eV, λ = 200 nm

Stopping potential or energy = E – ϕ = $\frac{WC}{\lambda} - \phi$

Minimum 1.7 V is necessary to stop the electron

The minimum K.E. = 2eV

[Since the electric potential of 2 V is reqd. to accelerate the electron to reach the plates]

the maximum K.E. = (2+1, 7)ev = 3.7 ev.

31. Given

$\sigma = 1 \times 10^{-9}$ cm^{-2}, W_0 (C_s) = 1.9 eV, d = 20 cm = 0.20 m, λ = 400 nm

we know $\rightarrow$ Electric potential due to a charged plate = V = E $\times$ d

Where E $\rightarrow$ elelctric field due to the charged plate = σ/E_0

d $\rightarrow$ Separation between the plates.

$$V = \frac{\sigma}{E_0} \times d = \frac{1 \times 10^{-9} \times 20}{8.85 \times 10^{-12} \times 100} = 22.598 \text{ V} = 22.6$$

$$V_0 e = h\nu - w_0 = \frac{hc}{\lambda} - w_0 = \frac{4.14 \times 10^{-15} \times 3 \times 10^8}{4 \times 10^{-7}} - 1.9$$

= 3.105 – 1.9 = 1.205 ev

or, V_0 = 1.205 V

As V_0 is much less than 'V'

Hence the minimum energy required to reach the charged plate must be = 22.6 eV

For maximum KE, the V must be an accelerating one.

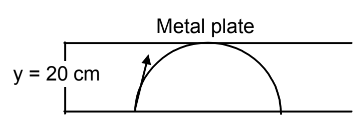
Hence max KE = V_0 + V = 1.205 + 22.6 = 23.8005 ev

32. Here electric field of metal plate = E = P/E_0

$$= \frac{1 \times 10^{-19}}{8.85 \times 10^{-12}} = 113 \text{ v/m}$$

accl. de = ϕ = qE / m

$$= \frac{1.6 \times 10^{-19} \times 113}{9.1 \times 10^{-31}} = 19.87 \times 10^{12}$$



$$t = \frac{\sqrt{2y}}{a} = \frac{\sqrt{2 \times 20 \times 10^{-2}}}{19.87 \times 10^{-31}} = 1.41 \times 10^{-7} \text{ sec}$$

$$\text{K.E.} = \frac{hc}{\lambda} - w = 1.2 \text{ eV}$$

= 1.2 $\times$ 1.6 $\times$ 10^{-19} J [because in previous problem i.e. in problem 31 : KE = 1.2 ev]

$$\therefore V = \frac{\sqrt{2KE}}{m} = \frac{\sqrt{2 \times 1.2 \times 1.6 \times 10^{-19}}}{4.1 \times 10^{-31}} = 0.665 \times 10^{-6}$$

$\therefore$ Horizontal displacement = $V_t \times t$

= 0.655 $\times$ 10^{-6} $\times$ 1.4 $\times$ 10^{-7} = 0.092 m = 9.2 cm.

33. When λ = 250 nm

$$\text{Energy of photon} = \frac{hc}{\lambda} = \frac{1240}{250} = 4.96 \text{ ev}$$

$$\therefore \text{K.E.} = \frac{hc}{\lambda} - w = 4.96 - 1.9 \text{ ev} = 3.06 \text{ ev.}$$

Velocity to be non positive for each photo electron

The minimum value of velocity of plate should be = velocity of photo electron

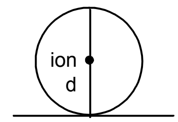
$\therefore$ Velocity of photo electron = $\sqrt{2KE/m}$

$= \sqrt{\dfrac{3.06}{9.1\times10^{-31}}} = \sqrt{\dfrac{3.06\times1.6\times10^{-19}}{9.1\times10^{-31}}}$ = 1.04 × 10^6 m/sec.

34. Work function = ϕ, distance = d

The particle will move in a circle

When the stopping potential is equal to the potential due to the singly charged ion at that point.



$eV_0 = \dfrac{hc}{\lambda} - \phi$

$\Rightarrow V_0 = \left(\dfrac{hc}{\lambda} - \phi\right)\dfrac{1}{e} \Rightarrow \dfrac{ke}{2d} = \left(\dfrac{hc}{\lambda} - \phi\right)\dfrac{1}{e}$

$\Rightarrow \dfrac{Ke^2}{2d} = \dfrac{hc}{\lambda} - \phi \Rightarrow \dfrac{hc}{\lambda} = \dfrac{Ke^2}{2d} + \phi = \dfrac{Ke^2 + 2d\phi}{2d}$

$\Rightarrow \lambda = \dfrac{hc\ 2d}{Ke^2 + 2d\phi} = \dfrac{2hcd}{\dfrac{1}{4\pi\varepsilon_0 e^2} + 2d\phi} = \dfrac{8\pi\varepsilon_0 hcd}{e^2 + 8\pi\varepsilon_0 d\phi}$.

35. a) When λ = 400 nm

Energy of photon = $\dfrac{hc}{\lambda} = \dfrac{1240}{400}$ = 3.1 eV

This energy given to electron

But for the first collision energy lost = 3.1 ev × 10% = 0.31 ev

for second collision energy lost = 3.1 ev × 10% = 0.31 ev

Total energy lost the two collision = 0.31 + 0.31 = 0.62 ev

K.E. of photon electron when it comes out of metal

= hc/λ – work function – Energy lost due to collision

= 3.1 ev – 2.2 – 0.62 = 0.31 ev

b) For the 3rd collision the energy lost = 0.31 ev

Which just equative the KE lost in the 3rd collision electron. It just comes out of the metal

Hence in the fourth collision electron becomes unable to come out of the metal

Hence maximum number of collision = 4.

♠ ♠ ♠

BOHR'S THEORY AND PHYSICS OF ATOM
CHAPTER 43

1. $a_0 = \frac{\varepsilon_0 h^2}{\pi m e^2} = \frac{A^2T^2(ML^2T^{-1})^2}{L^2MLT^{-2}M(AT)^2} = \frac{M^2L^4T^{-2}}{M^2L^3T^{-2}} = L$

   $\therefore a_0$ has dimensions of length.

2. We know, $\bar{\lambda} = 1/\lambda$ = $1.1 \times 10^7 \times (1/n_1^2 - 1/n_2^2)$

   a) $n_1 = 2$, $n_2 = 3$

   or, $1/\lambda = 1.1 \times 10^7 \times (1/4 - 1/9)$

   or, $\lambda = \frac{36}{5 \times 1.1 \times 10^7} = 6.54 \times 10^{-7} = 654$ nm

   b) $n_1 = 4$, $n_2 = 5$

   $\bar{\lambda} = 1/\lambda = 1.1 \times 10^7\ (1/16 - 1/25)$

   or, $\lambda = \frac{400}{1.1 \times 10^7 \times 9} = 40.404 \times 10^{-7}$ m = 4040.4 nm

   for $R = 1.097 \times 10^7$, $\lambda = 4050$ nm

   c) $n_1 = 9$, $n_2 = 10$

   $1/\lambda = 1.1 \times 10^7\ (1/81 - 1/100)$

   or, $\lambda = \frac{8100}{19 \times 1.1 \times 10^7} = 387.5598 \times 10^{-7} = 38755.9$ nm

   for $R = 1.097 \times 10^7$; $\lambda = 38861.9$ nm

3. Small wave length is emitted i.e. longest energy

   $n_1 = 1$, $n_2 = \infty$

   a) $\frac{1}{\lambda} = R\left(\frac{1}{n_1^2 - n_2^2}\right)$

   $\Rightarrow \frac{1}{\lambda} = 1.1 \times 10^7 \left(\frac{1}{1} - \frac{1}{\infty}\right)$

   $\Rightarrow \lambda = \frac{1}{1.1 \times 10^7} = \frac{1}{1.1} \times 10^{-7} = 0.909 \times 10^{-7} = 90.9 \times 10^{-8} = 91$ nm.

   b) $\frac{1}{\lambda} = z^2R\left(\frac{1}{n_1^2 - n_2^2}\right)$

   $\Rightarrow \lambda = \frac{1}{1.1 \times 10^{-7} z^2} = \frac{91\text{ nm}}{4} = 23$ nm

   c) $\frac{1}{\lambda} = z^2R\left(\frac{1}{n_1^2 - n_2^2}\right)$

   $\Rightarrow \lambda = \frac{91\text{ nm}}{z^2} = \frac{91}{9} = 10$ nm

4. Rydberg's constant = $\frac{me^4}{8h^3C\varepsilon_0^2}$

   $m_e = 9.1 \times 10^{-31}$ kg, $e = 1.6 \times 10^{-19}$ c, $h = 6.63 \times 10^{-34}$ J-S, $C = 3 \times 10^8$ m/s, $\varepsilon_0 = 8.85 \times 10^{-12}$

   or, $R = \frac{9.1 \times 10^{-31} \times (1.6 \times 10^{-19})^4}{8 \times (6.63 \times 10^{-34})^3 \times 3 \times 10^8 \times (8.85 \times 10^{-12})^2} = 1.097 \times 10^7\ m^{-1}$

5. $n_1 = 2$, $n_2 = \infty$

   $E = \frac{-13.6}{n_1^2} - \frac{-13.6}{n_2^2} = 13.6\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

   $= 13.6\ (1/\infty - 1/4) = -13.6/4 = -3.4$ eV

6. a) n = 1, r = $\frac{\varepsilon_0 h^2 n^2}{\pi m Z e^2} = \frac{0.53 n^2}{Z}$ A°

$= \frac{0.53 \times 1}{2}$ = 0.265 A°

$\varepsilon = \frac{-13.6 z^2}{n^2} = \frac{-13.6 \times 4}{1}$ = –54.4 eV

b) n = 4, r = $\frac{0.53 \times 16}{2}$ = 4.24 A

$\varepsilon = \frac{-13.6 \times 4}{164}$ = –3.4 eV

c) n = 10, r = $\frac{0.53 \times 100}{2}$ = 26.5 A

$\varepsilon = \frac{-13.6 \times 4}{100}$ = –0.544 A

7. As the light emitted lies in ultraviolet range the line lies in hyman series.

$\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

$\Rightarrow \frac{1}{102.5 \times 10^{-9}} = 1.1 \times 10^7 \, (1/1^2 - 1/n_2^2)$

$\Rightarrow \frac{10^9}{102.5} = 1.1 \times 10^7 (1 - 1/n_2^2) \Rightarrow \frac{10^2}{102.5} = 1.1 \times 10^7 (1 - 1/n_2^2)$

$\Rightarrow 1 - \frac{1}{n_2^2} = \frac{100}{102.5 \times 1.1} \Rightarrow \frac{1}{n_2^2} = \frac{1 - 100}{102.5 \times 1.1}$

$\Rightarrow n_2 = 2.97 = 3.$

8. a) First excitation potential of

$He^+ = 10.2 \times z^2 = 10.2 \times 4 = 40.8$ V

b) Ionization potential of L_1^{++}

$= 13.6 \text{ V} \times z^2 = 13.6 \times 9 = 122.4$ V

9. $n_1 = 4 \rightarrow n_2 = 2$

$n_1 = 4 \rightarrow 3 \rightarrow 2$

$\frac{1}{\lambda} = 1.097 \times 10^7 \left(\frac{1}{16} - \frac{1}{4}\right)$

$\Rightarrow \frac{1}{\lambda} = 1.097 \times 10^7 \left(\frac{1-4}{16}\right) \Rightarrow \frac{1.097 \times 10^7 \times 3}{16}$

$\Rightarrow \lambda = \frac{16 \times 10^{-7}}{3 \times 1.097} = 4.8617 \times 10^{-7}$

$= 1.861 \times 10^{-9}$ = 487 nm

$n_1 = 4$ and $n_2 = 3$

$\frac{1}{\lambda} = 1.097 \times 10^7 \left(\frac{1}{16} - \frac{1}{9}\right)$

$\Rightarrow \frac{1}{\lambda} = 1.097 \times 10^7 \left(\frac{9-16}{144}\right) \Rightarrow \frac{1.097 \times 10^7 \times 7}{144}$

$\Rightarrow \lambda = \frac{144}{7 \times 1.097 \times 10^7}$ = 1875 nm

$n_1 = 3 \rightarrow n_2 = 2$

$\frac{1}{\lambda} = 1.097 \times 10^7 \left(\frac{1}{9} - \frac{1}{4}\right)$

$\Rightarrow\ \frac{1}{\lambda} = 1.097 \times 10^7 \left(\frac{4-9}{36}\right) \Rightarrow \frac{1.097 \times 10^7 \times 5}{66}$

$\Rightarrow\ \lambda = \frac{36 \times 10^{-7}}{5 \times 1.097} = 656$ nm

10. λ = 228 A°

$E = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{228 \times 10^{-10}} = 0.0872 \times 10^{-16}$

The transition takes place form n = 1 to n = 2

Now, ex. $13.6 \times 3/4 \times z^2 = 0.0872 \times 10^{-16}$

$\Rightarrow\ z^2 = \frac{0.0872 \times 10^{-16} \times 4}{13.6 \times 3 \times 1.6 \times 10^{-19}} = 5.3$

$z = \sqrt{5.3} = 2.3$

The ion may be Helium.

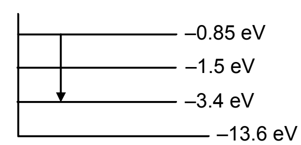
11. $F = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2}$

[Smallest dist. Between the electron and nucleus in the radius of first Bohrs orbit]

$= \frac{(1.6 \times 10^{-19}) \times (1.6 \times 10^{-19}) \times 9 \times 10^9}{(0.53 \times 10^{-10})^2} = 82.02 \times 10^{-9} = 8.202 \times 10^{-8} = 8.2 \times 10^{-8}$ N

12. a) From the energy data we see that the H atom transists from binding energy of 0.85 ev to exitation energy of 10.2 ev = Binding Energy of –3.4 ev.

So, n = 4 to n = 2



b) We know = $1/\lambda = 1.097 \times 10^7$ (1/4 – 1/16)

$\Rightarrow\ \lambda = \frac{16}{1.097 \times 3 \times 10^7} = 4.8617 \times 10^{-7} = 487$ nm.

13. The second wavelength is from Balmer to hyman i.e. from n = 2 to n = 1

$n_1 = 2$ to $n_2 = 1$

$\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

$\Rightarrow\ \frac{1}{\lambda} = 1.097 \times 10^7 \left(\frac{1}{2^2} - \frac{1}{1^2}\right) \Rightarrow 1.097 \times 10^7 \left(\frac{1}{4} - 1\right)$

$\Rightarrow\ \lambda = \frac{4}{1.097 \times 3} \times 10^{-7}$

$= 1.215 \times 10^{-7} = 121.5 \times 10^{-9} = 122$ nm.

14. Energy at n = 6, $E = \frac{-13.6}{36} = -0.3777777$

Energy in groundstate = –13.6 eV

Energy emitted in Second transition = –13.6 –(0.37777 + 1.13)

= –12.09 = 12.1 eV

b) Energy in the intermediate state = 1.13 ev + 0.0377777

$= 1.507777 = \frac{13.6 \times z^2}{n^2} = \frac{13.6}{n^2}$

or, $n = \sqrt{\frac{13.6}{1.507}} = 3.03 = 3 = n$.

15. The potential energy of a hydrogen atom is zero in ground state.

An electron is board to the nucleus with energy 13.6 ev.,

Show we have to give energy of 13.6 ev. To cancel that energy.

Then additional 10.2 ev. is required to attain first excited state.

Total energy of an atom in the first excited state is = 13.6 ev. + 10.2 ev. = 23.8 ev.

16. Energy in ground state is the energy acquired in the transition of 2nd excited state to ground state.
As 2nd excited state is taken as zero level.

$E = \frac{hc}{\lambda_1} = \frac{4.14 \times 10^{-15} \times 3 \times 10^8}{46 \times 10^{-9}} = \frac{1242}{46}$ = 27 ev.

Again energy in the first excited state

$E = \frac{hc}{\lambda_{II}} = \frac{4.14 \times 10^{-15} \times 3 \times 10^8}{103.5}$ = 12 ev.

17. a) The gas emits 6 wavelengths, let it be in nth excited state.

$\Rightarrow \frac{n(n-1)}{2} = 6 \Rightarrow n = 4$ $\therefore$ The gas is in 4th excited state.

b) Total no.of wavelengths in the transition is 6. We have $\frac{n(n-1)}{2} = 6 \Rightarrow n = 4$.

18. a) We know, $m \upsilon r = \frac{nh}{2\pi} \Rightarrow mr^2 w = \frac{nh}{2\pi} \Rightarrow w = \frac{hn}{2\pi \times m \times r^2}$

$= \frac{1 \times 6.63 \times 10^{-34}}{2 \times 3.14 \times 9.1 \times 10^{-31} \times (0.53)^2 \times 10^{-20}} = 0.413 \times 10^{17}$ rad/s $= 4.13 \times 10^{17}$ rad/s.

19. The range of Balmer series is 656.3 nm to 365 nm. It can resolve λ and $\lambda + \Delta\lambda$ if $\lambda/\Delta\lambda = 8000$.

$\therefore$ No.of wavelengths in the range $= \frac{656.3 - 365}{8000} = 36$

Total no.of lines 36 + 2 = 38 [extra two is for first and last wavelength]

20. a) $n_1 = 1$, $n_2 = 3$, $E = 13.6\ (1/1 - 1/9) = 13.6 \times 8/9 = hc/\lambda$

or, $\frac{13.6 \times 8}{9} = \frac{4.14 \times 10^{-15} \times 3 \times 10^8}{\lambda} \Rightarrow \lambda = \frac{4.14 \times 3 \times 10^{-7}}{13.6 \times 8} = 1.027 \times 10^{-7}$ = 103 nm.

b) As 'n' changes by 2, we may consider n = 2 to n = 4

then $E = 13.6 \times (1/4 - 1/16) = 2.55$ ev and $2.55 = \frac{1242}{\lambda}$ or λ = 487 nm.

21. Frequency of the revolution in the ground state is $\frac{V_0}{2\pi r_0}$

[r_0 = radius of ground state, V_0 = velocity in the ground state]

$\therefore$ Frequency of radiation emitted is $\frac{V_0}{2\pi r_0} = f$

$\therefore C = f\lambda \Rightarrow \lambda = C/f = \frac{C2\pi r_0}{V_0}$

$\therefore \lambda = \frac{C2\pi r_0}{V_0}$ = 45.686 nm = 45.7 nm.

22. KE = 3/2 KT = 1.5 KT, $K = 8.62 \times 10^{-5}$ eV/k, Binding Energy $= -13.6\ (1/\infty - 1/1) = 13.6$ eV.
According to the question, 1.5 KT = 13.6

$\Rightarrow 1.5 \times 8.62 \times 10^{-5} \times T = 13.6$

$\Rightarrow T = \frac{13.6}{1.5 \times 8.62 \times 10^{-5}} = 1.05 \times 10^5$ K

No, because the molecule exists an H_2^+ which is impossible.

23. $K = 8.62 \times 10^{-5}$ eV/k
K.E. of H_2 molecules = 3/2 KT
Energy released, when atom goes from ground state to no = 3

$\Rightarrow 13.6\ (1/1 - 1/9) \Rightarrow 3/2\ KT = 13.6(1/1 - 1/9)$

$\Rightarrow 3/2 \times 8.62 \times 10^{-5}\ T = \frac{13.6 \times 8}{9}$

$\Rightarrow T = 0.9349 \times 10^5 = 9.349 \times 10^4 = 9.4 \times 10^4$ K.

24. n = 2, T = 10^{-8} s

Frequency = $\frac{me^4}{4\varepsilon_0^2 n^3 h^3}$

So, time period = 1/f = $\frac{4\varepsilon o^2 n^3 h^3}{me^4} \Rightarrow \frac{4 \times (8.85)^2 \times 2^3 \times (6.63)^3}{9.1 \times (1.6)^4} \times \frac{10^{-24} - 10^{-102}}{10^{-76}}$

= 12247.735 × 10^{-19} sec.

No.of revolutions = $\frac{10^{-8}}{12247.735 \times 10^{-19}}$ = 8.16 × 10^5

= 8.2 × 10^6 revolution.

25. Dipole moment (μ)

= n i A = 1 × q/t A = qfA

= $e \times \frac{me^4}{4\varepsilon_0^2 h^3 n^3} \times (\pi r_0^2 n^2) = \frac{me^5 \times (\pi r_0^2 n^2)}{4\varepsilon_0^2 h^3 n^3}$

= $\frac{(9.1 \times 10^{-31})(1.6 \times 10^{-19})^5 \times \pi \times (0.53)^2 \times 10^{-20} \times 1}{4 \times (8.85 \times 10^{-12})^2 (6.64 \times 10^{-34})^3 (1)^3}$

= 0.0009176 × 10^{-20} = 9.176 × 10^{-24} A - m^2.

26. Magnetic Dipole moment = n i A = $\frac{e \times me^4 \times \pi r_n^2 n^2}{4\varepsilon_0^2 h^3 n^3}$

Angular momentum = mvr = $\frac{nh}{2\pi}$

Since the ratio of magnetic dipole moment and angular momentum is independent of Z.
Hence it is an universal constant.

Ratio = $\frac{e^5 \times m \times \pi r_0^2 n^2}{24\varepsilon_0 h^3 n^3} \times \frac{2\pi}{nh} \Rightarrow \frac{(1.6 \times 10^{-19})^5 \times (9.1 \times 10^{-31}) \times (3.14)^2 \times (0.53 \times 10^{-10})^2}{2 \times (8.85 \times 10^{-12})^2 \times (6.63 \times 10^{-34})^4 \times 1^2}$

= 8.73 × 10^{10} C/kg.

27. The energies associated with 450 nm radiation = $\frac{1242}{450}$ = 2.76 ev

Energy associated with 550 nm radiation = $\frac{1242}{550}$ = 2.258 = 2.26 ev.

The light comes under visible range
Thus, n_1 = 2, n_2 = 3, 4, 5, ……
$E_2 - E_3$ = 13.6 (1/2^2 – 1/3^2) = 1.9 ev
$E_2 - E_4$ = 13.6 (1/4 – 1/16) = 2.55 ev
$E_2 - E_5$ = 13.6 (1/4 – 1/25) = 2.856 ev
Only $E_2 - E_4$ comes in the range of energy provided. So the wavelength corresponding to that energy will be absorbed.

λ = $\frac{1242}{2.55}$ = 487.05 nm = 487 nm

487 nm wavelength will be absorbed.

28. From transitions n =2 to n =1.
E = 13.6 (1/1 – 1/4) = 13.6 × 3/4 = 10.2 eV
Let in check the transitions possible on He. n = 1 to 2
E_1 = 4 × 13.6 (1 – 1/4) = 40.8 eV [E_1 > E hence it is not possible]
n = 1 to n = 3
E_2 = 4 × 13.6 (1 – 1/9) = 48.3 eV [E_2 > E hence impossible]
Similarly n = 1 to n = 4 is also not possible.
n = 2 to n = 3
E_3 = 4 × 13.6 (1/4 – 1/9) = 7.56 eV

n = 2 to n = 4

$E_4 = 4 \times 13.6\ (1/4 - 1/16) = 10.2$ eV

As, $E_3 < E$ and $E_4 = E$

Hence E_3 and E_4 can be possible.

29. $\lambda = 50$ nm

Work function = Energy required to remove the electron from $n_1 = 1$ to $n_2 = \infty$.

$E = 13.6\ (1/1 - 1/\infty) = 13.6$

$\frac{hc}{\lambda} - 13.6 = KE$

$\Rightarrow \frac{1242}{50} - 13.6 = KE \Rightarrow KE = 24.84 - 13.6 = 11.24$ eV.

30. $\lambda = 100$ nm

$E = \frac{hc}{\lambda} = \frac{1242}{100} = 12.42$ eV

a) The possible transitions may be E_1 to E_2

E_1 to E_2, energy absorbed = 10.2 eV

Energy left = 12.42 – 10.2 = 2.22 eV

$2.22\text{ eV} = \frac{hc}{\lambda} = \frac{1242}{\lambda}$ or $\lambda = 559.45 = 560$ nm

E_1 to E_3, Energy absorbed = 12.1 eV

Energy left = 12.42 – 12.1 = 0.32 eV

$0.32 = \frac{hc}{\lambda} = \frac{1242}{\lambda}$ or $\lambda = \frac{1242}{0.32} = 3881.2 = 3881$ nm

E_3 to E_4, Energy absorbed = 0.65

Energy left = 12.42 – 0.65 = 11.77 eV

$11.77 = \frac{hc}{\lambda} = \frac{1242}{\lambda}$ or $\lambda = \frac{1242}{11.77} = 105.52$

b) The energy absorbed by the H atom is now radiated perpendicular to the incident beam.

$\rightarrow 10.2 = \frac{hc}{\lambda}$ or $\lambda = \frac{1242}{10.2} = 121.76$ nm

$\rightarrow 12.1 = \frac{hc}{\lambda}$ or $\lambda = \frac{1242}{12.1} = 102.64$ nm

$\rightarrow 0.65 = \frac{hc}{\lambda}$ or $\lambda = \frac{1242}{0.65} = 1910.76$ nm

31. $\phi = 1.9$ eV

a) The hydrogen is ionized

$n_1 = 1$, $n_2 = \infty$

Energy required for ionization = $13.6\ (1/n_1^2 - 1/n_2^2) = 13.6$

$\frac{hc}{\lambda} - 1.9 = 13.6 \Rightarrow \lambda = 80.1$ nm = 80 nm.

b) For the electron to be excited from $n_1 = 1$ to $n_2 = 2$

$E = 13.6\ (1/n_1^2 - 1/n_2^2) = 13.6(1 - ¼) = \frac{13.6 \times 3}{4}$

$\frac{hc}{\lambda} - 1.9 = \frac{13.6 \times 3}{4} \Rightarrow \lambda = 1242 / 12.1 = 102.64 = 102$ nm.

32. The given wavelength in Balmer series.

The first line, which requires minimum energy is from $n_1 = 3$ to $n_2 = 2$.

$\therefore$ The energy should be equal to the energy required for transition from ground state to n = 3.

i.e. E = 13.6 [1 – (1/9)] = 12.09 eV

$\therefore$ Minimum value of electric field = 12.09 v/m = 12.1 v/m

33. In one dimensional elastic collision of two bodies of equal masses.
The initial velocities of bodies are interchanged after collision.
$\therefore$ Velocity of the neutron after collision is zero.
Hence, it has zero energy.

34. The hydrogen atoms after collision move with speeds v_1 and v_2.
$mv = mv_1 + mv_2$ …(1)
$\frac{1}{2}mv^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 + \Delta E$ …(2)
From (1) $v^2 = (v_1 + v_2)^2 = v_1^2 + v_2^2 + 2v_1v_2$
From (2) $v^2 = v_1^2 + v_2^2 + 2\Delta E/m$
$= 2v_1v_2 = \frac{2\Delta E}{m}$ …(3)
$(v_1 - v_2)^2 = (v_1 + v_2)^2 - 4v_1v_2$
$\Rightarrow (v_1 - v_2) = v^2 - 4\Delta E/m$
For minimum value of 'v'
$v_1 = v_2 \Rightarrow v^2 - (4\Delta E/m) = 0$
$\Rightarrow v^2 = \frac{4\Delta E}{m} = \frac{4 \times 13.6 \times 1.6 \times 10^{-19}}{1.67 \times 10^{-27}}$
$\Rightarrow v = \sqrt{\frac{4 \times 13.6 \times 1.6 \times 10^{-19}}{1.67 \times 10^{-27}}} = 7.2 \times 10^4$ m/s.

35. Energy of the neutron is $\frac{1}{2} mv^2$.
The condition for inelastic collision is $\Rightarrow \frac{1}{2} mv^2 > 2\Delta E$
$\Rightarrow \Delta E = \frac{1}{4} mv^2$
ΔE is the energy absorbed.
Energy required for first excited state is 10.2 ev.
$\therefore \Delta E < 10.2$ ev
$\therefore 10.2 \text{ ev} < \frac{1}{4} mv^2 \Rightarrow V_{min} = \sqrt{\frac{4 \times 10.2}{m}}$ ev
$\Rightarrow v = \sqrt{\frac{10.2 \times 1.6 \times 10^{-19} \times 4}{1.67 \times 10^{-27}}} = 6 \times 10^4$ m/sec.

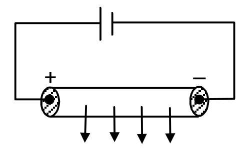
36. a) $\lambda = 656.3$ nm
Momentum $P = E/C = \frac{hc}{\lambda} \times \frac{1}{c} = \frac{h}{\lambda} = \frac{6.63 \times 10^{-34}}{656.3 \times 10^{-9}} = 0.01 \times 10^{-25} = 1 \times 10^{-27}$ kg-m/s
b) $1 \times 10^{-27} = 1.67 \times 10^{-27} \times v$
$\Rightarrow v = 1/1.67 = 0.598 = 0.6$ m/s
c) KE of atom $= \frac{1}{2} \times 1.67 \times 10^{-27} \times (0.6)^2 = \frac{0.3006 \times 10^{-27}}{1.6 \times 10^{-19}}$ ev $= 1.9 \times 10^{-9}$ ev.

37. Difference in energy in the transition from n = 3 to n = 2 is 1.89 ev.
Let recoil energy be E.
$\frac{1}{2} m_e [V_2^2 - V_3^2] + E = 1.89 \text{ ev} \Rightarrow 1.89 \times 1.6 \times 10^{-19}$ J
$\therefore \frac{1}{2} \times 9.1 \times 10^{-31}\left[\left(\frac{2187}{2}\right)^2 - \left(\frac{2187}{3}\right)^2\right] + E = 3.024 \times 10^{-19}$ J
$\Rightarrow E = 3.024 \times 10^{-19} - 3.0225 \times 10^{-25}$

38. $n_1 = 2$, $n_2 = 3$
Energy possessed by H_α light
$= 13.6\ (1/n_1^2 - 1/n_2^2) = 13.6 \times (1/4 - 1/9) = 1.89$ eV.
For $H\alpha$ light to be able to emit photoelectrons from a metal the work function must be greater than or equal to 1.89 ev.

39. The maximum energy liberated by the Balmer Series is $n_1 = 2$, $n_2 = \infty$
$E = 13.6(1/n_1^2 - 1/n_2^2) = 13.6 \times 1/4 = 3.4$ eV
3.4 ev is the maximum work function of the metal.

40. Wocs = 1.9 eV



The radiations coming from the hydrogen discharge tube consist of photons of energy = 13.6 eV.
Maximum KE of photoelectrons emitted
= Energy of Photons – Work function of metal.
= 13.6 eV – 1.9 eV = 11.7 eV

41. λ = 440 nm, e = Charge of an electron, ϕ = 2 eV, V_0 = stopping potential.

We have, $\frac{hc}{\lambda} - \phi = eV_0 \Rightarrow \frac{4.14 \times 10^{-15} \times 3 \times 10^8}{440 \times 10^{-9}} - 2eV = eV_0$

$\Rightarrow eV_0 = 0.823$ eV $\Rightarrow V_0 = 0.823$ volts.

42. Mass of Earth = Me = 6.0×10^{24} kg
Mass of Sun = Ms = 2.0×10^{30} kg
Earth – Sun dist = 1.5×10^{11} m

$mvr = \frac{nh}{2\pi}$ or, $m^2 v^2 r^2 = \frac{n^2h^2}{4\pi^2}$ …(1)

$\frac{GMeMs}{r^2} = \frac{Mev^2}{r}$ or $v^2 = GMs/r$ …(2)

Dividing (1) and (2)

We get $Me^2 r = \frac{n^2h^2}{4\pi^2 GMs}$

for n = 1

$r = \sqrt{\frac{h^2}{4\pi^2 GMsMe^2}} = 2.29 \times 10^{-138}$ m $= 2.3 \times 10^{-138}$ m.

b) $n^2 = \frac{Me^2 \times r \times 4 \times \pi^2 \times G \times Ms}{h^2} = 2.5 \times 10^{74}$.

43. $m_e Vr = \frac{nh}{z\pi}$ …(1)

$\frac{GM_nM_e}{r^2} = \frac{m_eV^2}{r} \Rightarrow \frac{GM_n}{r} = \nu^2$ …(2)

Squaring (2) and dividing it with (1)

$\frac{m_e^2 v^2 r^2}{\nu^2} = \frac{n^2h^2r}{4\pi^2 Gm_n} \Rightarrow me^2 r = \frac{n^2h^2r}{4\pi^2 Gm_n} \Rightarrow r = \frac{n^2h^2r}{4\pi^2 Gm_n me^2}$

$\Rightarrow \nu = \frac{nh}{2\pi r m_e}$ from (1)

$\Rightarrow \nu = \frac{nh 4\pi^2 GM_nM_e^2}{2\pi M_e n^2h^2} = \frac{2\pi GM_nM_e}{nh}$

$KE = \frac{1}{2}m_eV^2 = \frac{1}{2}m_e \frac{(2\pi GM_nM_e)^2}{nh} = \frac{4\pi^2 G^2 M_n^2 M_e^3}{2n^2h^2}$

$PE = \frac{-GM_nM_e}{r} = \frac{-GM_nM_e 4\pi^2 GM_nM_e^2}{n^2h^2} = \frac{-4\pi^2 G^2 M_n^2 M_e^3}{n^2h^2}$

Total energy = KE + PE $= \frac{2\pi^2 G^2 M_n^2 M_e^3}{2n^2h^2}$

44. According to Bohr's quantization rule

$mvr = \frac{nh}{2\pi}$

'r' is less when 'n' has least value i.e. 1

or, $mv = \frac{nh}{2\pi R}$ …(1)

Again, $r = \frac{mv}{qB}$, or, $mv = rqB$ …(2)

From (1) and (2)

$rqB = \frac{nh}{2\pi r}$ [q = e]

$\Rightarrow r^2 = \frac{nh}{2\pi eB} \Rightarrow r = \sqrt{h/2\pi\ eB}$ [here n = 1]

b) For the radius of nth orbit, $r = \sqrt{\frac{nh}{2\pi eB}}$.

c) $mvr = \frac{nh}{2\pi}$, $r = \frac{mv}{qB}$

Substituting the value of 'r' in (1)

$mv \times \frac{mv}{qB} = \frac{nh}{2\pi}$

$\Rightarrow m^2v^2 = \frac{nheB}{2\pi}$ [n = 1, q = e]

$\Rightarrow v^2 = \frac{heB}{2\pi m^2} \Rightarrow$ or $v = \sqrt{\frac{heB}{2\pi m^2}}$.

45. even quantum numbers are allowed

$n_1 = 2$, $n_2 = 4 \rightarrow$ For minimum energy or for longest possible wavelength.

$E = 13.6\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) = 13.6\left(\frac{1}{2^2} - \frac{1}{4^2}\right) = 2.55$

$\Rightarrow 2.55 = \frac{hc}{\lambda}$

$\Rightarrow \lambda = \frac{hc}{2.55} = \frac{1242}{2.55} = 487.05$ nm = 487 nm

46. Velocity of hydrogen atom in state 'n' = u

Also the velocity of photon = u

But u << C

Here the photon is emitted as a wave.

So its velocity is same as that of hydrogen atom i.e. u.

∴ According to Doppler's effect

frequency $v = v_0\left(\frac{1+u/c}{1-u/c}\right)$

as u <<< C $1 - \frac{u}{c} = q$

$\therefore v = v_0\left(\frac{1+u/c}{1}\right) = v_0\left(1+\frac{u}{c}\right) \Rightarrow v = v_0\left(1+\frac{u}{c}\right)$

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X - RAYS
CHAPTER 44

1. λ = 0.1 nm

   a) Energy = $\frac{hc}{\lambda} = \frac{1242 \text{ ev.nm}}{0.1\text{ nm}}$

   = 12420 ev = 12.42 Kev = 12.4 kev.

   b) Frequency = $\frac{C}{\lambda} = \frac{3\times10^8}{0.1\times10^{-9}} = \frac{3\times10^8}{10^{-10}} = 3\times10^{18}\text{Hz}$

   c) Momentum = E/C = $\frac{12.4\times10^3\times1.6\times10^{-19}}{3\times10^8}$ = 6.613 $\times$ 10^{-24} kg-m/s = 6.62 $\times$ 10^{-24} kg-m/s.

2. Distance = 3 km = 3 $\times$ 10^3 m

   C = 3 $\times$ 10^8 m/s

   t = $\frac{\text{Dist}}{\text{Speed}} = \frac{3\times10^3}{3\times10^8} = 10^{-5}$ sec.

   $\Rightarrow$ 10 $\times$ 10^{-8} sec = 10 μs in both case.

3. V = 30 KV

   λ = $\frac{hc}{E} = \frac{hc}{eV} = \frac{1242 \text{ ev}-\text{nm}}{e\times30\times10^3}$ = 414 $\times$ 10^{-4} nm = 41.4 Pm.

4. λ = 0.10 nm = 10^{-10} m ; h = 6.63 $\times$ 10^{-34} J-s

   C = 3 $\times$ 10^8 m/s ; e = 1.6 $\times$ 10^{-19} C

   λ_{min} = $\frac{hc}{eV}$ or V = $\frac{hc}{e\lambda}$

   = $\frac{6.63\times10^{-34}\times3\times10^8}{1.6\times10^{-19}\times10^{-10}}$ = 12.43 $\times$ 10^3 V = 12.4 KV.

   Max. Energy = $\frac{hc}{\lambda} = \frac{6.63\times10^{-34}\times3\times10^8}{10^{-10}}$ = 19.89 $\times$ 10^{-18} = 1.989 $\times$ 10^{-15} = 2 $\times$ 10^{-15} J.

5. λ = 80 pm, E = $\frac{hc}{\lambda} = \frac{1242}{80\times10^{-3}}$ = 15.525 $\times$ 10^3 eV = 15.5 KeV

6. We know λ = $\frac{hc}{V}$

   Now λ = $\frac{hc}{1.01V} = \frac{\lambda}{1.01}$

   $\lambda - \lambda'$ = $\frac{0.01}{1.01}\lambda$.

   % change of wave length = $\frac{0.01\times\lambda}{1.01\times\lambda}\times100 = \frac{1}{1.01}$ = 0.9900 = 1%.

7. d = 1.5 m, λ = 30 pm = 30 $\times$ 10^{-3} nm

   E = $\frac{hc}{\lambda} = \frac{1242}{30\times10^{-3}}$ = 41.4 $\times$ 10^3 eV

   Electric field = $\frac{V}{d} = \frac{41.4\times10^3}{1.5}$ = 27.6 $\times$ 10^3 V/m = 27.6 KV/m.

8. Given λ' = λ – 26 pm, V′ = 1.5 V

   Now, λ = $\frac{hc}{ev}$, λ' = $\frac{hc}{ev'}$

   or $\lambda V = \lambda' V'$

   $\Rightarrow \lambda V = (\lambda - 26\times10^{-12})\times1.5\text{ V}$

$\Rightarrow \lambda = 1.5\,\lambda - 1.5 \times 26 \times 10^{-12}$

$\Rightarrow \lambda = \dfrac{39 \times 10^{-12}}{0.5} = 78 \times 10^{-12}$ m

$V = \dfrac{hc}{e\lambda} = \dfrac{6.63 \times 3 \times 10^{-34} \times 10^{8}}{1.6 \times 10^{-19} \times 78 \times 10^{-12}} = 0.15937 \times 10^{5} = 15.93 \times 10^{3}$ V = 15.93 KV.

9. $V = 32$ KV $= 32 \times 10^{3}$ V
   When accelerated through 32 KV
   $E = 32 \times 10^{3}$ eV
   $\lambda = \dfrac{hc}{E} = \dfrac{1242}{32 \times 10^{3}} = 38.8 \times 10^{-3}$ nm = 38.8 pm.

10. $\lambda = \dfrac{hc}{eV}$; V = 40 kV, f = 9.7×10^{18} Hz
   or, $\dfrac{h}{c} = \dfrac{h}{eV}$; or, $\dfrac{i}{f} = \dfrac{h}{eV}$; or $h = \dfrac{eV}{f} V - s$
   $= \dfrac{eV}{f} V - s = \dfrac{40 \times 10^{3}}{9.7 \times 10^{18}} = 4.12 \times 10^{-15}$ eV-s.

11. $V = 40$ KV $= 40 \times 10^{3}$ V
   Energy = 40×10^{3} eV
   Energy utilized = $\dfrac{70}{100} \times 40 \times 10^{3} = 28 \times 10^{3}$ eV
   $\lambda = \dfrac{hc}{E} = \dfrac{1242 - \text{ev nm}}{28 \times 10^{3}\ \text{ev}} \Rightarrow 44.35 \times 10^{-3}$ nm = 44.35 pm.
   For other wavelengths,
   E = 70% (left over energy) = $\dfrac{70}{100} \times (40 - 28)10^{3} = 84 \times 10^{2}$.
   $\lambda' = \dfrac{hc}{E} = \dfrac{1242}{8.4 \times 10^{3}} = 147.86 \times 10^{-3}$ nm = 147.86 pm = 148 pm.
   For third wavelength,
   $E = \dfrac{70}{100} = (12 - 8.4) \times 10^{3} = 7 \times 3.6 \times 10^{2} = 25.2 \times 10^{2}$
   $\lambda' = \dfrac{hc}{E} = \dfrac{1242}{25.2 \times 10^{2}} = 49.2857 \times 10^{-2}$ nm = 493 pm.

12. $K_{\lambda} = 21.3 \times 10^{-12}$ pm, Now, $E_K - E_L = \dfrac{1242}{21.3 \times 10^{-3}} = 58.309$ kev
   $E_L = 11.3$ kev, $E_K = 58.309 + 11.3 = 69.609$ kev
   Now, Ve = 69.609 KeV, or V = 69.609 KV.

13. $\lambda = 0.36$ nm
   $E = \dfrac{1242}{0.36} = 3450$ eV $(E_M - E_K)$
   Energy needed to ionize an organ atom = 16 eV
   Energy needed to knock out an electron from K-shell
   = (3450 + 16) eV = 3466 eV = 3.466 KeV.

14. $\lambda_1 = 887$ pm
   $v = \dfrac{C}{\lambda} = \dfrac{3 \times 10^{8}}{887 \times 10^{-12}} = 3.382 \times 10^{7} = 33.82 \times 10^{16} = 5.815 \times 10^{8}$
   $\lambda_2 = 146$ pm
   $v = \dfrac{3 \times 10^{8}}{146 \times 10^{-12}} = 0.02054 \times 10^{20} = 2.054 \times 10^{18} = 1.4331 \times 10^{9}$.

We know, $\sqrt{v} = a(z-b)$

$$\Rightarrow \frac{\sqrt{5.815\times10^{8}} = a(13-b)}{\sqrt{1.4331\times10^{9}} = a(30-b)}$$

$$\Rightarrow \frac{13-b}{30-b} = \frac{5.815\times10^{-1}}{1.4331} = 0.4057.$$

$\Rightarrow 30 \times 0.4057 - 0.4057\ b = 13 - b$

$\Rightarrow 12.171 - 0.4.57\ b + b = 13$

$$\Rightarrow b = \frac{0.829}{0.5943} = 1.39491$$

$$\Rightarrow a = \frac{5.815\times10^{8}}{11.33} = 0.51323\times10^{8} = 5\times10^{7}.$$

For 'Fe',

$\sqrt{v} = 5\times10^{7}\,(26-1.39) = 5\times24.61\times10^{7} = 123.05\times10^{7}$

$c/\lambda = 15141.3\times10^{14}$

$$= \lambda = \frac{3\times10^{8}}{15141.3\times10^{14}} = 0.000198\times10^{-6}\ \text{m} = 198\times10^{-12} = 198\ \text{pm}.$$

15. E = 3.69 kev = 3690 eV

$$\lambda = \frac{hc}{E} = \frac{1242}{3690} = 0.33658\ \text{nm}$$

$\sqrt{c/\lambda} = a(z-b)$; $a = 5\times10^{7}\ \sqrt{Hz}$, b = 1.37 (from previous problem)

$$\sqrt{\frac{3\times10^{8}}{0.34\times10^{-9}}} = 5\times10^{7}(Z-1.37) \Rightarrow \sqrt{8.82\times10^{17}} = 5\times10^{7}(Z-1.37)$$

$\Rightarrow 9.39\times10^{8} = 5\times10^{7}\,(Z-1.37) \Rightarrow 93.9/5 = Z - 1.37$

$\Rightarrow Z = 20.15 = 20$

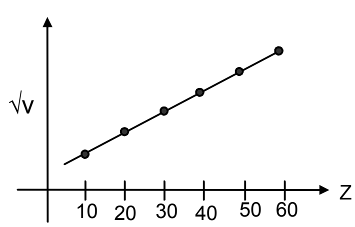
$\therefore$ The element is calcium.

16. K_B radiation is when the e jumps from n = 3 to n = 1 (here n is principal quantum no)

$$\Delta E = h\nu = Rhc\,(z-h)^{2}\left(\frac{1}{2^{2}} - \frac{1}{3^{2}}\right)$$

$$\Rightarrow \sqrt{v} = \sqrt{\frac{9RC}{8}}(z-h)$$

$\therefore \sqrt{v} \propto z$



Second method :

We can directly get value of v by `

hv = Energy

$$\Rightarrow v = \frac{\text{Energy(in kev)}}{h}$$

This we have to find out $\sqrt{v}$ and draw the same graph as above.

17. b = 1

For $\propto$ a (57)

$\sqrt{v} = a\,(Z-b)$

$\Rightarrow \sqrt{v} = a\,(57-1) = a\times56$ …(1)

For Cu(29)

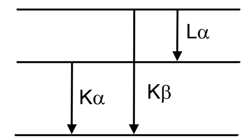
$\sqrt{1.88\times10^{78}} = a(29-1) = 28\,a$ …(2)

dividing (1) and (2)

$\sqrt{\dfrac{v}{1.88\times10^{18}}} = \dfrac{a\times56}{a\times28}$ = 2.

$\Rightarrow$ v = 1.88 × $10^{18}(2)^2$ = 4 × 1.88 × 10^{18} = 7.52 × 10^8 Hz.

18. $K_\alpha = E_K - E_L$,,,(1) λK_α = 0.71 A°
$K_\beta = E_K - E_M$,,,(2) λK_β = 0.63 A°
$L_\alpha = E_L - E_M$,,,(3)



Subtracting (2) from (1)

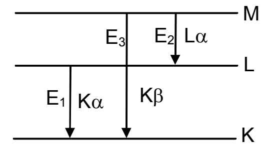
$K_\alpha - K_\beta = E_M - E_L = -L_\alpha$

or, $L_\alpha = K_\beta - K_\alpha = \dfrac{3\times10^8}{0.63\times10^{-10}} - \dfrac{3\times10^8}{0.71\times10^{-10}}$

= 4.761 × 10^{18} – 4.225 × 10^{18} = 0.536 × 10^{18} Hz.

Again $\lambda = \dfrac{3\times10^8}{0.536\times10^{18}}$ = 5.6 × 10^{-10} = 5.6 A°.

19. $E_1 = \dfrac{1242}{21.3\times10^{-3}}$ = 58.309 × 10^3 ev



$E_2 = \dfrac{1242}{141\times10^{-3}}$ = 8.8085 × 10^3 ev

$E_3 = E_1 + E_2 \Rightarrow$ (58.309 + 8.809) ev = 67.118 × 10^3 ev

$\lambda = \dfrac{hc}{E_3} = \dfrac{1242}{67.118\times10^3}$ = 18.5 × 10^{-3} nm = 18.5 pm.

20. E_K = 25.31 KeV, E_L = 3.56 KeV, E_M = 0.530 KeV
$K_\alpha = E_K - K_L$ = hv

$\Rightarrow v = \dfrac{E_K - E_L}{h} = \dfrac{25.31-3.56}{4.14\times10^{-15}}\times10^3$ = 5.25 × 10^{15} Hz

$K_\beta = E_K - K_M$ = hv

$\Rightarrow v = \dfrac{E_K - E_M}{h} = \dfrac{25.31-0.53}{4.14\times10^{-15}}\times10^3$ = 5.985 × 10^{18} Hz.

21. Let for, k series emission the potential required = v
∴ Energy of electrons = ev
This amount of energy ev = energy of L shell
The maximum potential difference that can be applied without emitting any electron is 11.3 ev.

22. V = 40 KV, i = 10 mA
1% of T_{KE} (Total Kinetic Energy) = X ray

i = ne or $n = \dfrac{10^{-2}}{1.6\times10^{-19}}$ = 0.625 × 10^{17} no.of electrons.

KE of one electron = eV = 1.6 × 10^{-19} × 40 × 10^3 = 6.4 × 10^{-15} J
T_{KE} = 0.625 × 6.4 × 10^{17} × 10^{-15} = 4 × 10^2 J.
a) Power emitted in X-ray = 4 × 10^2 × (–1/100) = 4w
b) Heat produced in target per second = 400 – 4 = 396 J.

23. Heat produced/sec = 200 w

$\Rightarrow \dfrac{neV}{t}$ = 200 $\Rightarrow$ (ne/t)V = 200

$\Rightarrow$ i = 200 /V = 10 mA.

24. Given : v = (25 × 10^{14} Hz)$(Z-1)^2$
Or C/λ = 25 × 10^{14} $(Z-1)^2$

a) $\dfrac{3\times10^8}{78.9\times10^{-12}\times25\times10^{14}} = (Z-1)^2$

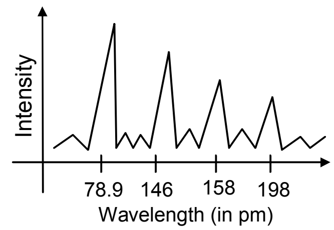
or, $(Z-1)^2$ = 0.001520 × 10^6 = 1520

$\Rightarrow$ Z – 1 = 38.98 or Z = 39.98 = 40. It is (Zr)

b) $\frac{3\times10^8}{146\times10^{-12}\times25\times10^{14}} = (Z-1)^2$

or, $(Z – 1)^2 = 0.0008219 \times 10^6$

$\Rightarrow$ Z – 1 = 28.669 or Z = 29.669 = 30. It is (Zn).



c) $\frac{3\times10^8}{158\times10^{-12}\times25\times10^{14}} = (Z-1)^2$

or, $(Z – 1)^2 = 0.0007594 \times 10^6$

$\Rightarrow$ Z – 1 = 27.5589 or Z = 28.5589 = 29. It is (Cu).

d) $\frac{3\times10^8}{198\times10^{-12}\times25\times10^{14}} = (Z-1)^2$

or, $(Z – 1)^2 = 0.000606 \times 10^6$

$\Rightarrow$ Z – 1 = 24.6182 or Z = 25.6182 = 26. It is (Fe).

25. Here energy of photon = E

E = 6.4 KeV = 6.4×10^3 ev

Momentum of Photon = E/C = $\frac{6.4\times10^3}{3\times10^8}$ = 3.41×10^{-24} m/sec.

According to collision theory of momentum of photon = momentum of atom

$\therefore$ Momentum of Atom = P = 3.41×10^{-24} m/sec

$\therefore$ Recoil K.E. of atom = P^2 / 2m

$\Rightarrow \frac{(3.41\times10^{-24})^2 eV}{(2)(9.3\times10^{-26}\times1.6\times10^{-19})} = 3.9$ eV [1 Joule = 1.6×10^{-19} ev]

26. $V_0 \to$ Stopping Potential, $\lambda \to$ Wavelength, $eV_0 = hv – hv_0$

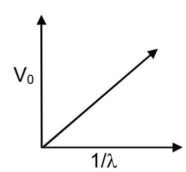
$eV_0 = hc/\lambda \Rightarrow V_0\lambda = hc/e$

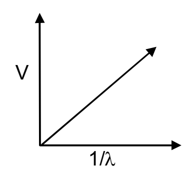
V $\to$ Potential difference across X-ray tube, $\lambda \to$ Cut of wavelength

λ = hc / eV or $V\lambda$ = hc / e

Slopes are same i.e. $V_0\lambda = V\lambda$

$\frac{hc}{e} = \frac{6.63\times10^{-34}\times3\times10^8}{1.6\times10^{-19}}$ = 1.242×10^{-6} Vm





27. λ = 10 pm = 100×10^{-12} m

D = 40 cm = 40×10^{-2} m

β = 0.1 mm = 0.1×10^{-3} m

$\beta = \frac{\lambda D}{d}$

$\Rightarrow$ d = $\frac{\lambda D}{\beta} = \frac{100\times10^{-12}\times40\times10^{-2}}{10^{-3}\times0.1}$ = 4×10^{-7} m.

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CHAPTER - 45
SEMICONDUCTOR AND SEMICONDUCTOR DEVICES

1. $f = 1013\ kg/m^3$, $V = 1\ m^3$

   $m = fV = 1013 \times 1 = 1013$ kg

   No.of atoms = $\frac{1013 \times 10^3 \times 6 \times 10^{23}}{23}$ = 264.26×10^{26}.

   a) Total no.of states = 2 N = $2 \times 264.26 \times 10^{26}$ = 528.52 = $5.3 \times 10^{28} \times 10^{26}$

   b) Total no.of unoccupied states = 2.65×10^{26}.

2. In a pure semiconductor, the no.of conduction electrons = no.of holes

   Given volume = 1 cm × 1 cm × 1 mm

   $= 1 \times 10^{-2} \times 1 \times 10^{-2} \times 1 \times 10^{-3} = 10^{-7}\ m^3$

   No.of electrons = $6 \times 10^{19} \times 10^{-7} = 6 \times 10^{12}$.

   Hence no.of holes = 6×10^{12}.

3. E = 0.23 eV, K = 1.38×10^{-23}

   KT = E

   $\Rightarrow 1.38 \times 10^{-23} \times T = 0.23 \times 1.6 \times 10^{-19}$

   $\Rightarrow T = \frac{0.23 \times 1.6 \times 10^{-19}}{1.38 \times 10^{-23}} = \frac{0.23 \times 1.6 \times 10^{4}}{1.38}$ = 0.2676×10^4 = 2670.

4. Bandgap = 1.1 eV, T = 300 K

   a) Ratio = $\frac{1.1}{KT} = \frac{1.1}{8.62 \times 10^{-5} \times 3 \times 10^2}$ = 42.53= 43

   b) 4.253′ = $\frac{1.1}{8.62 \times 10^{-5} \times T}$ or T = $\frac{1.1 \times 10^5}{4.253 \times 8.62}$ = 3000.47 K.

5. 2KT = Energy gap between acceptor band and valency band

   $\Rightarrow 2 \times 1.38 \times 10^{-23} \times 300$

   $\Rightarrow E = (2 \times 1.38 \times 3) \times 10^{-21}\ J = \frac{6 \times 1.38}{1.6} \times \frac{10^{-21}}{10^{-19}} eV = \left(\frac{6 \times 1.38}{1.6}\right) \times 10^{-2} eV$

   $= 5.175 \times 10^{-2}$ eV = 51.75 meV = 50 meV.

6. Given :

   Band gap = 3.2 eV,

   E = hc / λ = 1242 / λ = 3.2 or λ = 388.1 nm.

7. λ = 820 nm

   E = hc / λ = 1242/820 = 1.5 eV

8. Band Gap = 0.65 eV, λ =?

   E = hc / λ = 1242 / 0.65 = 1910.7×10^{-9} m = 1.9×10^{-5} m.

9. Band gap = Energy need to over come the gap

   $\frac{hc}{\lambda} = \frac{1242 eV - nm}{620 nm}$ = 2.0 eV.

10. Given n = $e^{-\Delta E/2KT}$, ΔE = Diamon → 6 eV ; ΔE Si → 1.1 eV

    Now, $n_1 = e^{-\Delta E_1/2KT} = e^{\frac{-6}{2 \times 300 \times 8.62 \times 10^{-5}}}$

    $n_2 = e^{-\Delta E_2/2KT} = e^{\frac{-1.1}{2 \times 300 \times 8.62 \times 10^{-5}}}$

    $\frac{n_1}{n_2} = \frac{4.14772 \times 10^{-51}}{5.7978 \times 10^{-10}}$ = 7.15×10^{-42}.

    Due to more ΔE, the conduction electrons per cubic metre in diamond is almost zero.

11. $\sigma = T^{3/2}\ e^{-\Delta E/2KT}$ at 4°K

$\sigma = 4^{3/2} = e^{\frac{-0.74}{2\times 8.62\times 10^{-5}\times 4}} = 8 \times e^{-1073.08}$.

At 300 K,

$\sigma = 300^{3/2} e^{\frac{-0.67}{2\times 8.62\times 10^{-5}\times 300}} = \frac{3\times 1730}{8} e^{-12.95}$.

Ratio = $\frac{8\times e^{-1073.08}}{[(3\times 1730)/8]\times e^{-12.95}} = \frac{64}{3\times 1730} e^{-1060.13}$.

12. Total no.of charge carriers initially = $2 \times 7 \times 10^{15} = 14 \times 10^{15}$/Cubic meter

Finally the total no.of charge carriers = 14×10^{17} / m^3

We know :

The product of the concentrations of holes and conduction electrons remains, almost the same.

Let x be the no.of holes.

So, $(7 \times 10^{15}) \times (7 \times 10^{15}) = x \times (14 \times 10^{17} - x)$

$\Rightarrow 14x \times 10^{17} - x^2 = 79 \times 10^{30}$

$\Rightarrow x^2 - 14x \times 10^{17} - 49 \times 10^{30} = 0$

x = $\frac{14\times 10^{17} \pm 14^2 \times \sqrt{10^{34} + 4\times 49\times 10^{30}}}{2}$ = 14.00035×10^{17}.

= Increased in no.of holes or the no.of atoms of Boron added.

$\Rightarrow$ 1 atom of Boron is added per $\frac{5\times 10^{28}}{1386.035\times 10^{15}} = 3.607 \times 10^{-3} \times 10^{13} = 3.607 \times 10^{10}$.

13. (No. of holes) (No.of conduction electrons) = constant.

At first :

No. of conduction electrons = 6×10^{19}

No.of holes = 6×10^{19}

After doping

No.of conduction electrons = 2×10^{23}

No. of holes = x.

$(6 \times 10^{19}) (6 \times 10^{19}) = (2 \times 10^{23})x$

$\Rightarrow \frac{6\times 6\times 10^{19+19}}{2\times 10^{23}} = x$

$\Rightarrow x = 18 \times 10^{15} = 1.8 \times 10^{16}$.

14. $\sigma = \sigma_0 e^{-\Delta E/2KT}$

$\Delta E = 0.650$ eV, T = 300 K

According to question, $K = 8.62 \times 10^{-5}$ eV

$\sigma_0 e^{-\Delta E/2KT} = 2\times \sigma_0 e^{\frac{-\Delta E}{2\times K\times 300}}$

$\Rightarrow e^{\frac{-0.65}{2\times 8.62\times 10^{-5}\times T}} = 6.96561 \times 10^{-5}$

Taking in on both sides,

We get, $\frac{-0.65}{2\times 8.62\times 10^{-5}\times T'} = -11.874525$

$\Rightarrow \frac{1}{T'} = \frac{11.574525\times 2\times 8.62\times 10^{-5}}{0.65}$

$\Rightarrow$ T' = 317.51178 = 318 K.

15. Given band gap = 1 eV
    Net band gap after doping = $(1 - 10^{-3})$eV = 0.999 eV
    According to the question, $KT_1 = 0.999/50$
    $\Rightarrow T_1 = 231.78 = 231.8$
    For the maximum limit $KT_2 = 2 \times 0.999$
    $$\Rightarrow T_2 = \frac{2 \times 1 \times 10^{-3}}{8.62 \times 10^{-5}} = \frac{2}{8.62} \times 10^2 = 23.2.$$
    Temperature range is (23.2 – 231.8).
16. Depletion region 'd' = 400 nm = 4×10^{-7} m
    Electric field $E = 5 \times 10^5$ V/m
    a) Potential barrier $V = E \times d = 0.2$ V
    b) Kinetic energy required = Potential barrier $\times$ e = 0.2 eV [Where e = Charge of electron]
17. Potential barrier = 0.2 Volt
    a) K.E. = (Potential difference) $\times$ e = 0.2 eV (in unbiased $cond^n$)
    b) In forward biasing
       KE + Ve = 0.2e
    $\Rightarrow$ KE = 0.2e – 0.1e = 0.1e.
    c) In reverse biasing
       KE – Ve = 0.2 e
    $\Rightarrow$ KE = 0.2e + 0.1e = 0.3e.
18. Potential barrier 'd' = 250 meV
    Initial KE of hole = 300 meV
    We know : KE of the hole decreases when the junction is forward biased and increases when reverse blased in the given 'Pn' diode.
    So,
    a) Final KE = (300 – 250) meV = 50 meV
    b) Initial KE = (300 + 250) meV = 550 meV
19. $i_1 = 25\ \mu A$, V = 200 mV, $i_2 = 75\ \mu A$
    a) When in unbiased condition drift current = diffusion current
       $\therefore$ Diffusion current = 25 μA.
    b) On reverse biasing the diffusion current becomes 'O'.
    c) On forward biasing the actual current be x.
       x – Drift current = Forward biasing current
    $\Rightarrow$ x – 25 μA = 75 μA
    $\Rightarrow$ x = (75 + 25) μA = 100 μA.
20. Drift current = 20 μA = 20×10^{-6} A.
    Both holes and electrons are moving
    $$\text{So, no.of electrons} = \frac{20 \times 10^{-6}}{2 \times 1.6 \times 10^{-19}} = 6.25 \times 10^{13}.$$
21. a) $e^{aV/KT} = 100$
    $$\Rightarrow e^{\frac{V}{8.62 \times 10^{-5} \times 300}} = 100$$
    $$\Rightarrow \frac{V}{8.62 \times 10^{-5} \times 300} = 4.605 \Rightarrow V = 4.605 \times 8.62 \times 3 \times 10^{-3} = 119.08 \times 10^{-3}$$
    $$R = \frac{V}{I} = \frac{V}{I_0(e^{ev/KT-1})} = \frac{119.08 \times 10^{-3}}{10 \times 10^{-6} \times (100-1)} = \frac{119.08 \times 10^{-3}}{99 \times 10^{-5}} = 1.2 \times 10^2.$$
    $V_0 = I_0 R$
    $\Rightarrow 10 \times 10^{-6} \times 1.2 \times 10^2 = 1.2 \times 10^{-3} = 0.0012$ V.

c) $0.2 = \frac{KT}{ei_0} e^{-eV/KT}$

$K = 8.62 \times 10^{-5}$ eV/K, T = 300 K

$i_0 = 10 \times 10^{-5}$ A.

Substituting the values in the equation and solving

We get V = 0.25

22. a) $i_0 = 20 \times 10^{-6}$A, T = 300 K, V = 300 mV

$i = i_0 e^{\frac{ev}{KT} - 1} = 20 \times 10^{-6}(e^{\frac{100}{8.62}} - 1) = 2.18$ A = 2 A.

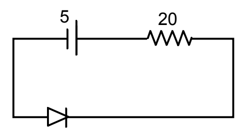
b) $4 = 20 \times 10^{-6}(e^{\frac{V}{8.62 \times 3 \times 10^{-2}}} - 1) \Rightarrow e^{\frac{V \times 10^3}{8.62 \times 3}} - 1 = \frac{4 \times 10^6}{20}$

$\Rightarrow e^{\frac{V \times 10^3}{8.62 \times 3}} = 200001 \Rightarrow \frac{V \times 10^3}{8.62 \times 3} = 12.2060$

$\Rightarrow$ V = 315 mV = 318 mV.

23. a) Current in the circuit = Drift current

(Since, the diode is reverse biased = 20 μA)

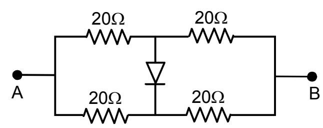


b) Voltage across the diode = $5 - (20 \times 20 \times 10^{-6})$

$= 5 - (4 \times 10^{-4}) = 5$ V.

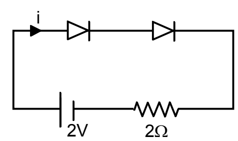
24. From the figure :

According to wheat stone bridge principle, there is no current through the diode.



Hence net resistance of the circuit is $\frac{40}{2} = 20\ \Omega$.

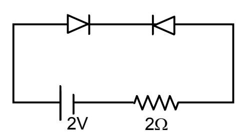
25. a) Since both the diodes are forward biased net resistance = 0



$i = \frac{2V}{2\Omega} = 1$ A

b) One of the diodes is forward biased and other is reverse biase.

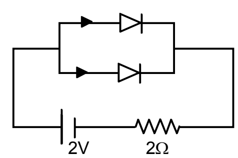
Thus the resistance of one becomes ∞.



$i = \frac{2}{2 + \infty} = 0$ A.

Both are forward biased.

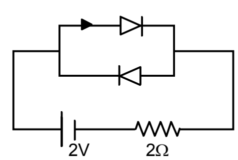
Thus the resistance is 0.



$i = \frac{2}{2} = 1$ A.

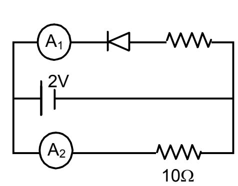
One is forward biased and other is reverse biased.

Thus the current passes through the forward biased diode.



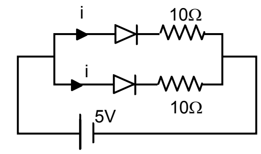
$\therefore i = \frac{2}{2} = 1$ A.

26. The diode is reverse biased. Hence the resistance is infinite. So, current through A_1 is zero.



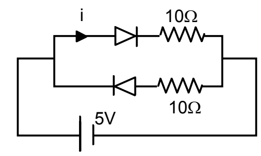
For A_2, current = $\frac{2}{10} = 0.2$ Amp.

27. Both diodes are forward biased. Thus the net diode resistance is 0.



$i = \dfrac{5}{(10+10)/10.10} = \dfrac{5}{5} = 1$ A.

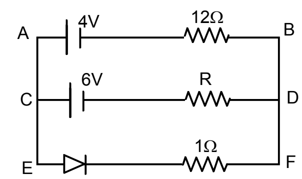
One diode is forward biased and other is reverse biased.



Current passes through the forward biased diode only.

$i = \dfrac{V}{R_{net}} = \dfrac{5}{10+0}$ = 1/2 = 0.5 A.

28. a) When R = 12 Ω



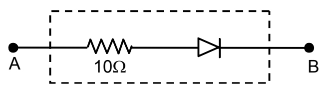
The wire EF becomes ineffective due to the net (–)ve voltage.

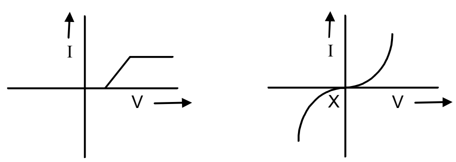
Hence, current through R = 10/24 = 0.4166 = 0.42 A.

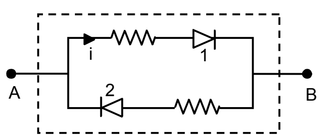
b) Similarly for R = 48 Ω.

$i = \dfrac{10}{(48+12)}$ = 10/60 = 0.16 A.

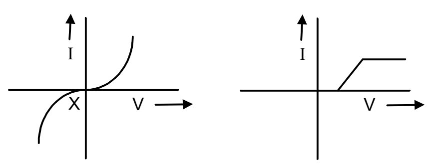
29.



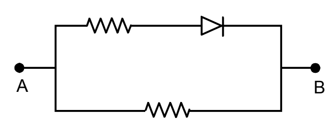




Since the diode 2 is reverse biased no current will pass through it.



30. Let the potentials at A and B be V_A and V_B respectively.



i) If $V_A > V_B$

Then current flows from A to B and the diode is in forward biased.

Eq. Resistance = 10/2 = 5 Ω.

ii) If $V_A < V_B$

Then current flows from B to A and the diode is reverse biased.

Hence Eq.Resistance = 10 Ω.

31. $\delta I_b = 80\ \mu A - 30\ \mu A = 50\ \mu A = 50 \times 10^{-6}$ A

$\delta I_c = 3.5\ mA - 1\ mA = -2.5\ mA = 2.5 \times 10^{-3}$ A

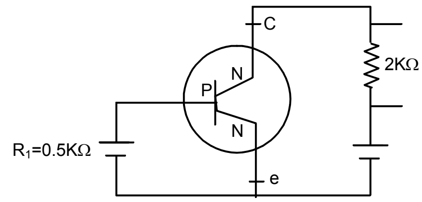
$\beta = \left(\dfrac{\delta I_c}{\delta I_b}\right) V_{ce}$ = constant

$\Rightarrow \dfrac{2.5 \times 10^{-3}}{50 \times 10^{-6}} = \dfrac{2500}{50}$ = 50.

Current gain = 50.

32. $\beta = 50$, $\delta I_b = 50\ \mu A$,

$V_0 = \beta \times RG = 50 \times 2/0.5 = 200$.



a) $VG = V_0/V_1 = \frac{V_0}{V_i} = \frac{V_0}{\delta I_b \times R_i} = \frac{200}{50 \times 10^{-6} \times 5 \times 10^2}$ = 8000 V.

b) $\delta V_i = \delta I_b \times R_i = 50 \times 10^{-6} \times 5 \times 10^2 = 0.00025$ V = 25 mV.

c) Power gain = $\beta^2 \times RG = \beta^2 \times \frac{R_0}{R_i} = 2500 \times \frac{2}{0.5} = 10^4$.

33. X = $A\overline{BC} + B\overline{CA} + C\overline{AB}$

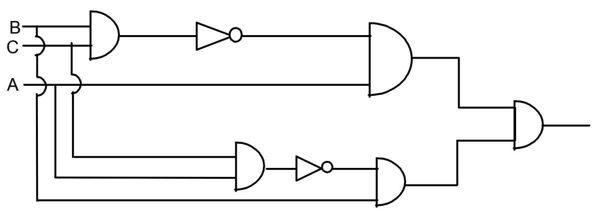
a) A = 1, B = 0, C = 1

X = 1.

b) A = B = C = 1

X = 0.

34. For $A\overline{BC} + B\overline{CA}$



35. LHS = $AB \times \overline{AB} = X + \overline{X}$ [X = AB]

If X = 0, $\overline{X}$ = 1

If $\overline{X}$ = 0, X = 1

⇒ 1 + 0 or 0 + 1 = 1

⇒ RHS = 1 (Proved)

♠ ♠ ♠

THE NUCLEUS
CHAPTER - 46

1. $M = Am_p$, $f = M/V$, $m_p = 1.007276$ u
$R = R_0A^{1/3} = 1.1 \times 10^{-15} A^{1/3}$, $u = 1.6605402 \times 10^{-27}$ kg
$$= \frac{A \times 1.007276 \times 1.6605402 \times 10^{-27}}{4/3 \times 3.14 \times R^3} = 0.300159 \times 10^{18} = 3 \times 10^{17} \text{ kg/m}^3.$$
'f' in CGS = Specific gravity = 3×10^{14}.

2. $f = \frac{M}{v} \Rightarrow V = \frac{M}{f} = \frac{4 \times 10^{30}}{2.4 \times 10^{17}} = \frac{1}{0.6} \times 10^{13} = \frac{1}{6} \times 10^{14}$
$V = 4/3\ \pi R^3$.
$\Rightarrow \frac{1}{6} \times 10^{14} = 4/3\ \pi \times R^3 \Rightarrow R^3 = \frac{1}{6} \times \frac{3}{4} \times \frac{1}{\pi} \times 10^{14}$
$\Rightarrow R^3 = \frac{1}{8} \times \frac{100}{\pi} \times 10^{12}$
$\therefore R = ½ \times 10^4 \times 3.17 = 1.585 \times 10^4$ m = 15 km.

3. Let the mass of 'α' particle be xu.
'α' particle contains 2 protons and 2 neutrons.
$\therefore$ Binding energy = $(2 \times 1.007825 \text{ u} \times 1 \times 1.00866 \text{ u} - \text{xu})C^2$ = 28.2 MeV (given).
$\therefore$ x = 4.0016 u.

4. $Li^7 + p \rightarrow l + \alpha + E$; $Li^7 = 7.016u$
$\alpha = {}^4He = 4.0026u$; p = 1.007276 u
$E = Li^7 + P - 2\alpha = (7.016 + 1.007276)u - (2 \times 4.0026)u = 0.018076$ u.
$\Rightarrow 0.018076 \times 931 = 16.828 = 16.83$ MeV.

5. $B = (Zm_p + Nm_n - M)C^2$
Z = 79 ; N = 118 ; m_p = 1.007276u ; M = 196.96 u ; m_n = 1.008665u
$B = [(79 \times 1.007276 + 118 \times 1.008665)u - Mu]c^2$
$= 198.597274 \times 931 - 196.96 \times 931 = 1524.302094$
so, Binding Energy per nucleon = 1524.3 / 197 = 7.737.

6. a) $U^{238}\ {}_2He^4 + Th^{234}$
$E = [M_u - (N_{HC} + M_{Th})]u = 238.0508 - (234.04363 + 4.00260)]u = 4.25487$ Mev = 4.255 Mev.
b) $E = U^{238} - [Th^{234} + 2n'_0 + 2p'_1]$
$= \{238.0508 - [234.64363 + 2(1.008665) + 2(1.007276)]\}u$
= 0.024712u = 23.0068 = 23.007 MeV.

7. ${}^{223}R_a$ = 223.018 u ; ${}^{209}Pb$ = 208.981 u ; ${}^{14}C$ = 14.003 u.
${}^{223}R_a \rightarrow {}^{209}Pb + {}^{14}C$
Δm = mass ${}^{223}R_a$ – mass $({}^{209}Pb + {}^{14}C)$
$\Rightarrow$ = 223.018 – (208.981 + 14.003) = 0.034.
Energy = $\Delta M \times u = 0.034 \times 931 = 31.65$ Me.

8. $E_{Z.N.} \rightarrow E_{Z-1}, N + P_1 \Rightarrow E_{Z.N.} \rightarrow E_{Z-1}, N + {}_1H^1$ [As hydrogen has no neutrons but protons only]
$\Delta E = (M_{Z-1,\,N} + N_H - M_{Z,N})c^2$

9. $E_2N = E_{Z,N-1} + {}^1_0n$.
Energy released = (Initial Mass of nucleus – Final mass of nucleus)c^2 = $(M_{Z.N-1} + M_0 - M_{ZN})c^2$.

10. $P^{32} \rightarrow S^{32} + {}_0\bar{v}^0 + {}_1\beta^0$
Energy of antineutrino and β-particle
= (31.974 – 31.972)u = 0.002 u = 0.002 × 931 = 1.862 MeV = 1.86.

11. $In \rightarrow P + e^-$
We know : Half life = 0.6931 / λ (Where λ = decay constant).
Or $\lambda = 0.6931 / 14 \times 60 = 8.25 \times 10^{-4}$ S [As half life = 14 min = 14 × 60 sec].
Energy = $[M_n - (M_P + M_e)]u = [(M_{nu} - M_{pu}) - M_{pu}]c^2 = [0.00189u - 511 \text{ KeV}/c^2]$
$= [1293159 \text{ ev}/c^2 - 511000 \text{ ev}/c^2]c^2 = 782159$ eV = 782 Kev.

12. $^{226}_{58}Ra \rightarrow ^{4}_{2}\alpha + ^{222}_{26}Rn$

$^{19}_{8}O \rightarrow ^{19}_{9}F + ^{0}_{n}\beta + ^{0}_{0}\overline{v}$

$^{13}_{25}Al \rightarrow ^{25}_{12}MG + ^{0}_{-1}e + ^{0}_{0}\overline{v}$

13. $^{64}Cu \rightarrow ^{64}Ni + e^{-} + v$

Emission of nutrino is along with a positron emission.

a) Energy of positron = 0.650 MeV.

Energy of Nutrino = 0.650 – KE of given position = 0.650 – 0.150 = 0.5 MeV = 500 Kev.

b) Momentum of Nutrino = $\frac{500 \times 1.6 \times 10^{-19}}{3 \times 10^{8}} \times 10^{3}$ J = 2.67×10^{-22} kg m/s.

14. a) ${}_{19}K^{40} \rightarrow {}_{20}Ca^{40} + {}_{-1}e^{0} + {}_{0}\overline{v}^{0}$

${}_{19}K^{40} \rightarrow {}_{18}Ar^{40} + {}_{-1}e^{0} + {}_{0}\overline{v}^{0}$

${}_{19}K^{40} + {}_{-1}e^{0} \rightarrow {}_{18}Ar^{40}$

${}_{19}K^{40} \rightarrow {}_{20}Ca^{40} + {}_{-1}e^{0} + {}_{0}v^{0}$.

b) Q = [Mass of reactants – Mass of products]c^2

= [39.964u – 39.9626u] = [39.964u – 39.9626]uc^2 = (39.964 – 39.9626) 931 Mev = 1.3034 Mev.

${}_{19}K^{40} \rightarrow {}_{18}Ar^{40} + {}_{-1}e^{0} + {}_{0}\overline{v}^{0}$

Q = (39.9640 – 39.9624)uc^2 = 1.4890 = 1.49 Mev.

${}_{19}K^{40} + {}_{-1}e^{0} \rightarrow {}_{18}Ar^{40}$

Q_{value} = (39.964 – 39.9624)uc^2.

15. $^{6}_{3}Li + n \rightarrow ^{7}_{3}Li$; $^{7}_{3}Li + r \rightarrow ^{8}_{3}Li$

$^{8}_{3}Li \rightarrow ^{8}_{4}Be + e^{-} + v^{-}$

$^{8}_{4}Be \rightarrow ^{4}_{2}He + ^{4}_{2}He$

16. $''C \rightarrow ''B + \beta^{+} + v$

mass of C″ = 11.014u ; mass of B″ = 11.0093u

Energy liberated = (11.014 – 11.0093)u = 29.5127 Mev.

For maximum K.E. of the positron energy of v may be assumed as 0.

∴ Maximum K.E. of the positron is 29.5127 Mev.

17. Mass ^{238Th} = 228.028726 u ; ^{224}Ra = 224.020196 u ; α = $^{4}_{2}He \rightarrow 4.00260u$

$^{238}Th \rightarrow ^{224}Ra^{*} + \alpha$

$^{224}Ra^{*} \rightarrow ^{224}Ra + v(217\ Kev)$

Now, Mass of $^{224}Ra^{*}$ = 224.020196 × 931 + 0.217 Mev = 208563.0195 Mev.

KE of α = E ^{226Th} – E($^{224}Ra^{*} + \alpha$)

= 228.028726 × 931 – [208563.0195 + 4.00260 × 931] = 5.30383 Mev= 5.304 Mev.

18. $^{12}N \rightarrow ^{12}C^{*} + e^{+} + v$

$^{12}C^{*} \rightarrow ^{12}C + v(4.43\ Mev)$

Net reaction : $^{12}N \rightarrow ^{12}C + e^{+} + v + v(4.43\ Mev)$

Energy of (e^{+} + v) = N^{12} – (c^{12} + v)

= 12.018613u – (12)u – 4.43 = 0.018613 u – 4.43 = 17.328 – 4.43 = 12.89 Mev.

Maximum energy of electron (assuming 0 energy for v) = 12.89 Mev.

19. a) $t_{1/2} = 0.693 / \lambda$ [$\lambda \rightarrow$ Decay constant]

$\Rightarrow t_{1/2}$ = 3820 sec = 64 min.

b) Average life = $t_{1/2}$ / 0.693 = 92 min.

c) $0.75 = 1\ e^{-\lambda t} \Rightarrow \ln 0.75 = -\lambda t \Rightarrow t = \ln 0.75 / -0.00018$ = 1598.23 sec.

20. a) 198 grams of Ag contains → N_0 atoms.

1 μg of Ag contains → $N_0/198 \times 1$ μg = $\frac{6 \times 10^{23} \times 1 \times 10^{-6}}{198}$ atoms

Activity = $\lambda N = \frac{0.963}{t_{1/2}} \times N = \frac{0.693 \times 6 \times 10^{17}}{198 \times 2.7}$ disintegrations/day.

= $\frac{0.693 \times 6 \times 10^{17}}{198 \times 2.7 \times 3600 \times 24}$ disintegration/sec = $\frac{0.693 \times 6 \times 10^{17}}{198 \times 2.7 \times 36 \times 24 \times 3.7 \times 10^{10}}$ curie = 0.244 Curie.

b) A = $\frac{A_0}{2t_{1/2}} = \frac{0.244}{2 \times \frac{7}{2.7}}$ = 0.0405 = 0.040 Curie.

21. $t_{1/2}$ = 8.0 days ; A_0 = 20 μ CI

a) t = 4.0 days ; λ = 0.693/8

$A = A_0 e^{-\lambda t} = 20 \times 10^{-6} \times e^{(-0.693/8)\times 4} = 1.41 \times 10^{-5}$ Ci = 14 μ Ci

b) $\lambda = \frac{0.693}{8 \times 24 \times 3600} = 1.0026 \times 10^{-6}$.

22. $\lambda = 4.9 \times 10^{-18}\ s^{-1}$

a) Avg. life of $^{238}U = \frac{1}{\lambda} = \frac{1}{4.9 \times 10^{-18}} = \frac{1}{4.9} \times 10^{-18}$ sec.

= 6.47×10^3 years.

b) Half life of uranium = $\frac{0.693}{\lambda} = \frac{0.693}{4.9 \times 10^{-18}} = 4.5 \times 10^9$ years.

c) A = $\frac{A_0}{2^{t/t_{1/2}}} \Rightarrow \frac{A_0}{A} = 2^{t/t_{1/2}} = 2^2 = 4$.

23. A = 200, A_0 = 500, t = 50 min

$A = A_0\ e^{-\lambda t}$ or 200 = 500 × $e^{-50 \times 60 \times \lambda}$

$\Rightarrow \lambda = 3.05 \times 10^{-4}$ s.

b) $t_{1/2} = \frac{0.693}{\lambda} = \frac{0.693}{0.000305}$ = 2272.13 sec = 38 min.

24. $A_0 = 4 \times 10^5$ disintegration / sec

$A' = 1 \times 10^6$ dis/sec ; t = 20 hours.

$A' = \frac{A_0}{2^{t/t_{1/2}}} \Rightarrow 2^{t/t_{1/2}} = \frac{A_0}{A'} \Rightarrow 2^{t/t_{1/2}} = 4$

$\Rightarrow t/t_{1/2} = 2 \Rightarrow t^{1/2}$ = t/2 = 20 hours / 2 = 10 hours.

$A'' = \frac{A_0}{2^{t/t_{1/2}}} \Rightarrow A'' = \frac{4 \times 10^6}{2^{100/10}} = 0.00390625 \times 10^6 = 3.9 \times 10^3$ dintegrations/sec.

25. $t_{1/2}$ = 1602 Y ; Ra = 226 g/mole ; Cl = 35.5 g/mole.

1 mole $RaCl_2$ = 226 + 71 = 297 g

297g = 1 mole of Ra.

0.1 g = $\frac{1}{297} \times 0.1$ mole of Ra = $\frac{0.1 \times 6.023 \times 10^{23}}{297} = 0.02027 \times 10^{22}$

$\lambda = 0.693 / t_{1/2} = 1.371 \times 10^{-11}$.

Activity = $\lambda N = 1.371 \times 10^{-11} \times 2.027 \times 10^{20} = 2.779 \times 10^9 = 2.8 \times 10^9$ disintegrations/second.

26. $t_{1/2}$ = 10 hours, A_0 = 1 ci

Activity after 9 hours = $A_0\ e^{-\lambda t} = 1 \times e^{\frac{-0.693}{10} \times 9}$ = 0.5359 = 0.536 Ci.

No. of atoms left after 9th hour, $A_9 = \lambda N_9$

$\Rightarrow N_9 = \frac{A_9}{\lambda} = \frac{0.536 \times 10 \times 3.7 \times 10^{10} \times 3600}{0.693} = 28.6176 \times 10^{10} \times 3600 = 103.023 \times 10^{13}$.

Activity after 10 hours = $A_0\ e^{-\lambda t} = 1 \times e^{\frac{-0.693}{10} \times 9}$ = 0.5 Ci.

No. of atoms left after 10th hour

$A_{10} = \lambda N_{10}$

$\Rightarrow N_{10} = \frac{A_{10}}{\lambda} = \frac{0.5 \times 3.7 \times 10^{10} \times 3600}{0.693/10}$ = 26.37 × 10^{10} × 3600 = 96.103 × 10^{13}.

No.of disintegrations = (103.023 – 96.103) × 10^{13} = 6.92 × 10^{13}.

27. $t_{1/2}$ = 14.3 days ; t = 30 days = 1 month

As, the selling rate is decided by the activity, hence A_0 = 800 disintegration/sec.

We know, $A = A_0 e^{-\lambda t}$ [λ = 0.693/14.3]

A = 800 × 0.233669 = 186.935 = 187 rupees.

28. According to the question, the emission rate of γ rays will drop to half when the β+ decays to half of its original amount. And for this the sample would take 270 days.

∴ The required time is 270 days.

29. a) $P \rightarrow n + e^+ + v$ Hence it is a β^+ decay.

b) Let the total no. of atoms be 100 N_0.

	Carbon	Boron
Initially	90 N_0	10 N_0
Finally	10 N_0	90 N_0

Now, $10 N_0 = 90 N_0 e^{-\lambda t} \Rightarrow 1/9 = e^{\frac{-0.693}{20.3} \times t}$ [because $t_{1/2}$ = 20.3 min]

$\Rightarrow \ln \frac{1}{9} = \frac{-0.693}{20.3} t \Rightarrow t = \frac{2.1972 \times 20.3}{0.693}$ = 64.36 = 64 min.

30. N = 4 × 10^{23} ; $t_{1/2}$ = 12.3 years.

a) Activity = $\frac{dN}{dt} = \lambda n = \frac{0.693}{t_{1/2}} N = \frac{0.693}{12.3} \times 4 \times 10^{23}$ dis/year.

= 7.146 × 10^{14} dis/sec.

b) $\frac{dN}{dt}$ = 7.146 × 10^{14}

No.of decays in next 10 hours = 7.146 × 10^{14} × 10 × 36.. = 257.256 × 10^{17} = 2.57 × 10^{19}.

c) $N = N_0 e^{-\lambda t} = 4 \times 10^{23} \times e^{\frac{-0.693}{20.3} \times 6.16}$ = 2.82 × 10^{23} = No.of atoms remained

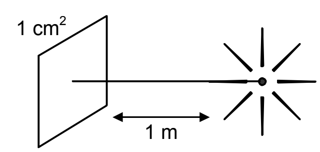
No. of atoms disintegrated = (4 – 2.82) × 10^{23} = 1.18 × 10^{23}.

31. Counts received per cm^2 = 50000 Counts/sec.

N = N_3o of active nucleic = 6 × 10^{16}

Total counts radiated from the source = Total surface area × 50000 counts/cm^2

= 4 × 3.14 × 1 × 10^4 × 5 × 10^4 = 6.28 × 10^9 Counts = dN/dt



We know, $\frac{dN}{dt} = \lambda N$

Or $\lambda = \frac{6.28 \times 10^9}{6 \times 10^{16}}$ = 1.0467 × 10^{-7} = 1.05 × 10^{-7} s^{-1}.

32. Half life period can be a single for all the process. It is the time taken for 1/2 of the uranium to convert to lead.

No. of atoms of U^{238} = $\frac{6 \times 10^{23} \times 2 \times 10^{-3}}{238}$ = $\frac{12}{238} \times 10^{20}$ = 0.05042 × 10^{20}

No. of atoms in Pb = $\frac{6 \times 10^{23} \times 0.6 \times 10^{-3}}{206} = \frac{3.6}{206} \times 10^{20}$

Initially total no. of uranium atoms = $\left(\frac{12}{235} + \frac{3.6}{206}\right) \times 10^{20}$ = 0.06789

$N = N_0 e^{-\lambda t} \Rightarrow N = N_0 e^{\frac{-0.693}{t/t_{1/2}}} \Rightarrow 0.05042 = 0.06789\ e^{\frac{-0.693}{4.47 \times 10^9}}$

$\Rightarrow \log\left(\frac{0.05042}{0.06789}\right) = \frac{-0.693t}{4.47 \times 10^9}$

$\Rightarrow$ t = 1.92 × 10^9 years.

33. $A_0 = 15.3$; $A = 12.3$; $t_{1/2} = 5730$ year

$\lambda = \frac{0.6931}{T_{1/2}} = \frac{0.6931}{5730} yr^{-1}$

Let the time passed be t,

We know A = $A_0 e^{-\lambda t} - \frac{0.6931}{5730} \times t \Rightarrow 12.3 = 15.3 \times e.$

$\Rightarrow$ t = 1804.3 years.

34. The activity when the bottle was manufactured = A_0

Activity after 8 years = $A_0 e^{\frac{-0.693}{12.5} \times 8}$

Let the time of the mountaineering = t years from the present

A = $A_0 e^{\frac{-0.693}{12.5} \times t}$; A = Activity of the bottle found on the mountain.

A = (Activity of the bottle manufactured 8 years before) $\times$ 1.5%

$\Rightarrow A_0 e^{\frac{-0.693}{12.5}} = A_0 e^{\frac{-0.693}{12.5} \times 8} \times 0.015$

$\Rightarrow \frac{-0.693}{12.5} t = \frac{-0.693 \times 8}{12.5} + \ln[0.015]$

$\Rightarrow$ 0.05544 t = 0.44352 + 4.1997 $\Rightarrow$ t = 83.75 years.

35. a) Here we should take R_0 at time is $t_0 = 30 \times 10^9$ s^{-1}

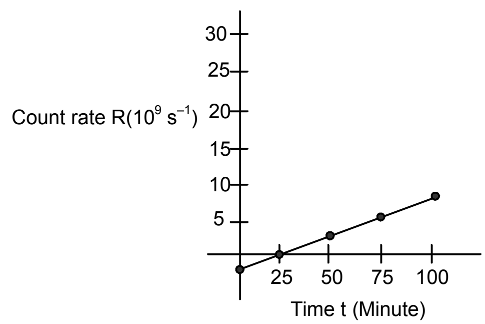
i) $\ln(R_0/R_1) = \ln\left(\frac{30 \times 10^9}{30 \times 10^9}\right) = 0$

ii) $\ln(R_0/R_2) = \ln\left(\frac{30 \times 10^9}{16 \times 10^9}\right) = 0.63$

iii) $\ln(R_0/R_3) = \ln\left(\frac{30 \times 10^9}{8 \times 10^9}\right) = 1.35$

iv) $\ln(R_0/R_4) = \ln\left(\frac{30 \times 10^9}{3.8 \times 10^9}\right) = 2.06$

v) $\ln(R_0/R_5) = \ln\left(\frac{30 \times 10^9}{2 \times 10^9}\right) = 2.7$



b) $\therefore$ The decay constant $\lambda = 0.028$ min^{-1}

c) $\therefore$ The half life period = $t_{1/2}$.

$t_{1/2} = \frac{0.693}{\lambda} = \frac{0.693}{0.028}$ = 25 min.

36. Given : Half life period $t_{1/2} = 1.30 \times 10^9$ year , A = 160 count/s = $1.30 \times 10^9 \times 365 \times 86400$

$\therefore$ A = λN $\Rightarrow 160 = \frac{0.693}{t_{1/2}} N$

$\Rightarrow N = \frac{160 \times 1.30 \times 365 \times 86400 \times 10^9}{0.693} = 9.5 \times 10^{18}$

$\therefore$ 6.023×10^{23} No. of present in 40 grams.

$6.023 \times 10^{23} = 40$ g $\Rightarrow 1 = \frac{40}{6.023 \times 10^{23}}$

$\therefore$ 9.5×10^{18} present in = $\frac{40 \times 9.5 \times 10^{18}}{6.023 \times 10^{23}} = 6.309 \times 10^{-4} = 0.00063.$

$\therefore$ The relative abundance at 40 k in natural potassium = $(2 \times 0.00063 \times 100)\% = 0.12\%$.

37. a) $P + e \rightarrow n + v$ neutrino [$a \rightarrow 4.95 \times 10^7\ s^{-1/2}$; $b \rightarrow 1$]

b) $\sqrt{f} = a(z - b)$

$\Rightarrow \sqrt{c/\lambda} = 4.95 \times 10^7 (79 - 1) = 4.95 \times 10^7 \times 78 \Rightarrow C/\lambda = (4.95 \times 78)^2 \times 10^{14}$

$\Rightarrow \lambda = \dfrac{3 \times 10^8}{14903.2 \times 10^{14}} = 2 \times 10^{-5} \times 10^{-6} = 2 \times 10^{-4}$ m = 20 pm.

38. Given : Half life period = $t_{1/2}$, Rate of radio active decay = $\dfrac{dN}{dt} = R \Rightarrow R = \dfrac{dN}{dt}$

Given after time t >> $t_{1/2}$, the number of active nuclei will become constant.

i.e. $(dN/dt)_{present} = R = (dN/dt)_{decay}$

$\therefore R = (dN/dt)_{decay}$

$\Rightarrow R = \lambda N$ [where, λ = Radioactive decay constant, N = constant number]

$\Rightarrow R = \dfrac{0.693}{t_{1/2}}(N) \Rightarrow Rt_{1/2} = 0.693\ N \Rightarrow N = \dfrac{Rt_{1/2}}{0.693}$.

39. Let N_0 = No. of radioactive particle present at time t = 0

N = No. of radio active particle present at time t.

$\therefore N = N_0\ e^{-\lambda t}$ [λ - Radioactive decay constant]

$\therefore$ The no.of particles decay = $N_0 - N = N_0 - N_0e^{-\lambda t} = N_0 (1 - e^{-\lambda t})$

We know, $A_0 = \lambda N_0$; $R = \lambda N_0$; $N_0 = R/\lambda$

From the above equation

$N = N_0 (1 - e^{-\lambda t}) = \dfrac{R}{\lambda}(1 - e^{-\lambda t})$ (substituting the value of N_0)

40. n = 1 mole = 6×10^{23} atoms, $t_{1/2}$ = 14.3 days

t = 70 hours, dN/dt in root after time t = λN

$N = No\ e^{-\lambda t} = 6 \times 10^{23} \times e^{\frac{-0.693 \times 70}{14.3 \times 24}} = 6 \times 10^{23} \times 0.868 = 5.209 \times 10^{23}$.

$5.209 \times 10^{23} \times \dfrac{-0.693}{14.3 \times 24} = \dfrac{0.0105 \times 10^{23}}{3600}$ dis/hour.

$= 2.9 \times 10^{-6} \times 10^{23}$ dis/sec $= 2.9 \times 10^{17}$ dis/sec.

Fraction of activity transmitted = $\left(\dfrac{1\mu ci}{2.9 \times 10^{17}}\right) \times 100\%$

$\Rightarrow \left(\dfrac{1 \times 3.7 \times 10^8}{2.9 \times 10^{11}} \times 100\right)\% = 1.275 \times 10^{-11}$ %.

41. V = 125 cm^3 = 0.125 L, P = 500 K pa = 5 atm.

T = 300 K, $t_{1/2}$ = 12.3 years = 3.82×10^8 sec. Activity = $\lambda \times N$

$N = n \times 6.023 \times 10^{23} = \dfrac{5 \times 0.125}{8.2 \times 10^{-2} \times 3 \times 10^2} \times 6.023 \times 10^{23} = 1.5 \times 10^{22}$ atoms.

$\lambda = \dfrac{0.693}{3.82 \times 10^8} = 0.1814 \times 10^{-8} = 1.81 \times 10^{-9}\ s^{-1}$

Activity = $\lambda N = 1.81 \times 10^{-9} \times 1.5 \times 10^{22} = 2.7 \times 10^3$ disintegration/sec

$= \dfrac{2.7 \times 10^{13}}{3.7 \times 10^{10}}$ Ci = 729 Ci.

42. ${}^{212}_{83}Bi \rightarrow {}^{208}_{81}Ti + {}^{4}_{2}He(\alpha)$

${}^{212}_{83}Bi \rightarrow {}^{212}_{84}Bi \rightarrow {}^{212}_{84}P_0 + e^-$

$t_{1/2}$ = 1 h. Time elapsed = 1 hour

at t = 0 Bi^{212} Present = 1 g

$\therefore$ at t = 1 Bi^{212} Present = 0.5 g

Probability α-decay and β-decay are in ratio 7/13.

$\therefore$ Tl remained = 0.175 g

$\therefore$ P_0 remained = 0.325 g

43. Activities of sample containing ^{108}Ag and ^{110}Ag isotopes = 8.0×10^8 disintegration/sec.

a) Here we take A = 8×10^8 dis./sec

$\therefore$ i) $\ln (A_1/A_{0_1}) = \ln (11.794/8) = 0.389$

ii) $\ln (A_2/A_{0_2}) = \ln(9.1680/8) = 0.1362$

iii) $\ln (A_3/A_{0_3}) = \ln(7.4492/8) = -0.072$

iv) $\ln (A_4/A_{0_4}) = \ln(6.2684/8) = -0.244$

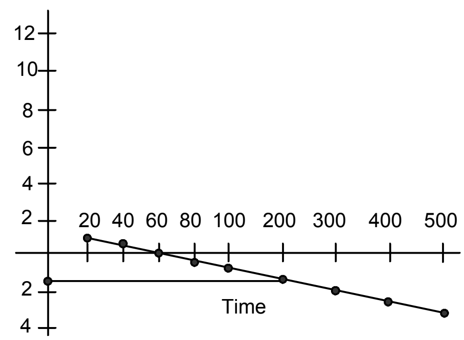
v) $\ln(5.4115/8) = -0.391$

vi) $\ln(3.0828/8) = -0.954$

vii) $\ln(1.8899/8) = -1.443$

viii) $\ln(1.167/8) = -1.93$

ix) $\ln(0.7212/8) = -2.406$



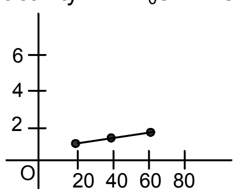
b) The half life of 110 Ag from this part of the plot is 24.4 s.

c) Half life of ^{110}Ag = 24.4 s.

$\therefore$ decay constant $\lambda = 0.693/24.4 = 0.0284 \Rightarrow t = 50$ sec,

The activity $A = A_0 e^{-\lambda t} = 8 \times 10^8 \times e^{-0.0284 \times 50} = 1.93 \times 10^8$

d)



e) The half life period of ^{108}Ag from the graph is 144 s.

44. $t_{1/2} = 24$ h

$\therefore t_{1/2} = \frac{t_1 t_2}{t_1 + t_2} = \frac{24 \times 6}{24 + 6} = 4.8$ h.

$A_0 = 6$ rci ; $A = 3$ rci

$\therefore A = \frac{A_0}{2^{t/t_{1/2}}} \Rightarrow 3 \text{ rci} = \frac{6 \text{ rci}}{2^{t/4.8h}} \Rightarrow \frac{t}{24.8h} = 2 \Rightarrow t = 4.8$ h.

45. $Q = qe^{-t/CR}$; $A = A_0 e^{-\lambda t}$

$\frac{\text{Energy}}{\text{Activity}} = \frac{1q^2 \times e^{-2t/cR}}{2\ CA_0 e^{-\lambda t}}$

Since the term is independent of time, so their coefficients can be equated,

So, $\frac{2t}{CR} = \lambda t$ or, $\lambda = \frac{2}{CR}$ or, $\frac{1}{\tau} = \frac{2}{CR}$ or, $R = 2\frac{\tau}{C}$ (Proved)

46. $R = 100\ \Omega$; $L = 100$ mH

After time t, $i = i_0\ (1 - e^{-t/Lr})$ $N = N_0\ (e^{-\lambda t})$

$\frac{i}{N} = \frac{i_0 (1 - e^{-tR/L})}{N_0 e^{-\lambda t}}$ i/N is constant i.e. independent of time.

Coefficients of t are equal $-R/L = -\lambda \Rightarrow R/L = 0.693/t_{1/2}$

$= t_{1/2} = 0.693 \times 10^{-3} = 6.93 \times 10^{-4}$ sec.

47. 1 g of 'I' contain 0.007 g U^{235} So, 235 g contains 6.023×10^{23} atoms.

So, 0.7 g contains $\frac{6.023 \times 10^{23}}{235} \times 0.007$ atom

1 atom given 200 Mev. So, 0.7 g contains $\frac{6.023 \times 10^{23} \times 0.007 \times 200 \times 10^6 \times 1.6 \times 10^{-19}}{235}$ J = 5.74×10^{-8} J.

48. Let n atoms disintegrate per second

Total energy emitted/sec = $(n \times 200 \times 10^6 \times 1.6 \times 10^{-19})$ J = Power

300 MW = 300×10^6 Watt = Power

$300 \times 10^6 = n \times 200 \times 10^6 \times 1.6 \times 10^{-19}$

$\Rightarrow n = \frac{3}{2 \times 1.6} \times 10^{19} = \frac{3}{3.2} \times 10^{19}$

6×10^{23} atoms are present in 238 grams

$\frac{3}{3.2} \times 10^{19}$ atoms are present in $\frac{238 \times 3 \times 10^{19}}{6 \times 10^{23} \times 3.2}$ = 3.7×10^{-4} g = 3.7 mg.

49. a) Energy radiated per fission = 2×10^8 ev

Usable energy = $2 \times 10^8 \times 25/100 = 5 \times 10^7$ ev = $5 \times 1.6 \times 10^{-12}$

Total energy needed = $300 \times 10^8 = 3 \times 10^8$ J/s

No. of fission per second = $\frac{3 \times 10^8}{5 \times 1.6 \times 10^{-12}}$ = 0.375×10^{20}

No. of fission per day = $0.375 \times 10^{20} \times 3600 \times 24 = 3.24 \times 10^{24}$ fissions.

b) From 'a' No. of atoms disintegrated per day = 3.24×10^{24}

We have, 6.023×10^{23} atoms for 235 g

for 3.24×10^{24} atom = $\frac{235}{6.023 \times 10^{23}} \times 3.24 \times 10^{24}$ g = 1264 g/day = 1.264 kg/day.

50. a) ${}^2_1H + {}^2_1H \rightarrow {}^3_1H + {}^1_1H$

Q value = $2M({}^2_1H) = [M({}^3_1H) + M({}^3_1H)]$

= [2 × 2.014102 – (3.016049 + 1.007825)]u = 4.0275 Mev = 4.05 Mev.

b) ${}^2_1H + {}^2_1H \rightarrow {}^3_2H + n$

Q value = $2[M({}^2_1H) - M({}^3_2He) + M_n]$

= [2 × 2.014102 – (3.016049 + 1.008665)]u = 3.26 Mev = 3.25 Mev.

c) ${}^2_1H + {}^3_1H \rightarrow {}^4_2H + n$

Q value = $[M({}^2_1H) + M({}^3_1He) - M({}^4_2He) + M_n]$

= (2.014102 + 3.016049) – (4.002603 + 1.008665)]u = 17.58 Mev = 17.57 Mev.

51. PE = $\frac{Kq_1q_2}{r} = \frac{9 \times 10^9 \times (2 \times 1.6 \times 10^{-19})^2}{r}$ …(1)

$1.5\ KT = 1.5 \times 1.38 \times 10^{-23} \times T$ …(2)

Equating (1) and (2) $1.5 \times 1.38 \times 10^{-23} \times T = \frac{9 \times 10^9 \times 10.24 \times 10^{-38}}{2 \times 10^{-15}}$

$\Rightarrow T = \frac{9 \times 10^9 \times 10.24 \times 10^{-38}}{2 \times 10^{-15} \times 1.5 \times 1.38 \times 10^{-23}}$ = 22.26087×10^9 K = 2.23×10^{10} K.

52. ${}^4H + {}^4H \rightarrow {}^8Be$

$M({}^2H) \rightarrow 4.0026$ u

$M({}^8Be) \rightarrow 8.0053$ u

Q value = $[2\ M({}^2H) - M({}^8Be)] = (2 \times 4.0026 - 8.0053)$ u

= –0.0001 u = –0.0931 Mev = –93.1 Kev.

53. In 18 g of N_0 of molecule = 6.023×10^{23}

In 100 g of N_0 of molecule = $\frac{6.023 \times 10^{26}}{18}$ = 3.346×10^{25}

$\therefore$ % of Deuterium = $3.346 \times 10^{26} \times 99.985$

Energy of Deuterium = $30.4486 \times 10^{25} = (4.028204 - 3.016044) \times 93$

= 942.32 ev = 1507×10^5 J = 1507 mJ

♠ ♠ ♠

THE SPECIAL THEORY OF RELATIVITY
CHAPTER - 47

1. S = 1000 km = 10^6 m

   The process requires minimum possible time if the velocity is maximum.

   We know that maximum velocity can be that of light i.e. = 3×10^8 m/s.

   So, time = $\frac{\text{Distance}}{\text{Speed}} = \frac{10^6}{3 \times 10^8} = \frac{1}{300}$ s.

2. ℓ = 50 cm, b = 25 cm, h = 10 cm, v = 0.6 c

   a) The observer in the train notices the same value of ℓ, b, h because relativity are in due to difference in frames.

   b) In 2 different frames, the component of length parallel to the velocity undergoes contraction but the perpendicular components remain the same. So length which is parallel to the x-axis changes and breadth and height remain the same.

   $e' = e\sqrt{1 - \frac{V^2}{C^2}} = 50\sqrt{1 - \frac{(0.6)^2 C^2}{C^2}}$

   $= 50\sqrt{1 - 0.36} = 50 \times 0.8 = 40$ cm.

   The lengths observed are 40 cm × 25 cm × 10 cm.

3. L = 1 m

   a) v 3×10^5 m/s

   $L' = 1\sqrt{1 - \frac{9 \times 10^{10}}{9 \times 10^{16}}} = \sqrt{1 - 10^{-6}}$ = 0.9999995 m

   b) v = 3 x 10^6 m/s

   $L' = 1\sqrt{1 - \frac{9 \times 10^{12}}{9 \times 10^{16}}} = \sqrt{1 - 10^{-4}}$ = 0.99995 m.

   c) v = 3×10^7 m/s

   $L' = 1\sqrt{1 - \frac{9 \times 10^{14}}{9 \times 10^{16}}} = \sqrt{1 - 10^{-2}}$ = 0.9949 = 0.995 m.

4. v = 0.6 cm/sec ; t = 1 sec

   a) length observed by the observer = vt ⇒ $0.6 \times 3 \times 10^6$ ⇒ 1.8×10^8 m

   b) $\ell = \ell_0 \sqrt{1 - v^2/c^2}$ ⇒ $1.8 \times 10^8 = \ell_0 \sqrt{1 - \frac{(0.6)^2 C^2}{C^2}}$

   $\ell_0 = \frac{1.8 \times 10^8}{0.8}$ = 2.25×10^8 m/s.

5. The rectangular field appears to be a square when the length becomes equal to the breadth i.e. 50 m.

   i.e. L′ = 50 ; L = 100 ; v = ?

   C = 3×10^8 m/s

   We know, $L' = L\sqrt{1 - v^2/c^2}$

   ⇒ $50 = 100\sqrt{1 - v^2/c^2}$ ⇒ $v = \sqrt{3/2}C$ = 0.866 C.

6. L_0 = 1000 km = 10^6 m

   v = 360 km/h = (360 × 5) / 18 = 100 m/sec.

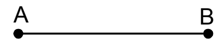
   a) $h' = h_0\sqrt{1 - v^2/c^2} = 10^6\sqrt{1 - \left(\frac{100}{3 \times 10^8}\right)^2} = 10^6\sqrt{1 - \frac{10^4}{9 \times 10^6}}$ = 10^9.

   Solving change in length = 56 nm.

   b) Δt = ΔL/v = 56 nm / 100 m = 0.56 ns.

7. v = 180 km/hr = 50 m/s

t = 10 hours



let the rest dist. be L.

$L' = L\sqrt{1-v^2/c^2} \Rightarrow L' = 10 \times 180 = 1800$ k.m.

$$1800 = L\sqrt{1-\frac{180^2}{(3\times10^5)^2}}$$

or, $1800 \times 1800 = L(1 - 36 \times 10^{-14})$

or, $L = \dfrac{3.24\times10^6}{1-36\times10^{-14}} = 1800 + 25 \times 10^{-12}$

or 25 nm more than 1800 km.

b) Time taken in road frame by Car to cover the dist = $\dfrac{1.8\times10^6 + 25\times10^{-9}}{50}$

$= 0.36 \times 10^5 + 5 \times 10^{-8} = 10$ hours + 0.5 ns.

8. a) u = 5c/13

$$\Delta t = \frac{t}{\sqrt{1-v^2/c^2}} = \frac{1y}{\sqrt{1-\frac{25c^2}{169c^2}}} = \frac{y\times13}{12} = \frac{13}{12}y\ .$$

The time interval between the consecutive birthday celebration is 13/12 y.

b) The fried on the earth also calculates the same speed.

9. The birth timings recorded by the station clocks is proper time interval because it is the ground frame. That of the train is improper as it records the time at two different places. The proper time interval ΔT is less than improper.

i.e. $\Delta T' = v\ \Delta T$

Hence for – (a) up train → Delhi baby is elder (b) down train → Howrah baby is elder.

10. The clocks of a moving frame are out of synchronization. The clock at the rear end leads the one at from by $L_0\ V/C^2$ where L_0 is the rest separation between the clocks, and v is speed of the moving frame.

Thus, the baby adjacent to the guard cell is elder.

11. v = 0.9999 C ; Δt = One day in earth ; $\Delta t'$ = One day in heaven

$$v = \frac{1}{\sqrt{1-v^2/c^2}} = \frac{1}{\sqrt{1-\frac{(0.9999)^2C^2}{C^2}}} = \frac{1}{0.014141782} = 70.712$$

$\Delta t' = v\ \Delta t$;

Hence, $\Delta t'$ = 70.7 days in heaven.

12. t = 100 years ; V = 60/100 K ; C = 0.6 C.

$$\Delta t = \frac{t}{\sqrt{1-V^2/C^2}} = \frac{100y}{\sqrt{1-\frac{(0.6)^2C^2}{C^2}}} = \frac{100y}{0.8} = 125\ y.$$

13. We know

$f' = f\sqrt{1-V^2/C^2}$

f′ = apparent frequency ;

f = frequency in rest frame

v = 0.8 C

$$f' = \sqrt{1-\frac{0.64C^2}{C^2}} = \sqrt{0.36} = 0.6\ s^{-1}$$

14. V = 100 km/h, Δt = Proper time interval = 10 hours

$$\Delta t' = \frac{\Delta t}{\sqrt{1-V^2/C^2}} = \frac{10\times 3600}{\sqrt{1-\left(\dfrac{1000}{36\times 3\times 10^8}\right)^2}}$$

$$\Delta t' - \Delta t = 10 \times 3600 \left[\frac{1}{1-\left(\dfrac{1000}{36\times 3\times 10^8}\right)^2} - 1\right]$$

By solving we get, $\Delta t' - \Delta t$ = 0.154 ns.

$\therefore$ Time will lag by 0.154 ns.

15. Let the volume (initial) be V.

$V' = V/2$

So, $V/2 = v\sqrt{1-V^2/C^2}$

$\Rightarrow C/2 = \sqrt{C^2 - V^2} \Rightarrow C^2/4 = C^2 - V^2$

$\Rightarrow V^2 = C^2 - \frac{C^2}{4} = \frac{3}{4}C^2 \Rightarrow V = \frac{\sqrt{3}}{2}C$.

16. d = 1 cm, v = 0.995 C

a) time in Laboratory frame = $\frac{d}{v} = \frac{1\times 10^{-2}}{0.995C}$

$= \frac{1\times 10^{-2}}{0.995\times 3\times 10^8} = 33.5 \times 10^{-12}$ = 33.5 PS

b) In the frame of the particle

$t' = \frac{t}{\sqrt{1-V^2/C^2}} = \frac{33.5\times 10^{-12}}{\sqrt{1-(0.995)^2}}$ = 335.41 PS.

17. x = 1 cm = 1×10^{-2} m ; K = 500 N/m, m = 200 g

Energy stored = ½ Kx^2 = ½ × 500 × 10^{-4} = 0.025 J

Increase in mass = $\frac{0.025}{C^2} = \frac{0.025}{9\times 10^{16}}$

Fractional Change of max = $\frac{0.025}{9\times 10^{16}} \times \frac{1}{2\times 10^{-1}}$ = 0.01388 × 10^{-16} = 1.4 × 10^{-8}.

18. Q = MS $\Delta\theta \Rightarrow$ 1 × 4200 (100 – 0) = 420000 J.

E = $(\Delta m)C^2$

$\Rightarrow \Delta m = \frac{E}{C^2} = \frac{Q}{C^2} = \frac{420000}{(3\times 10^8)^2}$

= 4.66 × 10^{-12} = 4.7 × 10^{-12} kg.

19. Energy possessed by a monoatomic gas = 3/2 nRdt.

Now dT = 10, n = 1 mole, R = 8.3 J/mol-K.

E = 3/2 × t × 8.3 × 10

Loss in mass = $\frac{1.5\times 8.3\times 10}{C^2} = \frac{124.5}{9\times 10^{15}}$

= 1383 × 10^{-16} = 1.38 × 10^{-15} Kg.

20. Let initial mass be m

½ mv^2 = E

$\Rightarrow E = \frac{1}{2}m\left(\frac{12\times 5}{18}\right)^2 = \frac{m\times 50}{9}$

$\Delta m = E/C^2$

$\Rightarrow \Delta m = \frac{m \times 50}{9 \times 9 \times 10^{16}} \Rightarrow \frac{\Delta m}{m} = \frac{50}{81 \times 10^{16}}$

$\Rightarrow 0.617 \times 10^{-16} = 6.17 \times 10^{-17}$.

21. Given : Bulb is 100 Watt = 100 J/s

So, 100 J in expended per 1 sec.

Hence total energy expended in 1 year = $100 \times 3600 \times 24 \times 365 = 3153600000$ J

Change in mass recorded = $\frac{\text{Total energy}}{C^2} = \frac{315360000}{9 \times 10^{16}}$

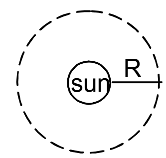
$= 3.504 \times 10^8 \times 10^{-16}$ kg $= 3.5 \times 10^{-8}$ Kg.

22. $I = 1400$ w/m^2

Power = 1400 w/m$^2 \times$ A

$= (1400 \times 4\pi R^2)w = 1400 \times 4\pi \times (1.5 \times 10^{11})^2$

$= 1400 \times 4\pi \times (1/5)^2 \times 10^{22}$



a) $\frac{E}{t} = \frac{\Delta m C^2}{t} = \frac{\Delta m}{t} = \frac{E/t}{C^2}$

$C^2 = \frac{1400 \times 4\pi \times 2.25 \times 10^{22}}{9 \times 10^{16}} = 1696 \times 10^{66} = 4.396 \times 10^9 = 4.4 \times 10^9$.

b) 4.4×10^9 Kg disintegrates in 1 sec.

2×10^{30} Kg disintegrates in $\frac{2 \times 10^{30}}{4.4 \times 10^9}$ sec.

$= \left(\frac{1 \times 10^{21}}{2.2 \times 365 \times 24 \times 3600}\right) = 1.44 \times 10^{-8} \times 10^{21}$ y $= 1.44 \times 10^{13}$ y.

23. Mass of Electron = Mass of positron = 9.1×10^{-31} Kg

Both are oppositely charged and they annihilate each other.

Hence, $\Delta m = m + m = 2 \times 9.1 \times 10^{-31}$ Kg

Energy of the resulting γ particle = $\Delta m\, C^2$

$= 2 \times 9.1 \times 10^{-31} \times 9 \times 10^{16}$ J $= \frac{2 \times 9.1 \times 9 \times 10^{-15}}{1.6 \times 10^{-19}}$ ev

$= 102.37 \times 10^4$ ev $= 1.02 \times 10^6$ ev $= 1.02$ Mev.

24. $m_e = 9.1 \times 10^{-31}$, $v_0 = 0.8$ C

a) $m' = \frac{Me}{\sqrt{1 - V^2/C^2}} = \frac{9.1 \times 10^{-31}}{\sqrt{1 - 0.64C^2/C^2}} = \frac{9.1 \times 10^{-31}}{0.6}$

$= 15.16 \times 10^{-31}$ Kg $= 15.2 \times 10^{-31}$ Kg.

b) K.E. of the electron : $m'C^2 - m_eC^2 = (m' - m_e)\, C^2$

$= (15.2 \times 10^{-31} - 9.1 \times 10^{-31})(3 \times 10^8)^2 = (15.2 \times 9.1) \times 9 \times 10^{-31} \times 10^{18}$ J

$= 54.6 \times 10^{-15}$ J $= 5.46 \times 10^{-14}$ J $= 5.5 \times 10^{-14}$ J.

c) Momentum of the given electron = Apparent mass $\times$ given velocity

$= 15.2 \times 10^{-31} - 0.8 \times 3 \times 10^8$ m/s $= 36.48 \times 10^{-23}$ kg m/s

$= 3.65 \times 10^{-22}$ kg m/s

25. a) $ev - m_0C^2 = \frac{m_0C^2}{2\sqrt{1 - \frac{V^2}{C^2}}} \Rightarrow ev - 9.1 \times 10^{-31} \times 9 \times 10^{16}$

$= \frac{9.1 \times 9 \times 10^{-31} \times 10^{16}}{2\sqrt{1 - \frac{0.36C^2}{C^2}}} \Rightarrow eV - 9.1 \times 9 \times 10^{-15}$

= $\frac{9.1 \times 9 \times 10^{-15}}{2 \times 0.8}$ $\Rightarrow$ eV – 9.1 × 9 × 10^{-15} = $\frac{9.1 \times 9 \times 10^{-15}}{1.6}$

$\Rightarrow$ eV = $\left(\frac{9.1 \times 9}{1.6} + 9.1 \times 9\right) \times 10^{-15}$ = eV$\left(\frac{81.9}{1.6} + 81.9\right) \times 10^{-15}$

eV = 133.0875 × 10^{-15} $\Rightarrow$ V = 83.179 × 10^4 = 831 KV.

b) $eV - m_0C^2 = \frac{m_0C^2}{2\sqrt{1-\frac{V^2}{C^2}}}$ $\Rightarrow$ eV – 9.1 × 9 × 10^{-19} × 9 × 10^{16} = $\frac{9.1 \times 9 \times 10^{-15}}{2\sqrt{1-\frac{0.81C^2}{C^2}}}$

$\Rightarrow$ eV – 81.9 × 10^{-15} = $\frac{9.1 \times 9 \times 10^{-15}}{2 \times 0.435}$

$\Rightarrow$ eV = 12.237 × 10^{-15}

$\Rightarrow$ V = $\frac{12.237 \times 10^{-15}}{1.6 \times 10^{-19}}$ = 76.48 kV.

V = 0.99 C = $eV - m_0C^2 = \frac{m_0C^2}{2\sqrt{1-\frac{V^2}{C^2}}}$

$\Rightarrow$ eV = $\frac{m_0C^2}{2\sqrt{1-\frac{V^2}{C^2}}} + m_0C^2 = \frac{9.1 \times 10^{-31} \times 9 \times 10^{16}}{2\sqrt{1-(0.99)^2}} + 9.1 \times 10^{-31} \times 9 \times 10^{16}$

$\Rightarrow$ eV = 372.18 × 10^{-15} $\Rightarrow$ V = $\frac{372.18 \times 20^{-15}}{1.6 \times 10^{-19}}$ = 272.6 × 10^4

$\Rightarrow$ V = 2.726 × 10^6 = 2.7 MeV.

26. a) $\frac{m_0C^2}{\sqrt{1-\frac{V^2}{C^2}}} - m_0C^2$ = 1.6 × 10^{-19}

$\Rightarrow$ $m_0C^2\left(\frac{1}{\sqrt{1-V^2/C^2}} - 1\right)$ = 1.6 × 10^{-19}

$\Rightarrow$ $\frac{1}{\sqrt{1-V^2/C^2}} - 1 = \frac{1.6 \times 10^{-19}}{9.1 \times 10^{-31} \times 9 \times 10^{16}}$

$\Rightarrow$ V = C × 0.001937231 = 3 × 0.001967231 × 0^8 = 5.92 × 10^5 m/s.

b) $\frac{m_0C^2}{\sqrt{1-\frac{V^2}{C^2}}} - m_0C^2$ = 1.6 × 10^{-19} × 10 × 10^3

$\Rightarrow$ $m_0C^2\left(\frac{1}{\sqrt{1-V^2/C^2}} - 1\right)$ = 1.6 × 10^{-15}

$\Rightarrow$ $\frac{1}{\sqrt{1-V^2/C^2}} - 1 = \frac{1.6 \times 10^{-15}}{9.1 \times 9 \times 10^{15}}$

$\Rightarrow$ V = 0.584475285 × 10^8 = 5.85 × 10^7 m/s.

c) K.E. = 10 Mev = 10 × 10^6 eV = 10^7 × 1.6 × 10^{-19} J = 1.6 × 10^{-12} J

$\Rightarrow$ $\frac{m_0C^2}{\sqrt{1-\frac{V^2}{C^2}}} - m_0C^2$ = 1.6 × 10^{-12} J

$\Rightarrow$ V^2 = 8..999991359 × 10^{16} $\Rightarrow$ V = 2.999987038 × 10^8.

27. $\Delta m = m - m_0 = 2m_0 - m_0 = m_0$

Energy $E = m_0c^2 = 9.1 \times 10^{-31} \times 9 \times 10^{16}$ J

E in e.v. = $\dfrac{9.1 \times 9 \times 10^{-15}}{1.6 \times 10^{-19}}$ = 51.18×10^4 ev = 511 Kev.

28. $$\frac{\left(\dfrac{m_0C^2}{\sqrt{1-\dfrac{V^2}{C^2}}} - m_0C^2\right) - \dfrac{1}{2}mv^2}{\dfrac{1}{2}m_0v^2} = 0.01$$

$$\Rightarrow \left[\frac{m_0C^2(1 + \dfrac{v^2}{2C^2} + \dfrac{1}{2}\times\dfrac{3}{4}\dfrac{V^2}{C^2} + \dfrac{1}{2}\times\dfrac{3}{4}\times\dfrac{5}{6}\dfrac{V^6}{C^6}) - m_0C^2}{\dfrac{1}{2}m_0v^2}\right] - \frac{1}{2}mv^2 = 0.1$$

$$\Rightarrow \frac{\dfrac{1}{2}m_0v^2 + \dfrac{3}{8}m_0\dfrac{V^4}{C^2} + \dfrac{15}{96}m_0\dfrac{V^4}{C^2} - \dfrac{1}{2}m_0v^2}{\dfrac{1}{2}m_0v^2} = 0.01$$

$$\Rightarrow \frac{3}{4}\frac{V^4}{C^2} + \frac{15}{96 \times 2}\frac{V^4}{C^4} = 0.01$$

Neglecting the v^4 term as it is very small

$$\Rightarrow \frac{3}{4}\frac{V^2}{C^2} = 0.01 \Rightarrow \frac{V^2}{C^2} = 0.04 / 3$$

$$\Rightarrow V/C = 0.2/\sqrt{3} = V = 0.2/\sqrt{3}\,C = \frac{0.2}{1.732} \times 3 \times 10^8$$

$= 0.346 \times 10^8$ m/s $= 3.46 \times 10^7$ m/s.

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